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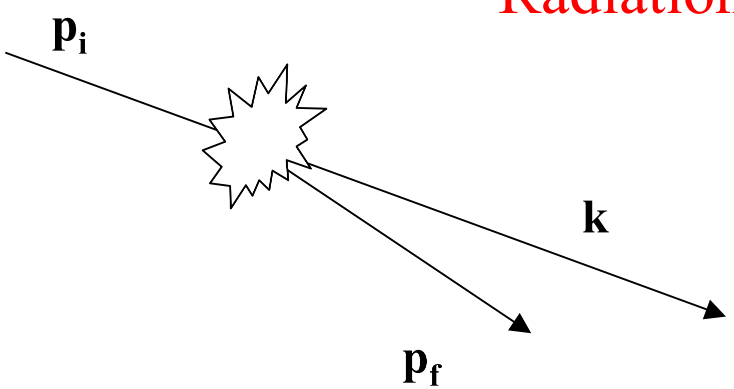
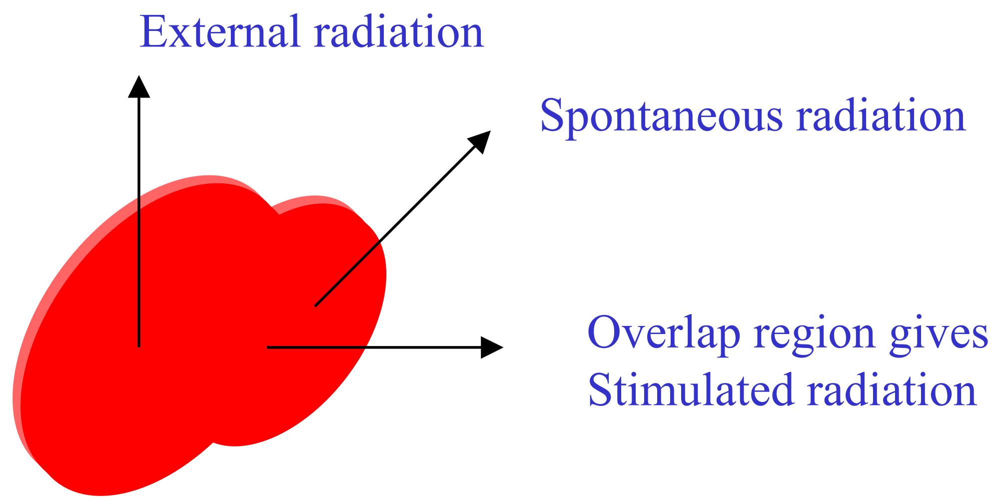
Scaling of brightness for SASE X-Ray FEL

Outline

- Radiation and Particle energy exchange
- 3D small signal gain
- Scaling of gain length for SASE FEL
- Fit for Kwan Je and Ming solution
- Optimization of FEL operation
- Low energy X-ray FEL
- Ponderomotive laser acceleration
- Expectation

Radiation and Particle energy exchange





Energy exchange between particle and field can be calculated in the near field by integrating the projection of he electric field on the trajectory, or in the far field by looking at the interference between the spontaneous radiation from the particle and the external field. Far field is not sensitive to structures smaller than diffraction limited size in the near field region. This greatly simplifies the calculation or estimation of the energy exchange.

In the far field region the total field is the sum of fields from an external source and from spontaneous emission of particles

$$E = E_{ex}(\omega,\theta) + E_s\,(\omega\text{-}\omega_o(E\ ),\theta)\ \Sigma\ e^{i\varphi j}$$

W ☢ ✐ ⌛$E_{ex}(\omega,\theta)$ +$E_s(\omega\text{-}\omega_o(E\ ),\theta)$ ⌛² dωdΩ

