A theoretical overview of multiscale mechanisms of rf breakdown in superconductors

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Outline

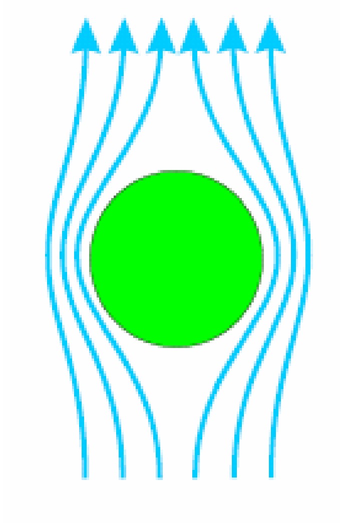
- **Introduction:** Meissner screening currents and the depairing limit. Vortex penetration and dc critical fields H_{c1}, H_c and H_{c2}.

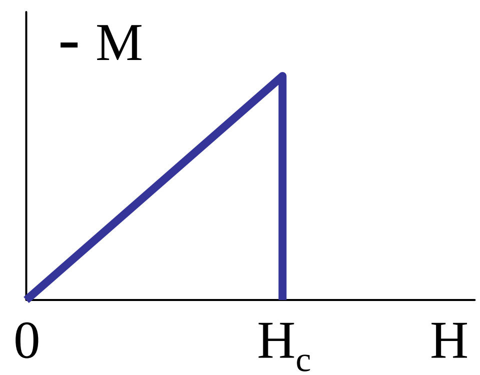
- **Multiscale mechanisms of rf breakdown:**
- **Nanoscale:** BCS surface resistance, normal and anomalous skin effects, effect of impurity scattering, nonlinearity due to rf pairbreaking

- **Microscale:** RF dissipation due to vortex penetration, surface barrier. RF resistance due to vortices penetrating along the grain boundary network. Josephson and mixed Abrikosov-Josephson vortices.

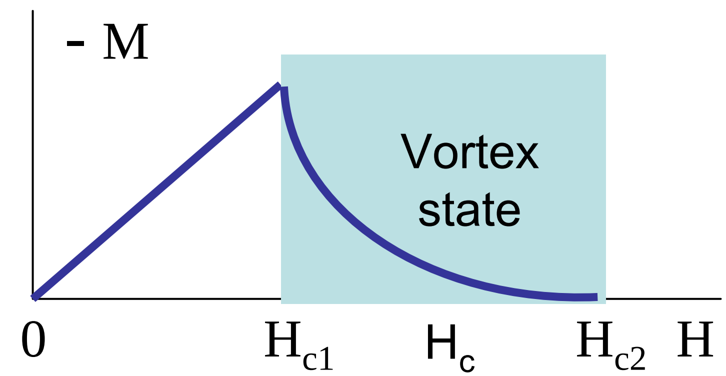
- **Macroscale:** Thermal mechanism of rf breakdown; minimum in the rf breakdown field $H_b(T)$, effect of cooling environment.

Type-I and type-II superconductors





Complete Meissner effect in type-I superconductors

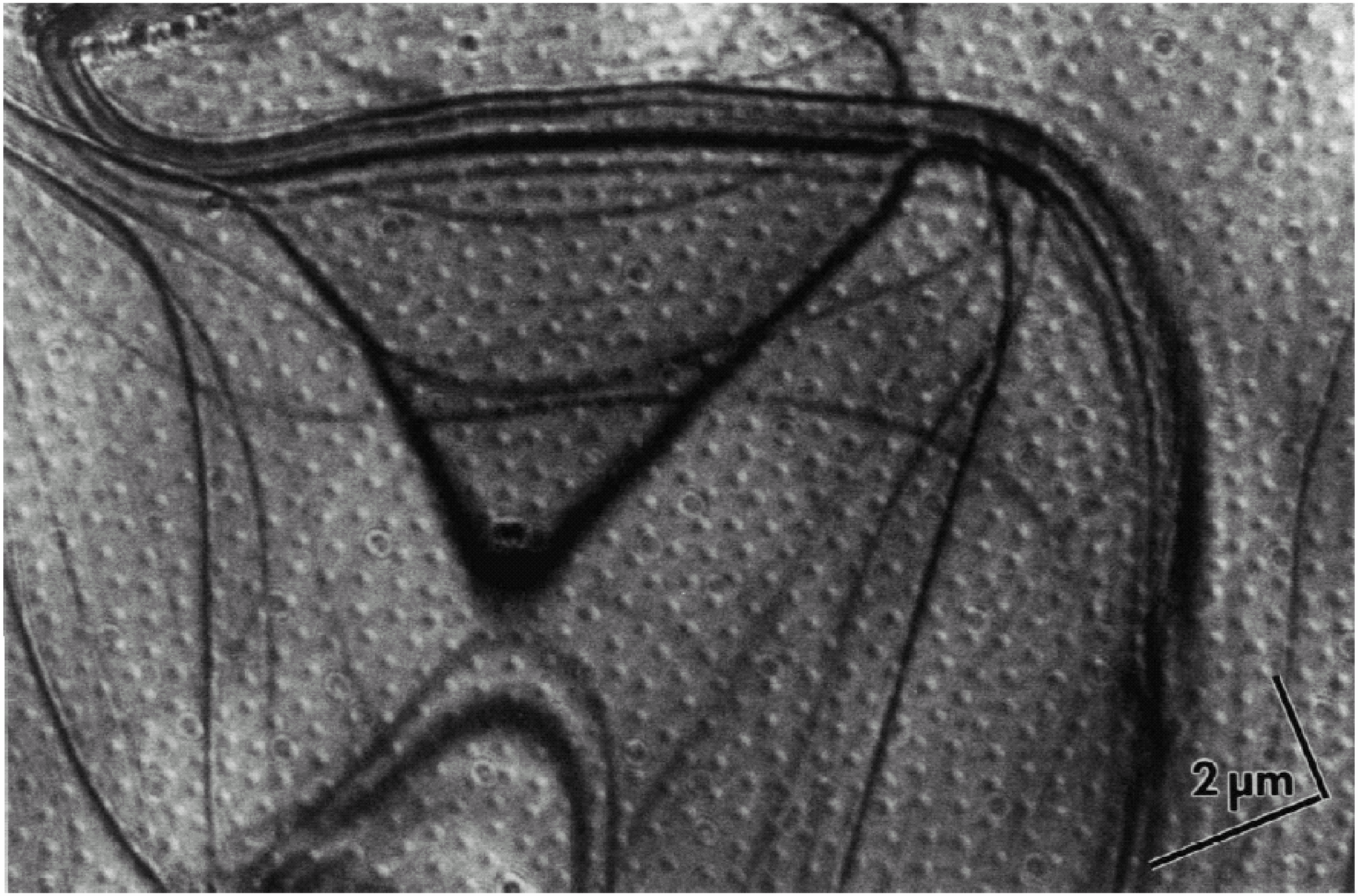


High-field partial Meissner effect in type-II superconductors

- **Type-I:** Meissner state $B = H + M = 0$ for $H < H_c$; normal state at $H > H_c$
- **Type-II:** Meissner state $B = H + M = 0$ for $H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

