Estimate of Path Length Change Due to Dipole Fringe Fields

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Michael Borland
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Accelerator Systems Division, Advanced Photon Source

1 Introduction

Recently N. Nakamura indicated that, based on analytical estimates and particle tracking with OPERA, that the path length computations for the `NIBEND` element in `elegant` [1] might be in error. In this note we provide estimates for a few cases, concluding that `elegant` is reasonably accurate in this regard.

For analytical simplicity, we'll work with a sector dipole with linear fringe of length f. Assuming a full gap g, f is related to the well-known edge integral K by $f = 6gK$. For completeness,

$$K = \int_{\infty}^{0} \frac{(B_y(z) - B_y(0))B_y(z)}{gB_y^2(0)} dz, \tag{1}$$

where $\hat{z}$ is perpendicular to the end of the magnet and $z = 0$ is a point well inside the magnet where the field has reached the full value $B_y(0)$.

As of this writing, the released version (25.0.2) of `elegant` assumes that the fringe is centered on the reference plane of the magnet. The next release allows the user to select two additional options: the fringe may be entirely inside or entirely outside the reference plane. In this note, we provide analytical estimates of two cases: centered fringe and fringe outside the reference plane.

2 Fringe Outside Reference Plane

Referring to Figure 1, the analytical calculation in this case is relatively simple. Let θ_0, ρ_0, and B_0 be the nominal bending angle, radius, and magnetic field of the dipole. When the fringe field is included, we assume that the overall magnet strength will be adjusted by a factor $F = B_1/B_0 = \rho_0/\rho_1$ in order to obtain exactly the same deflection angle. The reference particle enters from the right and experiences a vertical field given by

$$B(\xi) = \begin{cases} B_0 F & \xi \geq f \\ B_0 F \xi / f & f > \xi \geq 0 \\ 0 & 0 > \xi \end{cases} \tag{2}$$

The deflection of the particle in traversing the fringe region is

$$\phi_1 \approx \frac{1}{B_1 \rho_1} \int_0^f d\xi B(\xi) = \frac{f}{2\rho_1}. \tag{3}$$

The deflection in the body of the magnet is thus

$$\theta_1 = \theta_0 - 2\phi_1 \tag{4}$$

The offset of the particle at the end of the fringe region is

$$\Delta r \approx \frac{1}{B_1 \rho_1} \int_0^f d\eta \int_0^\eta d\xi B(\xi) = \frac{f^2}{6\rho_1}. \tag{5}$$

From the diagram, we see that

$$(\rho_0 - \Delta r) \sin \frac{\theta_0}{2} = \rho_1 \sin \frac{\theta_1}{2}. \tag{6}$$

Using results from above, this becomes

$$\left(\rho_0 - \frac{f^2}{6\rho_1}\right)\sin\frac{\theta_0}{2} = \rho_1 \sin\left(\frac{\theta_0}{2} - \frac{f}{2\rho_1}\right). \tag{7}$$

which can be solved numerically to obtain ρ_1. Once this is obtained, the path length change is given by

$$\Delta s = \rho_1\theta_1 - \rho_0\theta_0. \tag{8}$$

Using Nakamura's parameters, we take $\rho_0 = 1$ m, $\theta_0 = \pi/4$, $K = 0.36$, and $g = 0.06$ m, we find $\Delta s = -7.3$ mm. The result from `elegant`, $\Delta s = -7.2$ mm, is very close. The radius ρ_1 in the two cases is nearly identical at 1.15575 m (analytical) and 1.15583 m (`elegant`). As expected, the magnet must be made weaker to compensate for the bending in the fringe region.

3 Centered Fringe

This is the default mode in `elegant` and the only mode available in the released version. The deflection and offset of the particle in the fringe region are identical to what was found above, as is the amount of bending that occurs in the body of the magnet. What's changed is how we relate this to the radius.

Referring to Figure 2, if the particle traveled in a straight line through the fringe region, it would enter the body region with

$$X_1 = \rho_0 \sin\frac{\theta_0}{2} - \frac{f}{2}\cos\frac{\theta_0}{2}. \tag{9}$$

In reality, the particle is offset by Δr, approximately perpendicular to its original direction of travel, as shown in Figure 3. This implies that it enters the body region at

$$X_1 \approx \rho_0 \sin\frac{\theta_0}{2} - \frac{f}{2}\cos\frac{\theta_0}{2} - \Delta r \sin\frac{\theta_0}{2} \tag{10}$$

Given that

$$X_1 = \rho_1 \sin\frac{\theta_1}{2}, \tag{11}$$

we can once again solve numerically for ρ_1. The path length change is

$$\Delta s = \rho_1\theta_1 + f - \rho_0\theta_0. \tag{12}$$

The second term accounts for the fact that the arc of angle $\theta 1$ does not cover the entire distance from entrance reference plane to exit reference plane. For that, we need two drifts of length $f/2$ at each end.