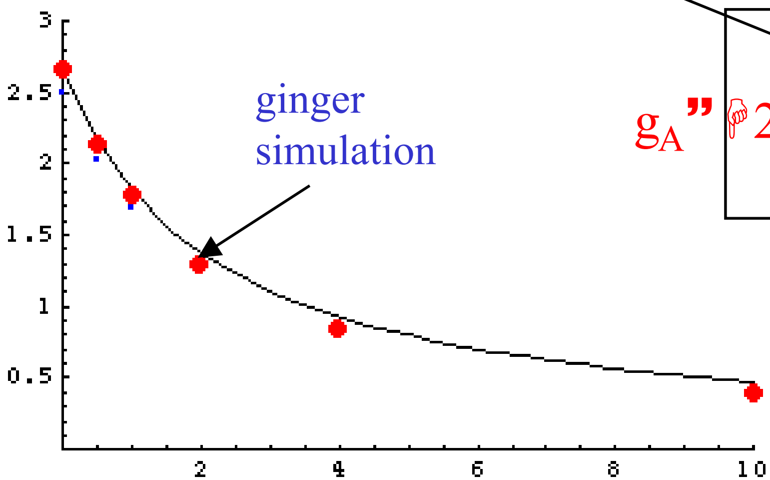
Δ W ☢2✐ ⌛$E_{ex}(\omega,\theta)$ ⌛⌛$E_s(\omega\text{-}\omega_o(E\ ),\theta)$ ⌛ dωdΩ $\text{Cos}\varphi_j$

$E_s\,(\omega\text{-}\omega_o(E\ ),\theta) \Rightarrow E_s\,(\omega\text{-}\omega_o(E\ ),\theta) + \partial/\partial E\ \ E_s\,(\omega\text{-}\omega_o(E\ ),\theta)\ \Delta$ Ej

$g_A = 2\,\pi\,\dot{N}\,\partial\,/\partial E$ ☜ $_s\,(\omega\text{-}\omega_o(E\ ),\theta)$ Madey theorem

Small signal gain with correction for final emittanc

Gain



Beam emittance/light emittance

g_A” ☞2 □M☹2 $\frac{WI}{\gamma J_A}$

↖☹2 □M♦

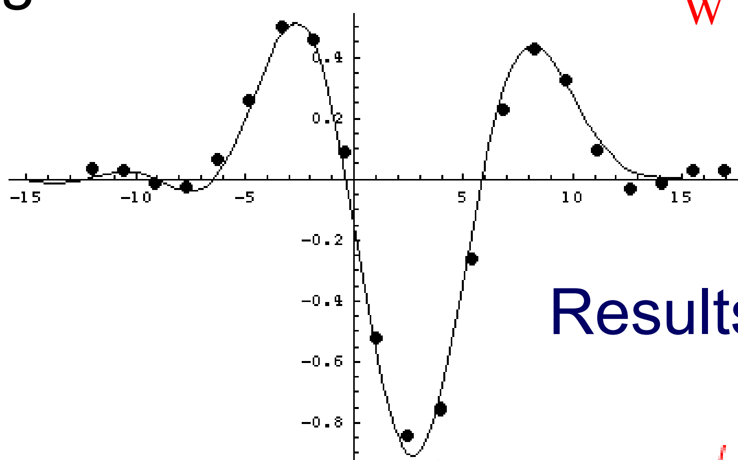
1 ☐ $\frac{ℳ_b}{2\ ℳ_x}$ ✋ $\frac{4\ k_w\ \delta}{2\ □M}$ ☐ $\frac{2\ □M}{4\ k_w\ \delta}$ 💣

Z_R ✍ $L_w/4$

$W = K^2/(1+K^2)$ helical w

$W = K^2/2/(1+K^2/2)\ JJ^2$ plane w

Gain



detuning

Results of slicing experime

Where, $JJ = J_0\left(\frac{1}{2}\,\frac{k^2/2}{1+k^2/2}\right) - J_1\left(\frac{1}{2}\,\frac{k^2/2}{1+k^2/2}\right)$

zero order estimation for gain length g_A~1, $\varepsilon_b > \varepsilon_x$

1 / ☟2 ❑M_A☹ 🖬 W I / ♑♌ A ♍x / ♍b 2 ↖✉2 ❑M♦ / (4 k_w ♌ / 2 ❑M ✉ 2 ❑M / 4 k_w ♌) 🖬 WI / ♑♌ A ♏ 2 ↖✉2 ❑M♦ / (♌ / ♌✉ ✉ ♌✉ / ♌)