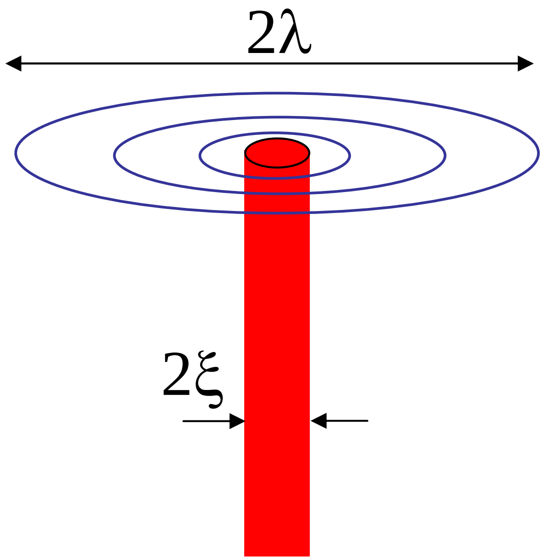
Pure Nb is weakly type II ($H_{c1} \approx 16$ mT, $H_c \approx 19$ mT, $H_{c2} \approx 30$ mT); Impurities decrease H_{c1} and increase H_{c2}, but do not affect H_c

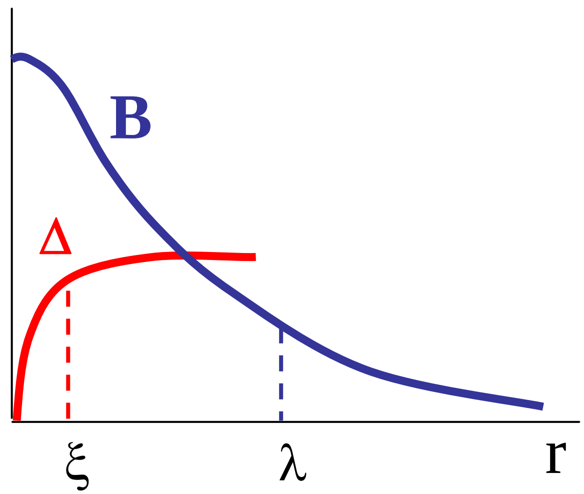
Lorentz electron microscopy of vortex structures



Nb film at 4.2K and 10mT, Harada et al, 1992

Single vortex line





- Small core region $r < \xi$ where $\Delta(r)$ is suppressed
- Region of circulating supercurrents, $r < \lambda$.
- Each vortex carries the flux quantum ϕ_0

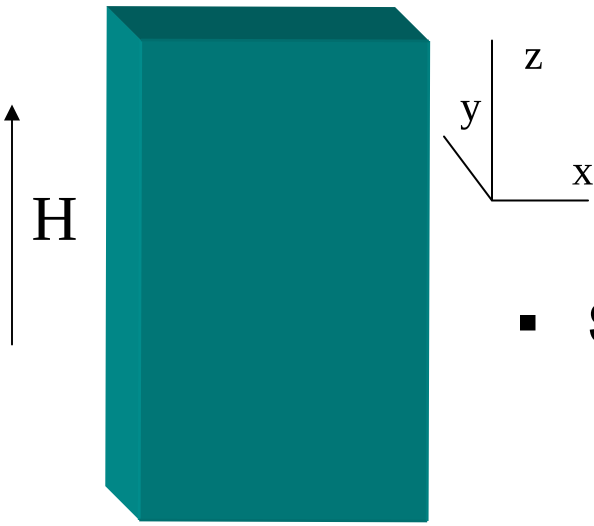
Important lengths and fields

- Coherence length ξ and magnetic (London) penetration depth λ

$$B_{c1} = \frac{\phi_0}{2\pi\lambda^2}\left(\ln\frac{\lambda}{\xi} + 0.5\right), \qquad B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}, \qquad B_{c2} = \frac{\phi_0}{2\pi\xi^2}$$

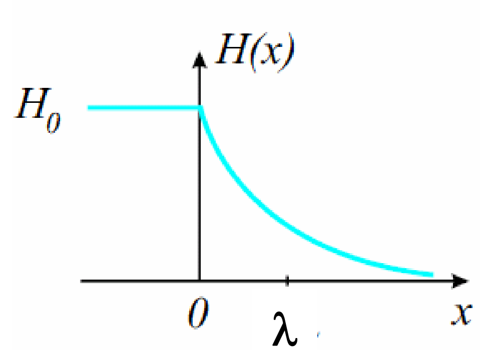
Type-II superconductors: $\lambda/\xi > 1/\sqrt{2}$: For clean Nb, $\lambda \approx 40$ nm, $\xi \approx 38$ nm

Dc screening of the magnetic field



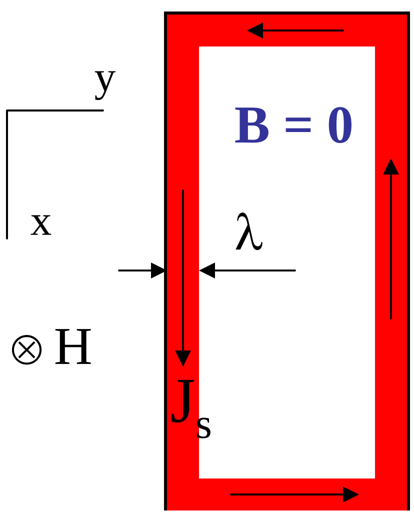
- London equation

$$\lambda^2 \frac{\partial^2 H_z}{\partial y^2} - H_z = 0$$



- Screening surface current density $J_s(y)$:

$$H(y) = H_0 e^{-y/\lambda}, \qquad J_s(y) = \frac{H_0}{\lambda} e^{-y/\lambda}$$



- Meissner effect: no magnetic induction B in the bulk.
- J(0) at the surface cannot exceed the depairing current density J_d :

$$J_d = \frac{H_c(T)}{\lambda(T)} \cong J_0 \left(1 - \frac{T^2}{T_c^2}\right)^{3/2}$$