Using Nakamura's parameters again, we find $\Delta s = -0.64$ mm, while `elegant` gives $\Delta s = -0.53$ mm, which is in reasonable agreement. The radius ρ_1 is 0.99919 m (analytical) and 0.99930 m (`elegant`).

4 Fringe Inside

The case with fringe inside can be treated the same way as the centered-fringe case, provided we replace $f/2$ with f in Eq. (10) and f with $2f$ in Eq. (12). This gives $\Delta s = 6.0$ mm, compared to 6.1 mm from `elegant`. Also, we find ρ_1 is 0.8426 m (analytical) and 0.8427 m (`elegant`). This agreement is very good.

5 Conclusion

We've developed a semi-analytical method for estimating the central bending radius and path length change in the presence of fringe fields. It involves numerically solving for ρ_1 in the equation

$$\rho_1 \sin\left(\frac{\theta_0}{2} - \frac{f}{2\rho_1}\right) = \left(\rho_0 - \frac{f^2}{6\rho_1}\right)\sin\frac{\theta_0}{2} - \alpha f \cos\frac{\theta_0}{2}, \tag{13}$$

where

$$\alpha = \begin{cases} 0 & \text{Fringe outside} \\ \frac{1}{2} & \text{Fringe centered} \\ 1 & \text{Fringe inside} \end{cases} \tag{14}$$

The path length change is given by

$$\Delta s = (\rho_1 - \rho_0)\theta_0 + (2\alpha - 1)f. \tag{15}$$

Although it is not readily apparent from the equations, the results are insensitive to ρ_0 except for the centered-fringe case.

Using these equations, we've tested `elegant`'s `NIBEND` element using a linear fringe model with various positions relative to the reference plane. In all cases, good agreement was found for the change in path length and the bending radius.

6 Revision Notes

None

References

[1] M. Borland. elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation. Technical Report LS-287, Advanced Photon Source, September 2000.

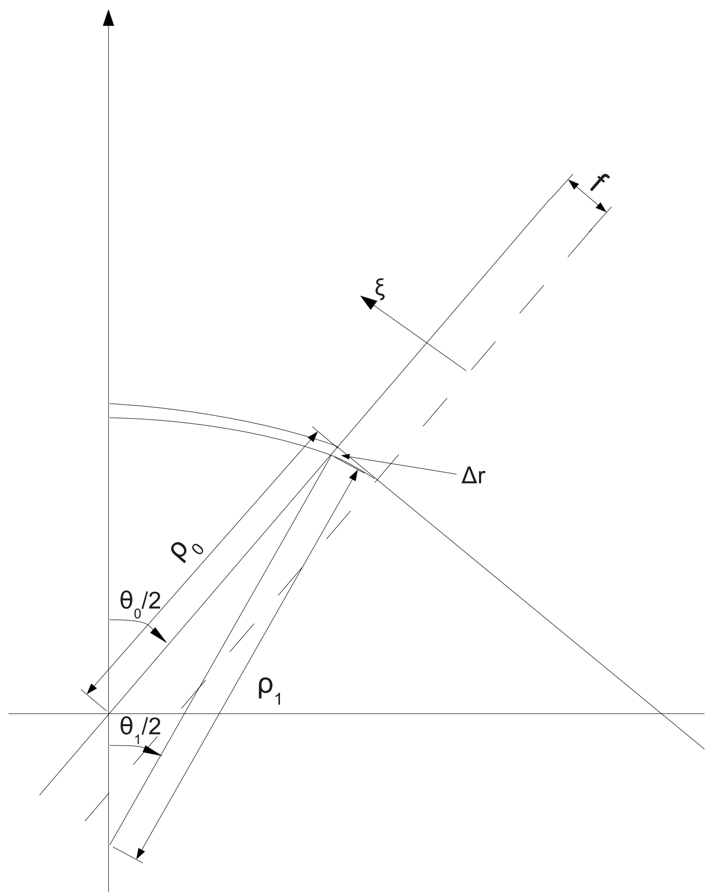

Figure 1: Diagram for semi-analytical calculation of path-lengthening when the fringe is completely outside the reference plane.