Now believers can try fit solutions of the Vlasov equations (by Ming Xie) in form:

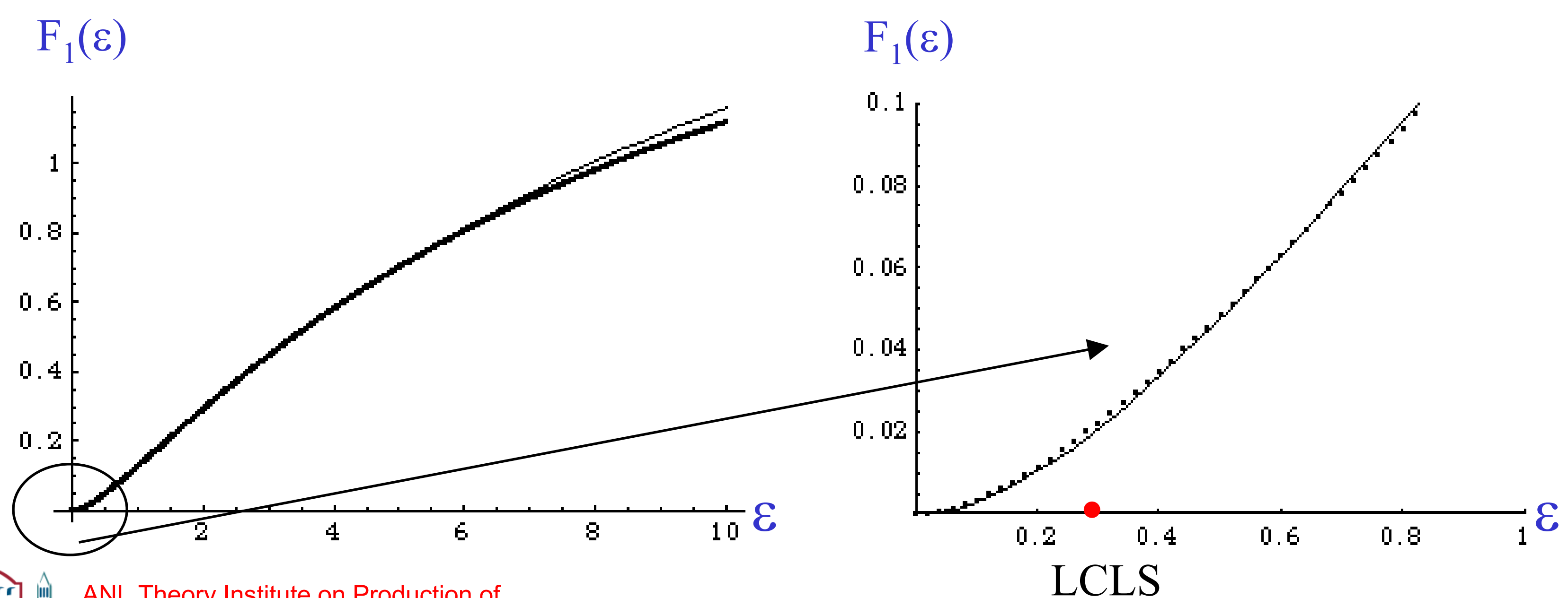
1 / ☟2 ❑M_A☹ 🖬 WI / ♑♌ A F_1 ☟♍☹ 2 ↖✉2 ❑M_A F_2 ☟♍☹♦ / (k_w ♌ / F_3 ☟♍☹2 ❑M_A ✉ F_3 ☟♍☹2 ❑M_A / k_w ♌)

Where F_1, F_2 and F_3 are 3 unknown function of a single parameter $\varepsilon = \varepsilon_x/\varepsilon_b$