$J_d(0) \approx 2.8$ MA/mm^2 for pure Nb

Depairing current density

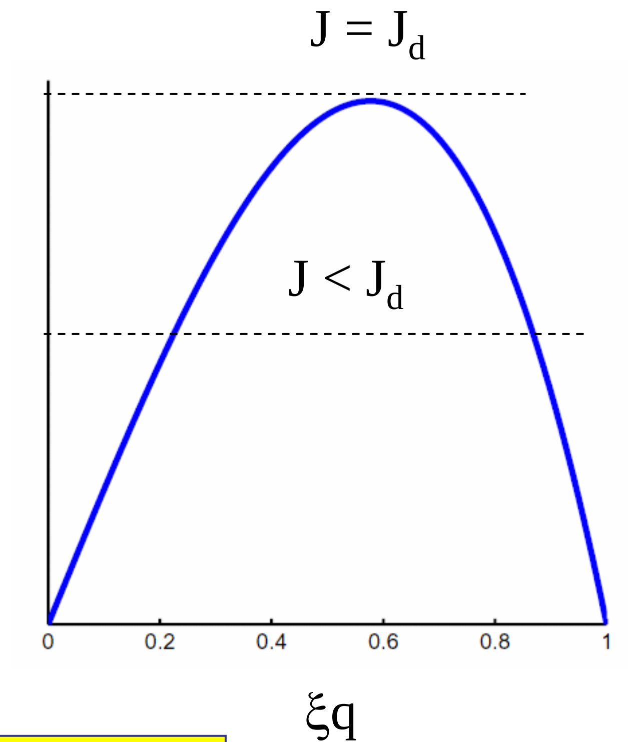
- What maximum current density J can flow in a superconductor?
- Current-carrying state with $\psi = \Delta \exp(-iqx)$, where q is proportional to the velocity of the Cooper pairs. At $T \approx T_c$, the GL equations give:

$$\psi_0^2 = 1 - \xi^2 q^2, \qquad J = \frac{\psi_0^2 \phi_0 q}{2\pi\lambda^2 \mu_0}$$

- Current density as a function of q:

$$J = \frac{\phi_0 q}{2\pi\lambda^2\mu_0}(1 - \xi^2 q^2)$$

Suppression of ψ by current

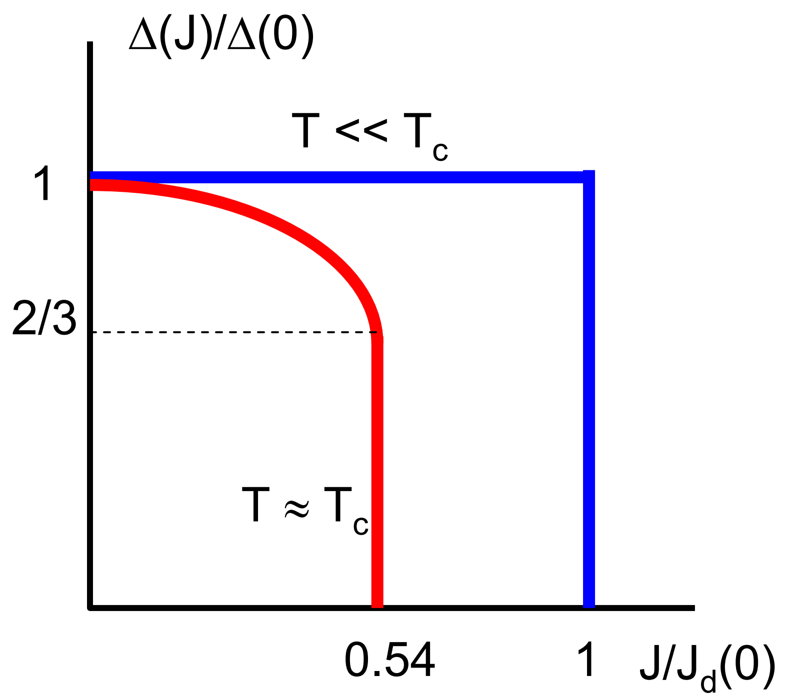


- Maximum J at $\xi q = 1/\sqrt{3}$ yields the depairing current density:

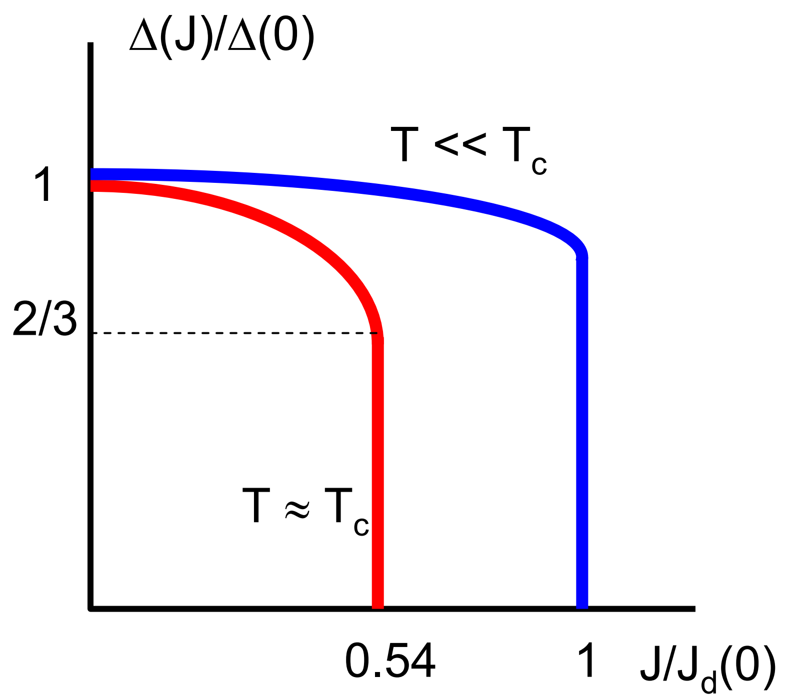
$$J_d = \frac{\phi_0}{3\sqrt{3}\pi\mu_0\lambda^2\xi} \cong 0.54\frac{H_c}{\lambda} \propto \left(1 - \frac{T}{T_c}\right)^{3/2}$$