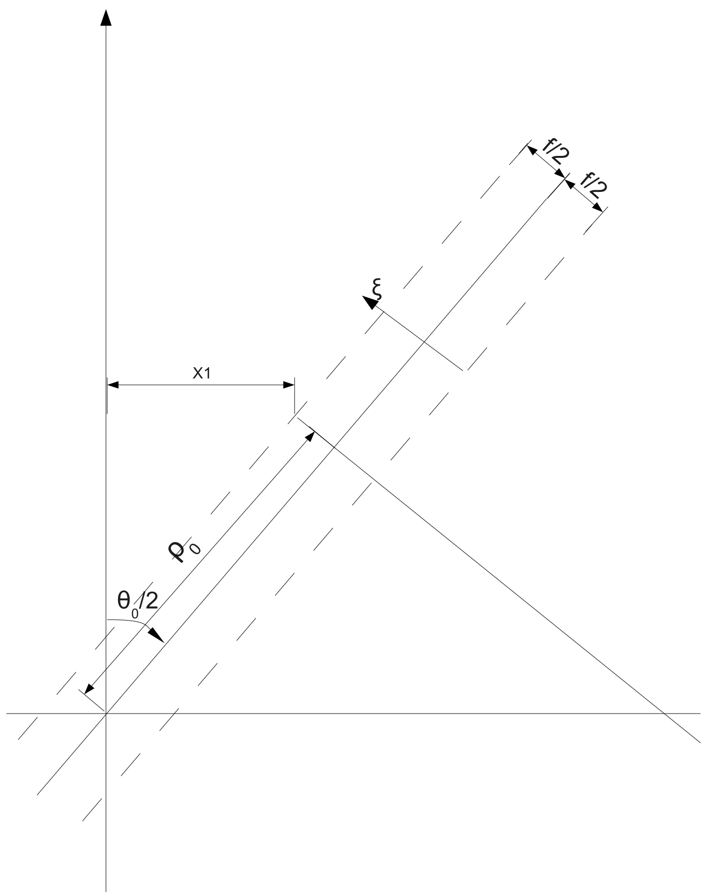

Figure 2: Diagram for semi-analytical calculation of path-lengthening when the fringe is centered on the reference plane.

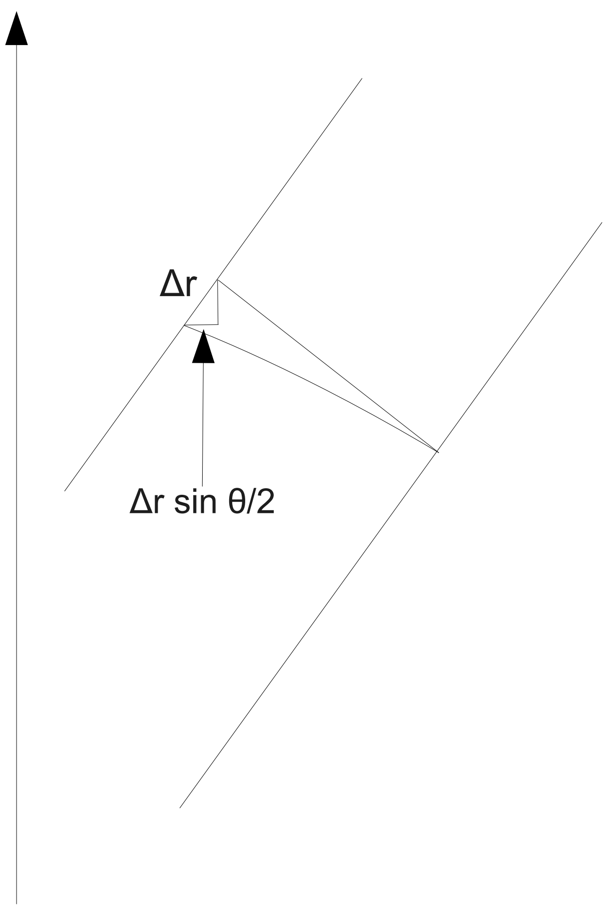

Figure 3: Showing the effect of the displacement of the beam in the fringe region on the calculation for centered fringe.