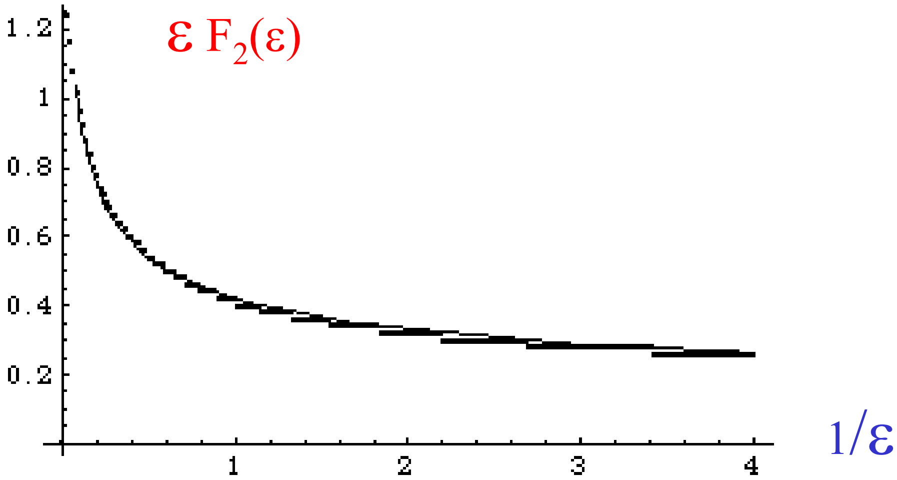
Fit for $F_1(\varepsilon)$

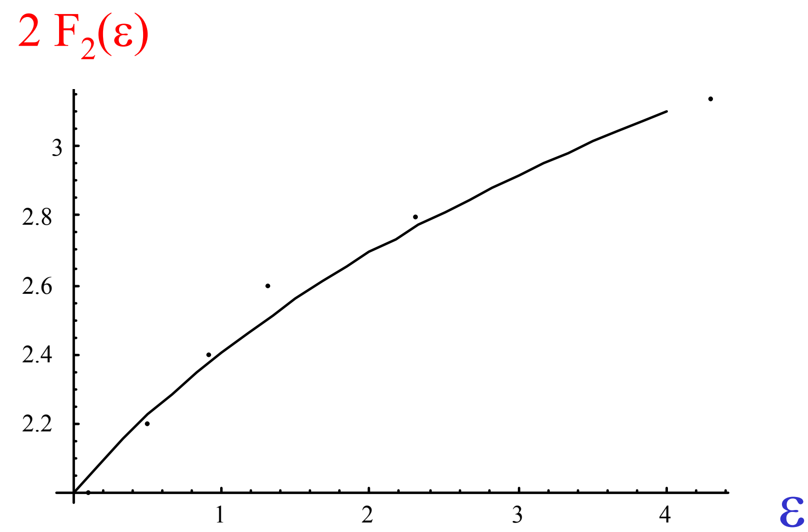
For $\sigma = 0$ and with optimal β

$$\frac{1}{(2\pi M_A)^2} = \frac{W\, I}{\gamma\, I_A} F_1(\epsilon)$$



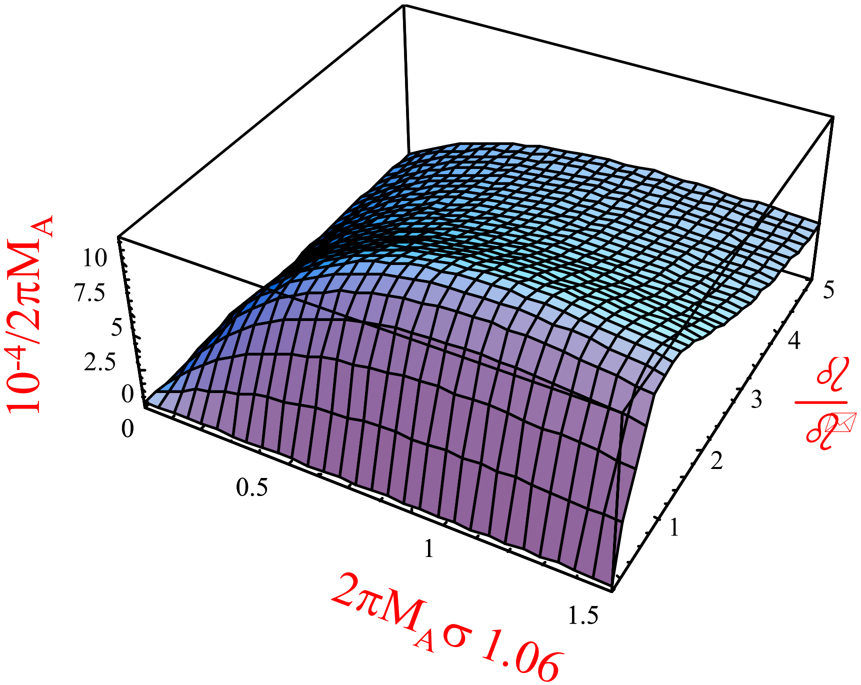
Fit for $F_2(\varepsilon)$ and $F_3(\varepsilon)$