Current dependence of the SC gap

- **Clean limit ($l \gg \xi_0$)**

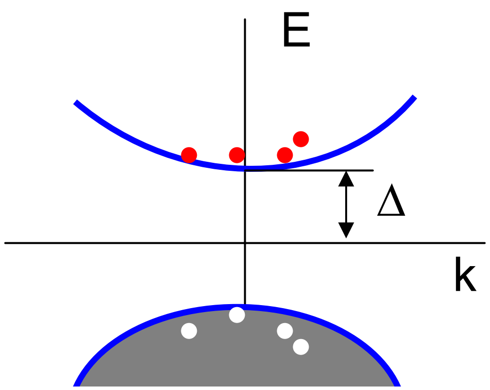


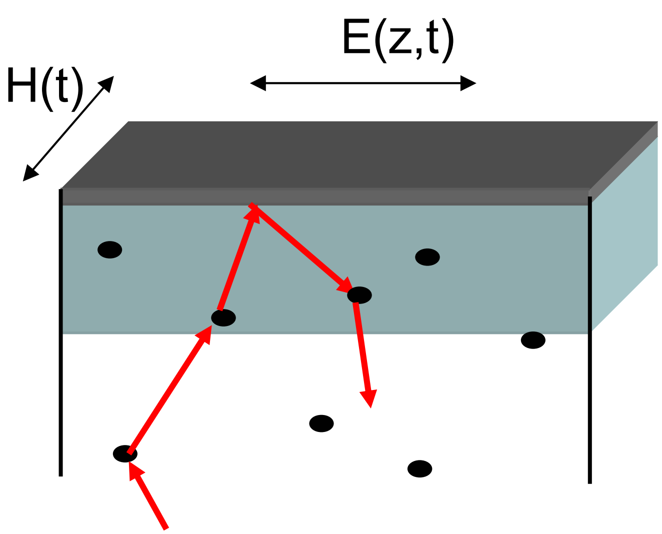
- **Dirty limit ($l \ll \xi_0$)**



Nb cavities usually correspond to the clean limit for which Δ(J) is independent of RF field at low T (J. Bardeen, Rev. Mod. Phys. 34, 667 (1962)).

RF dissipation





- **Thermal activation of normal electrons**
  $n_a = n_0(\pi T/2\Delta)^{1/2}\exp(-\Delta/T)$
- **Accelerating electric field**
  $E(z,t) = \mu_0\omega\lambda H_\omega e^{-\lambda|z|}\sin\omega t$
- **Scattering mechanisms and normal state conductivity**: $\sigma_n = e^2 n_0 l/p_F,\quad p_F = \hbar(3\pi^2 n_0)^{1/3}$

- **Surface:** from specular to diffusive
- **Normal skin effect** ($l \ll \lambda$): multiple impurity scattering in the λ - belt:
  $R_s \sim (\mu_0^2\omega^2\lambda^3\sigma_n\Delta/T)\exp(-\Delta/T)$
- **Anomalous skin effect** ($l \gg \lambda$): scattering by the gradient of the ac field E(z):
  Effective $\sigma_{eff} \sim e^2 n_0 \lambda /p_{F;}\qquad l \rightarrow \lambda$

Kinetics of normal electrons

- Material of Nb cavities is usually clean enough to ensure the conditions of the anomalous skin effect ($l \gg \lambda$). Typically, $l \sim 500$ nm, while $\lambda \sim 40$ nm.
- For $l \gg \lambda$, the normal state resistivity is irrelevant to the rf surface resistance.

- Quasi-static rf resistance $\omega \ll \Delta$ (good approximation for SC cavities)

- Quasi-equilibrium Fermi-Dirac distribution function for normal electrons: $2\pi\tau_r f \ll 1$

- Recombination time due to electron-phonon collisions (Kaplan et al, PRB 14, 8454 (1976))

$$\tau_r^{-1} = \tau_0^{-1}\left(\frac{\pi T}{T_c}\right)^{1/2}\left(\frac{2\Delta}{T_c}\right)^{5/2}\exp\left(-\frac{\Delta}{T}\right)$$

TESLA single cell cavity (f = 1.3 GHz, T = 2K, τ_0^{-1} = 6.7 GHz), $\tau_r^{-1} \approx 0.03$ GHz

Nonequilibrium effects are important for strong rf fields $H_a \sim H_c$

Linear surface resistance for $H_\omega << H_c$

- **Solution of the kinetic equation for type-II superconductor for the clean limit and diffusive surface scattering at $\omega^2 << \Delta T$:**

$$R_s = \frac{3\mu_0^2\lambda^3\Delta}{2T}\sigma_{eff}\omega^2 e^{-\Delta/T}\left[\ln\frac{1.12Tv_F^2}{\omega^2\lambda^2\Delta} - 1\right]$$

- **Effective conductivity** in the **nonlocal clean limit:**

$$\sigma_{eff} = \frac{n_0 e^2\lambda}{p_F}$$

No dependence of R_s on the normal resistivity and impurity scattering

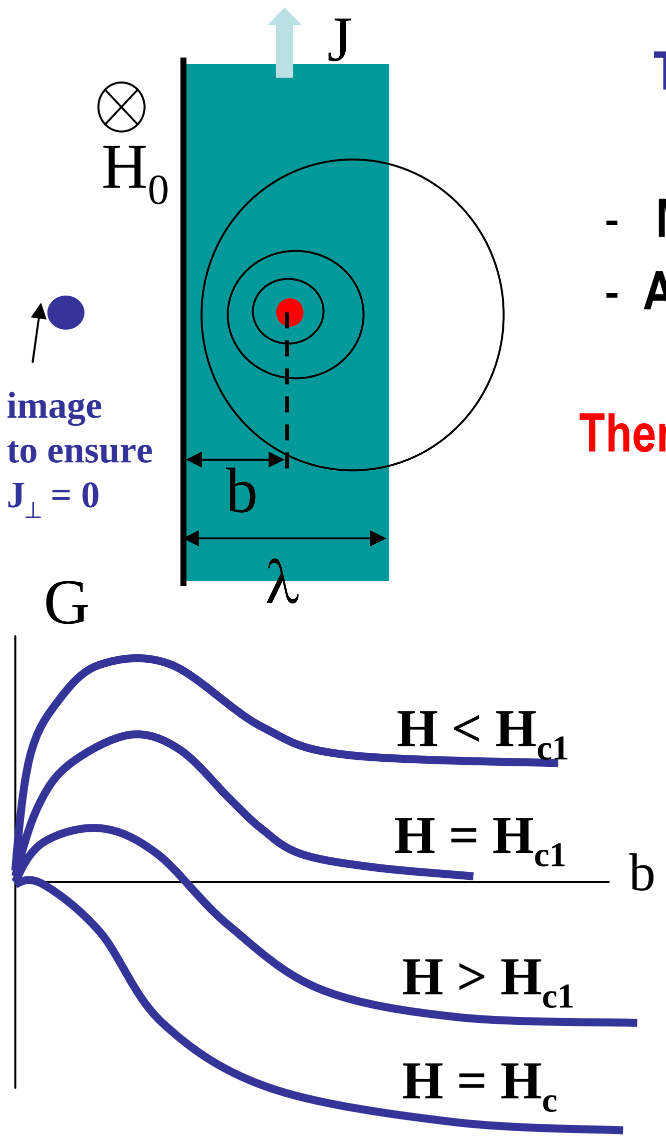
Nonlinear surface resistance

- RF dissipation was calculated for clean limit (l >> λ) from kinetic equations for a superconductor in a strong rf field superimposed on a dc field; $H(t) = H_\omega \cos\omega t + H_0$

$$P = \frac{1}{2} R_s(T) H_\omega^2 \left[1 + \frac{CT_c^2}{T^2 H_c^2} \left(H_0^2 + \frac{H_\omega^2}{4} \right) \right], \qquad C \approx 1$$

- Nonlinear correction due to rf pairbreaking increases as the temperature decreases
- At low T, the nonlinearity becomes important even for comparatively weak rf amplitudes $H_\omega \sim (T/T_c)H_c \ll H_c$
- RF power P depends quadratically on the dc magnetic field. Field dependence of $P(H_0)$ does not necessarily indicates vortex contribution.

