Tune response to quadrupole strength variation for a thick lens:

$$\frac{d\nu}{d|K_1|}=\frac{1+\text{FSE}}{16\pi k^2}\left[-4\alpha\sin^2 kL+\beta k\left(2kL+\sin 2kL\right)+\frac{\gamma}{k}\left(2kL-\sin 2kL\right)\right]\quad\text{in the focusing plane}$$

$$\frac{d\nu}{d|K_1|}=\frac{1+\text{FSE}}{16\pi k^2}\left[4\alpha\sinh^2 kL-\beta k\left(2kL+\sinh 2kL\right)+\frac{\gamma}{k}\left(2kL-\sinh 2kL\right)\right]\quad\text{in the defocusing plane}$$

here

$$k=\sqrt{|K_1|(1+\text{FSE})}$$

FSE is a value of fractional strength error for the quadrupole as specified in QUAD element; K_1 is the strength of the quadrupole; α,β,γ are the Twiss parameters at the beginning of the quadrupole (s_0).

$$Trace\left[M\left(s_0\right)\right]=2\cos\mu,\quad\mu=2\pi\nu$$

$$M\left(s_0\right)=R_{s_1 s_0}R_L\left(k\right)$$

$$R_{s_1 s_0}=M\left(s_0,k\right)R_L^{-1}\left(k\right)$$

$$\frac{dR_{s_1 s_0}}{dk}=0$$

$$\frac{d\mu}{dk}=-\frac{1}{2\sin\mu}Trace\left[M\left(s_0\right)R_L^{-1}\frac{dR_L}{dk}\right]$$