This function fits the numerical solution to the Vlasov equation to within a reasonable accuracy for a wide range of parameter values.

LCLS parameters



= 1.58

$2\pi M_A \sigma = .22$

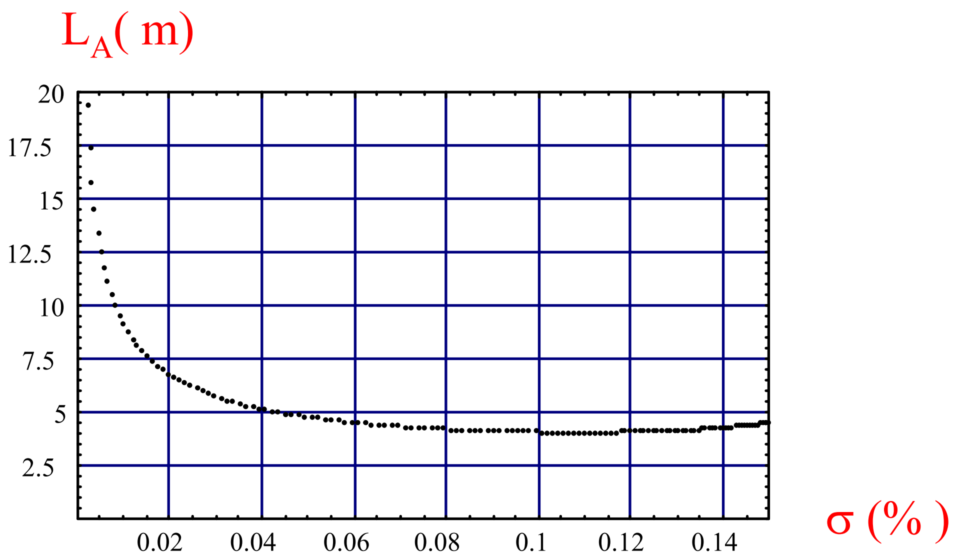
L_A= 9.6 m compare with
LCLS design report L_A= 9.5 m

For optimal performance $2\pi M_A \sigma = 1$

and = 1

M_A= 120 periods

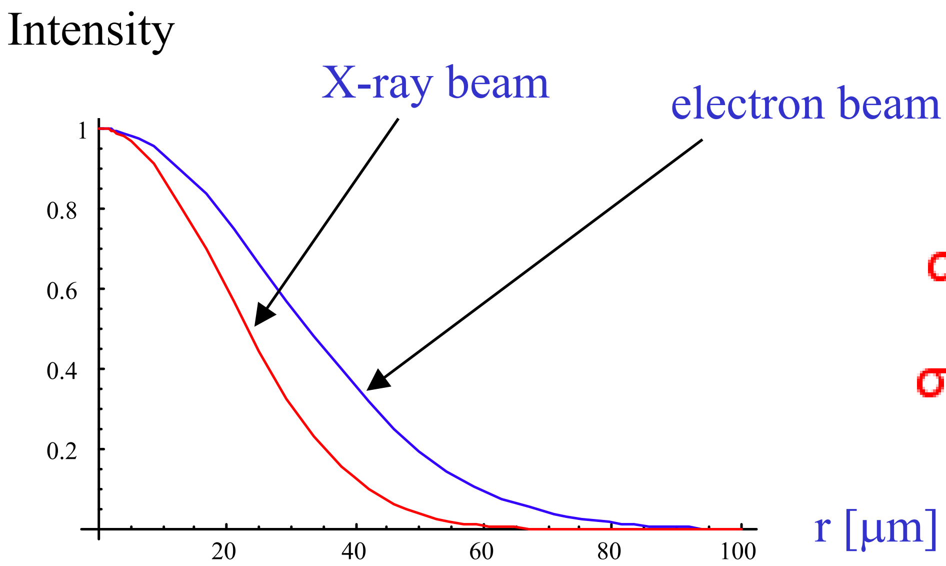
LCLS parameters



For fixed longitudinal invariant emittance $\varepsilon_{long} = \gamma\, \sigma_{long}\, \sigma_{energy} = .01\text{cm}$