Surface barrier: How do vortices get in a superconductor ?



Two forces acting on the vortex at the surface:

- Meissner currents push the vortex in the bulk
- Attraction to the antivortex image pushes the vortex out

Thermodynamic potential G(b) as a function of the position b:

$$G(b) = \phi_0 [H_0 e^{-b/\lambda} - \frac{\phi_0}{2\pi\mu_0\lambda^2} K_0\left(\frac{2b}{\lambda}\right) + H_{c1} - H_0]$$

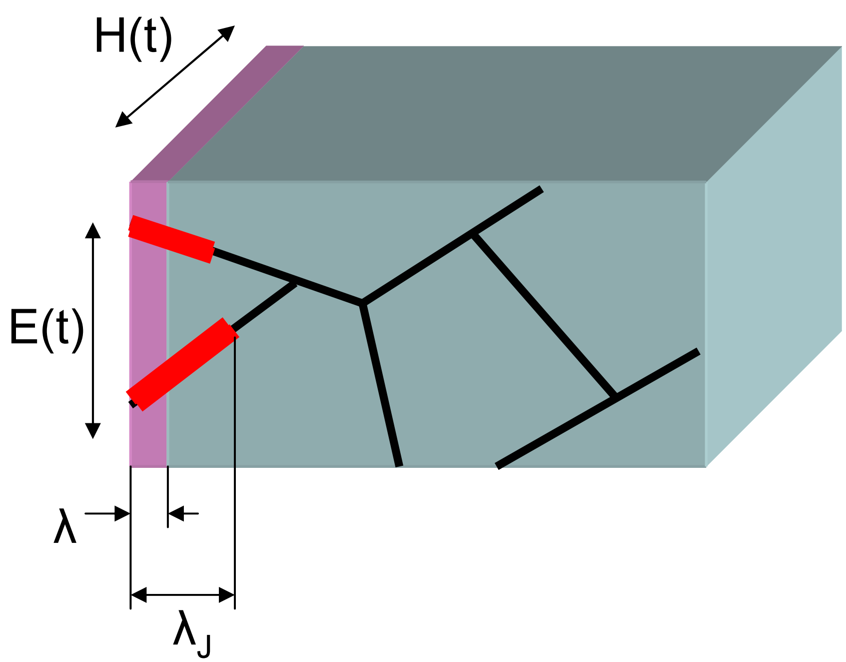
Meissner Image

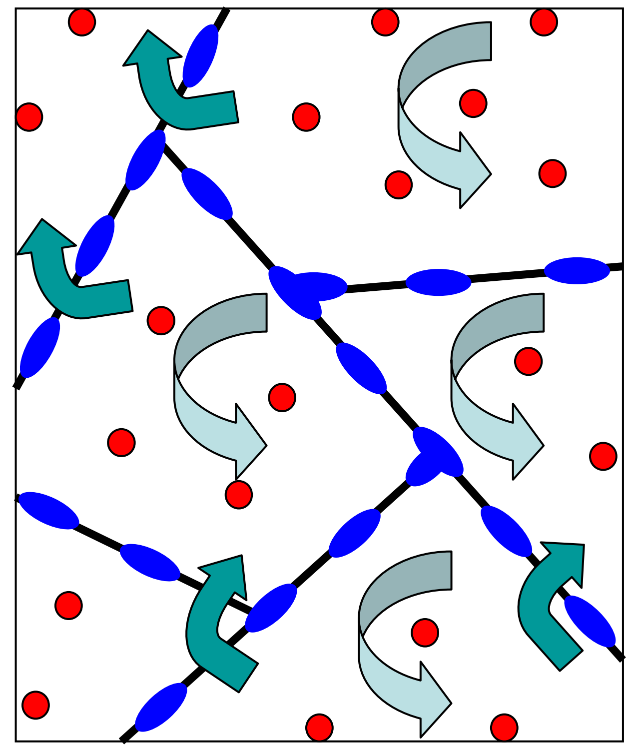
Vortices have to overcome the surface barrier even at $H > H_{c1}$ (Bean & Livingston, 1964)

Surface barrier disappears only at $H = H_c$

Surface barrier is reduced by defects

Effect of grain boundaries





$$\lambda_J = (\lambda\xi)^{1/2}\sqrt{\frac{J_d}{J_b}} >> \lambda$$

- GBs are planar weak links with excess resistance R_b and reduced critical current density J_b

- RF field leaks in through the GB network
- GB vortices are different from intragrain ones

$$H_p = \frac{2H_c}{\pi}\sqrt{\frac{2\xi J_d}{\lambda J_b}} << H_c$$

For $J_d \sim 3$ MA/mm^2 and $J_b \sim 10^{-2}$ MA/mm^2, $\lambda_J \sim 17\lambda$, and $H_p \sim 0.16H_c$

Dissipation due to grain boundaries.

- **Strong rf field, $H_p \ll H_\omega < H_c$. Nonlinear RF penetration depth and dissipation:**

$$L \approx \left(\frac{R_b}{2\mu_0\lambda\omega}\right)^{1/2} \ln\frac{H_\omega}{H_p}$$

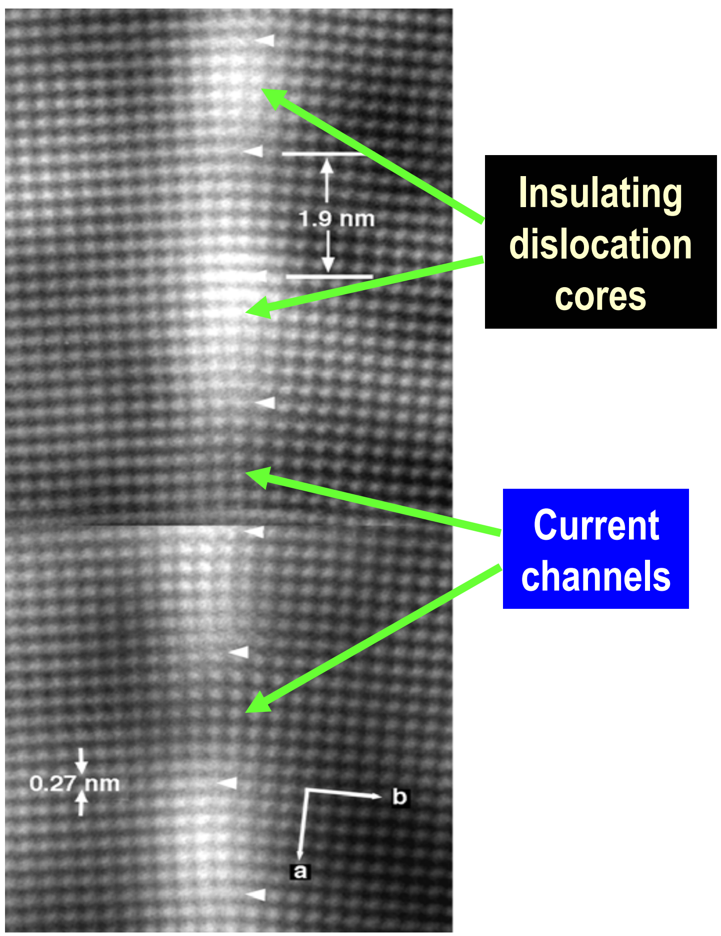
$$P \approx \mu_0^2\omega^2 H_\omega^2 \lambda^2 L^3 / R_b \propto H_\omega^2 \sqrt{\mu_0\omega\lambda R_b} \ln^3\left(\frac{H_\omega}{H_p}\right)$$

- **Weak rf field, $H_\omega < H_p$. RF penetration depth and dissipation:**

$$L \approx \lambda_J, \qquad P \approx \mu_0^2\omega^2 H_\omega^2 \lambda^2 \lambda_J^3 / R_b$$

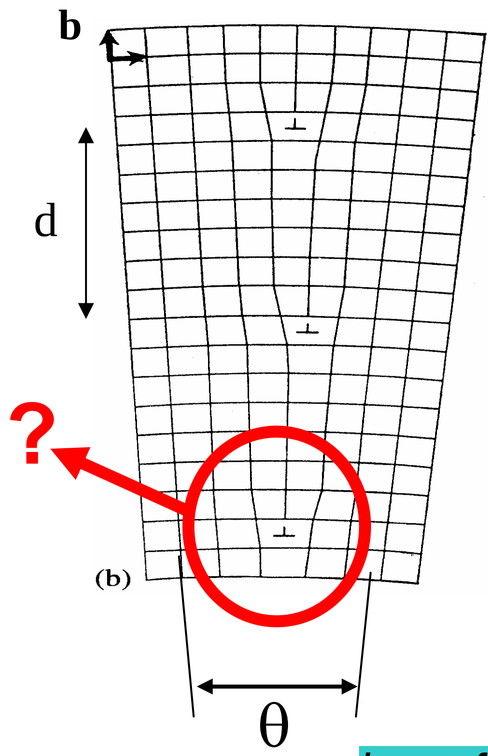
Different dependencies of RF dissipated power on R_b, J_b and ω for weak and strong rf fields.