For LCLS brightness, γ, and 1.5 A radiation, this yields optimal field gain length of 4.2 m. To obtain this gain length, an additional rotation in longitudinal phase space (shorter bunch length) is desirable. This holds only as long as the longitudinal emittance can be preserved, which may be very challenging.

LCLS parameters



$$\sigma_{A\,X\text{-}ray} = \sigma_e$$

$$\sigma_{I\,X\text{-}ray} = \frac{\sigma_{A\,X\text{-}ray}}{\sqrt{2}} = \frac{\sigma_e}{\sqrt{2}}$$

$\Rightarrow$ $Z_R = 32$ m

LCLS design report $\Rightarrow$ $Z_R = 32$ m

Cooperation length = M_A wave length = 48 nm
LCLS design report = 48 nm

For hard X-ray SASE FEL, $\varepsilon < 1$. If the beta function is optimized:

$$\frac{1}{\rho\, 2\pi M_A \varepsilon^2} \approx \sqrt{3}\,\frac{WI}{\gamma I_A}\,\varepsilon^2\, e^{-2\pi M \sigma}$$

$$\frac{1}{2\pi M_A} = \sqrt{3}\,\frac{W I \epsilon^2}{\gamma I_A \sigma}\, 2\pi M \sigma\, e^{-2\pi M \sigma}$$

Finally, if the energy spread is optimized:

$$\frac{1}{2\pi M_A} = \sqrt{\frac{3}{2\pi}}\,\frac{\alpha}{4e}\, W N\, \frac{\lambda_c^3}{\epsilon_x \epsilon_y \epsilon_z}\left(\frac{\mathcal{E}}{h\nu}\right)^2$$

$$\frac{1}{2\pi M_A} = \sqrt{\frac{3}{2\pi}}\,\frac{\alpha W}{4e}\, B_F \left(\frac{\mathcal{E}}{h\nu}\right)^2$$

$$B_F = \frac{N \lambda_c^3}{\epsilon_x \epsilon_y \epsilon_z}$$

B_F equals one for perfect beam (degenerate polarized beam of Fermi particle)

$B_F = 2.5\ 10^{-12}$ for LCLS . $B_F = 2$ for copper or any other metal.

Radiation from modern lasers have sufficient power to be used as electromagnetic wigglers with periods of 0.4-10 μm and strength parameters W up to ≈1 (or 1/2).

For electromagnetic wiggler, the resonance condition is: $\lambda = \frac{\lambda_W}{4\gamma^2}\left(1 + \frac{a^2}{2}\right)$

Backward Compton-scattered 0.4 μm photons by 15 MeV electrons become 10 keV X-rays

.

$$\frac{1}{2\pi M_A} = \sqrt{\frac{3}{2\pi}}\,\frac{\alpha W}{4e}\,B_F\left(\frac{\varepsilon}{h\nu}\right)^2$$

If we use the brightest source almost available today and drop electron energy by 3 orders of magnitude this increases M_A by 6 orders of magnitude to around 10^8. The laser energy required for this kind of electromagnetic wiggler is TJ (10^{12}) (we remember Nova is kJ 10^3).

Energy required from laser scales like $M_A{}^2$
If we can increase 6D brightness by 6 orders of magnitude
Laser energy required drops to 1J which is available now.

How can one achieve the required brightness?

$B_F = 2.5\ 10^{-12}$ for LCLS . $B_F = 2$ for copper or any other metal. We lose 12 orders of magnitude in 6D brightness when we extract electrons

Where does the brightness go?