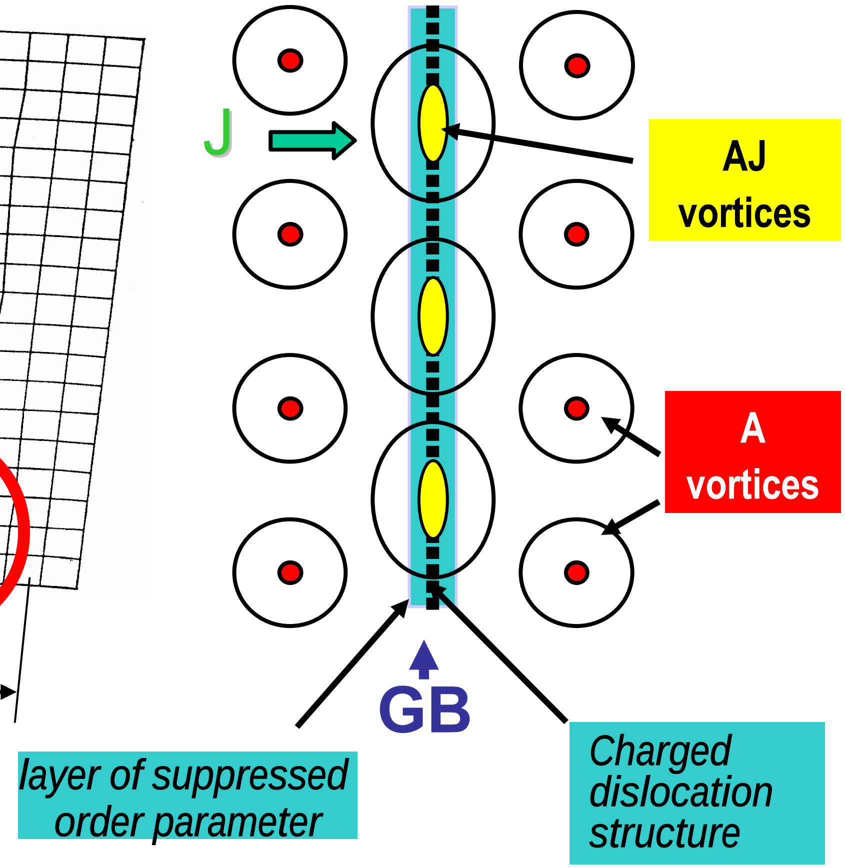
Vortices on low-angle GB



HRTEM image of 8°[001] tilt GB in $Bi_2Sr_2CaCu_2O_x$

S.E. Babcock et al



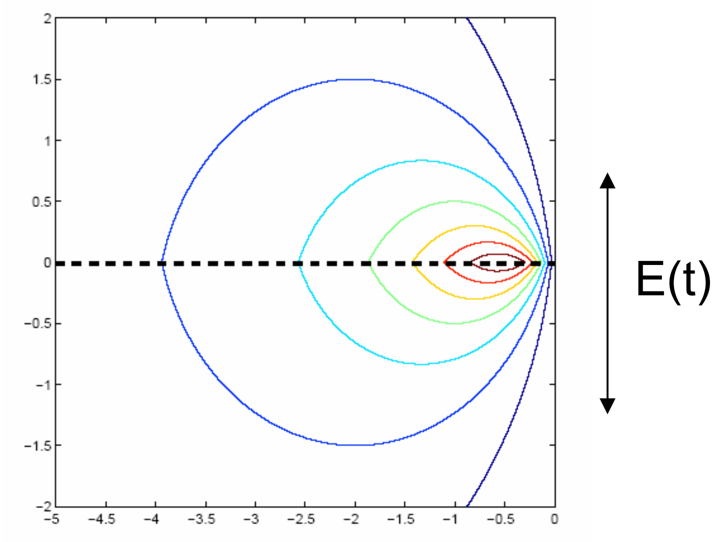


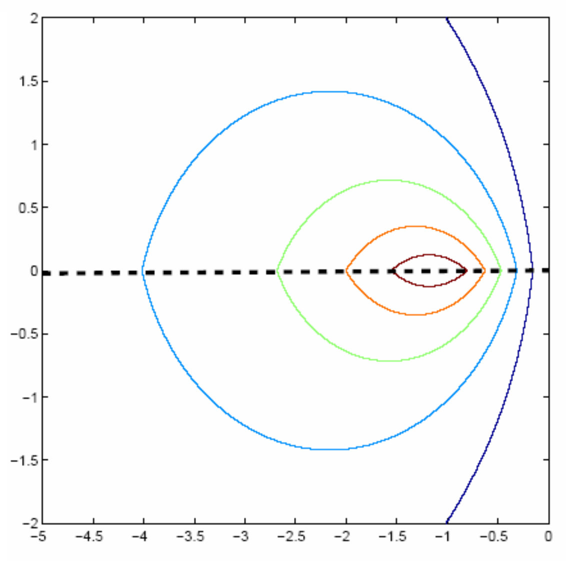
AG and E.A. Pashitskii, PRB, 57, 13875 (1998);

Global J_c through GB is determined by vortex pinning.

J_c is much smaller than the intrinsic J_b on the scale of few current channels.

Penetration of vortices along GBs through oscillating surface barrier



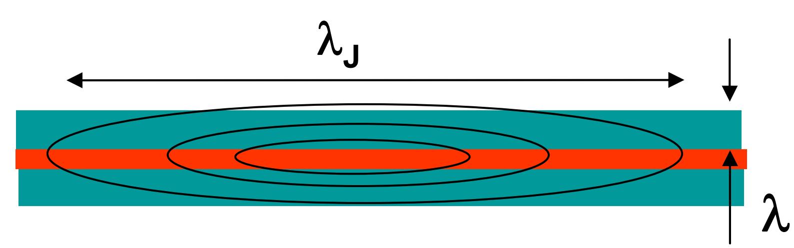


AJ vortex

Deformation of the vortex core during flux penetration along GBs.

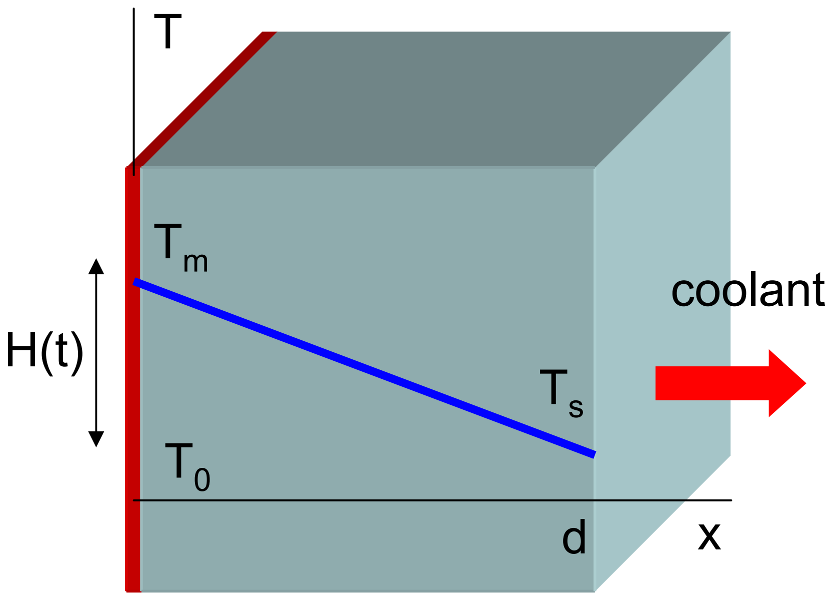
Transformation of the Abrikosov (A) to the Josephson (J) and mixed Abrikosov-Josephson (AJ) vortices

Dissipation due to vortex oscillations in RF field



J vortex on a high-angle grain boundary

Analytical thermal breakdown model



$$\frac{1}{2}H_\omega^2 R_s(T_m) = \kappa(T_0)(T_m - T_s)/d,$$

$$\kappa(T_0)(T_m - T_s)/d = h(T_0)(T_s - T_0)$$

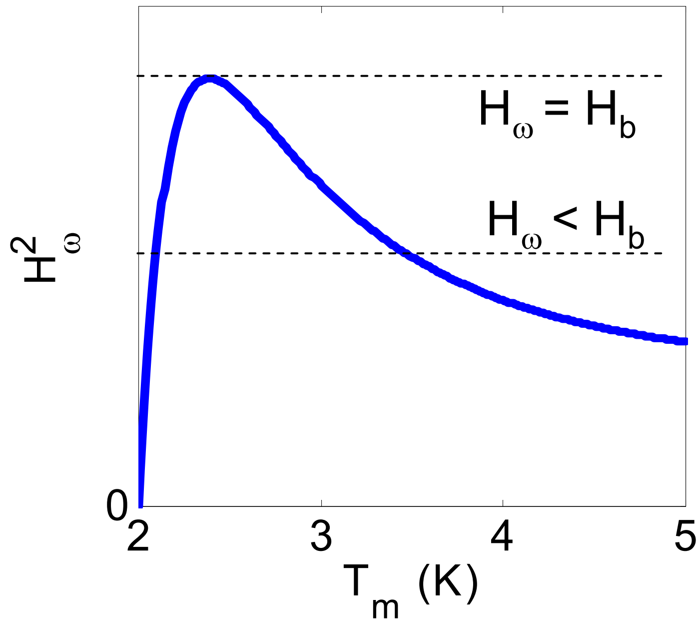
Since $T_m - T_0 \ll T_0$ even at the breakdown field H_b, thermal conductivity κ and the Kapitza resistance h are taken at $T = T_0$ (for a general case of thermal quench, see A. Gurevich and R. Mints, Rev. Mod. Phys. 59, 941 (1987)

Equation for the maximum temperature $T_m(H_\omega)$:

$$H_\omega^2 = \frac{2(T_m - T_0)}{R_s(T_m)} \frac{\kappa h}{(\kappa + dh)}$$

Breakdown rf field

$$H_\omega^2 = \frac{2h\kappa}{(\kappa + dh)R_0 T_c} T_m (T_m - T_0) \exp\left(\frac{\Delta}{T_m}\right)$$



Thermal runaway occurs at a rather weak overheating:

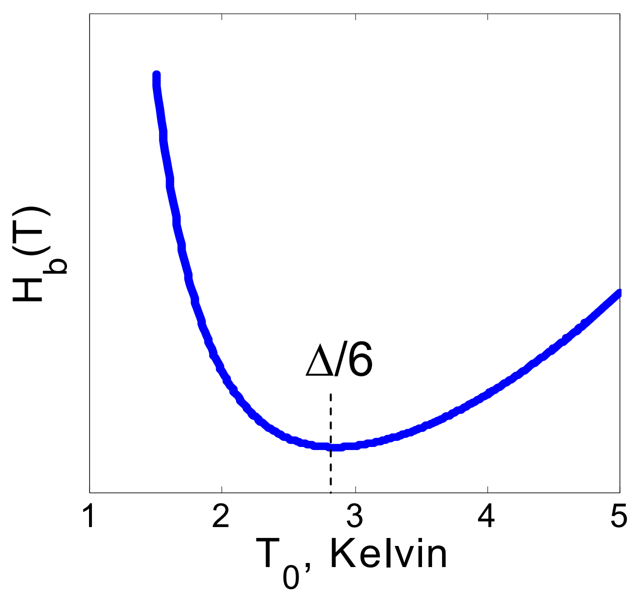
$$T_m - T_0 \approx \frac{T_0^2}{\Delta} = \frac{T_0^2}{1.86\,T_c} = 0.23\,K,$$

$$H_b^2 = \frac{2h\kappa T_0^3}{(\kappa + dh)R_0 T_c \Delta e} \exp\left(\frac{\Delta}{T_0}\right)$$

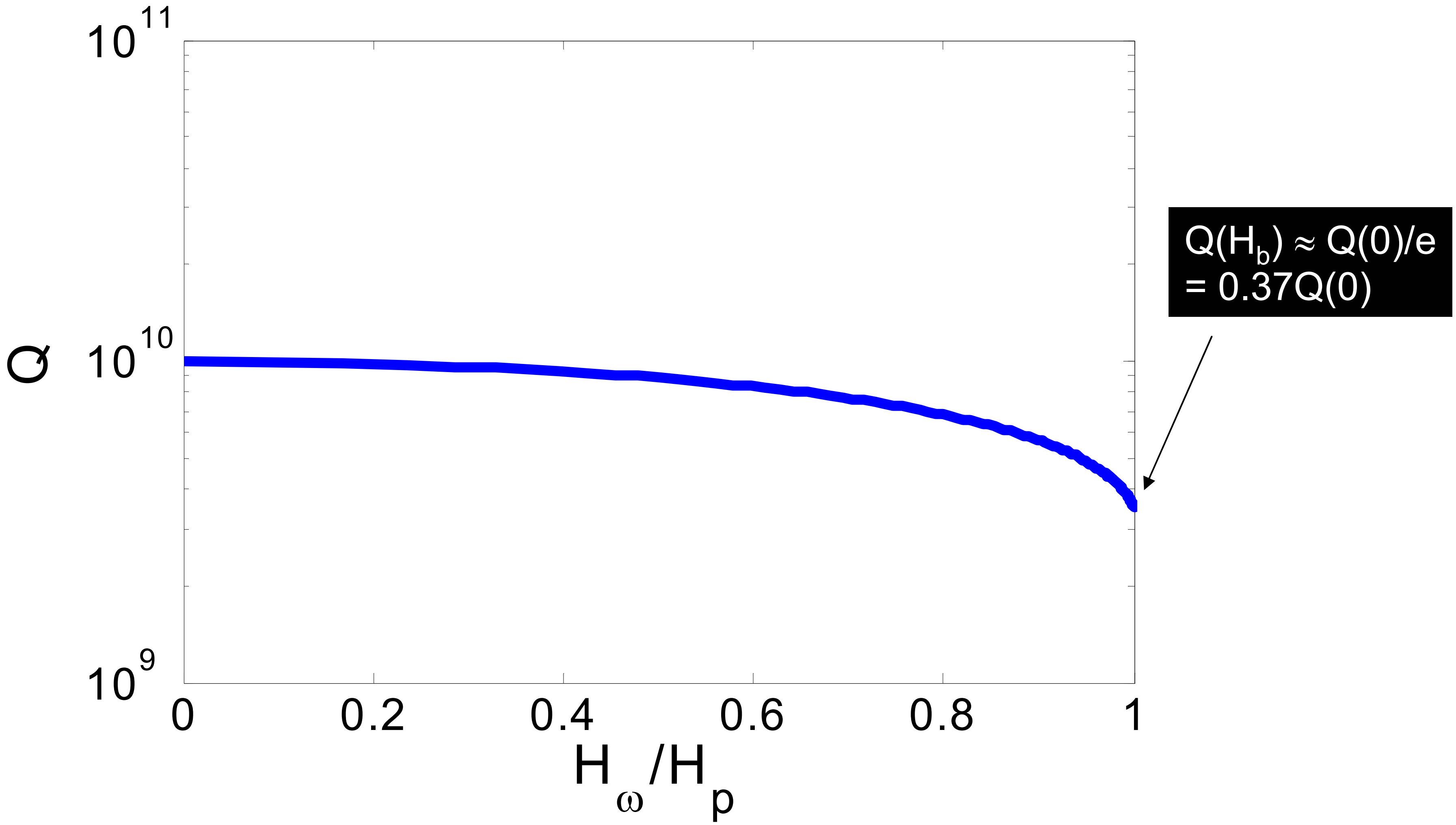
For κ >> dh, the breakdown field is limited by the Kapitza resistance, $h(T)=\alpha T_0^3$. Thus,

$$H_b = \left(\frac{2\alpha}{R_0 T_c e \Delta}\right)^{1/2} T_0^3 \exp\left(\frac{\Delta}{2T_0}\right)$$

is minimum at $T_0 = \Delta/6$



Quality factor (an example)



Conclusions

- Multiscale mechanisms of the rf breakdown.
- Anomalous skin effect in the clean limit; RRR does not affect R_s
- Importance of pairbreaking nonlinear effects and nonequilibrium superconductivity in strong RF field.
- Dependence of R_s on dc magnetic field even in the Meissner state.
- Grain boundaries can significantly reduce the field onset of vortex penetration and increase rf dissipation.
- R_s in Nb is determined by the first 40 nm from the surface: surface defects and impurity concentration profile on that scale are extremely important (C. Antoine (Saclay), P. Lee (UW)).

Theoretical challenges:

- Develop a theory of nonlinear surface resistance which can be used at high rf amplitudes $H_\omega \approx H_c$. Account of current pairbreaking and nonequilibrium superconductivity in strong rf field.
- Develop a theory of rf dissipation due to vortex penetration through oscillating surface barrier with the account of the grain boundary network.