A reason for the loss in brightness may be that inside metal there are ions that compensate space charge field, on the order of magnitude of atomic field $\sim 10^9$ V/cm, which is much greater than 10^6V/cm the field on the surface of the cathode.

Place for speculation

How can the required brightness be reached?

- Traditional Technology:

a. Needle cathode (tungsten, graphite nanotube) achieve $B_F \sim 2\ 10^{-6}$ but necessary to manipulate in 6 D phase space

b. To achieve this brightness in Photo RF guns will require electric field on the same order of atomic field, electrons will be extracted long before the field reaches the desired strength

- Plasma ($\omega_p \tau_L \approx 1$) laser field can be applied in very short time ~10 fs

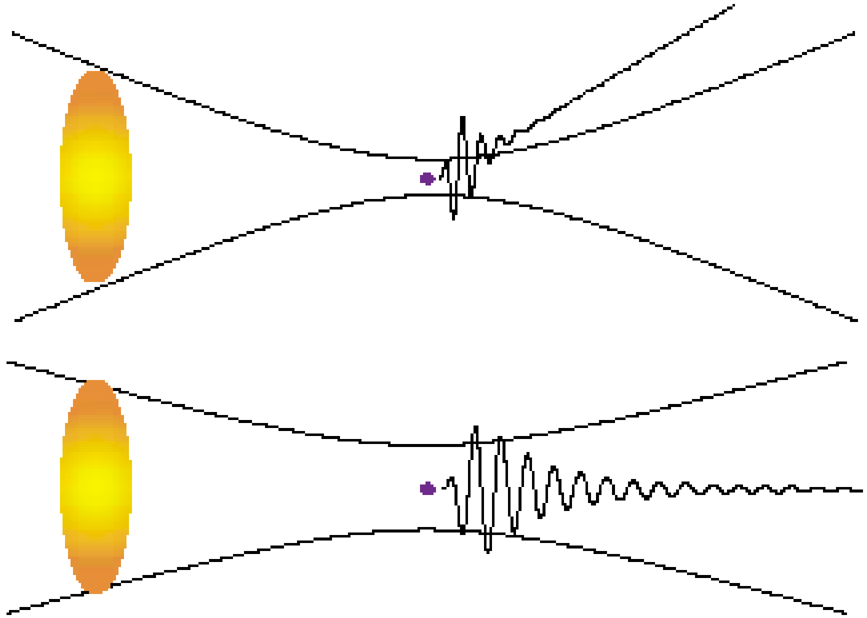
  Preliminary simulation indicates that this method gives increase in brightness, but not orders of magnitude more research is required.

- Laser Vacuum ($\omega_p \tau_L << 1$). Ponderomotive acceleration

  Estimates suggest that this method might very well be the solution, however more work is still required.

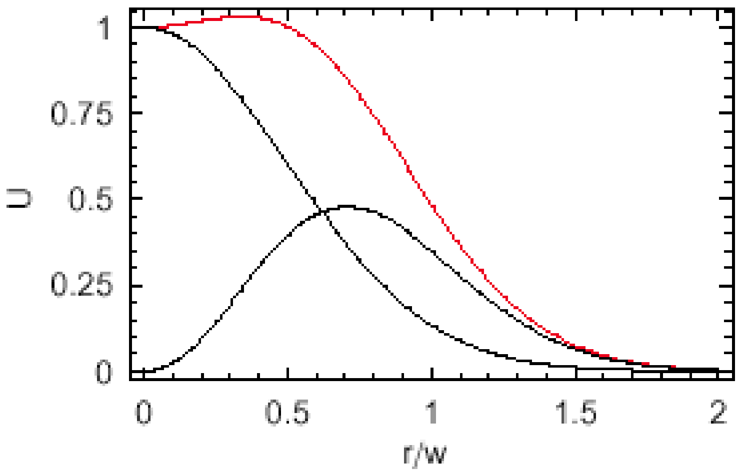
Ponderomotive Laser Acceleration and Focusing in Vacuum

G.V. Stupakov and M.S. Zolotorev, PRL, **86**, p. 5274, 2001



To obtain a high-brightness beam, we want to avoid scattering during acceleration.

Radial distribution of light intensity with crater in center creates strong focussing channel



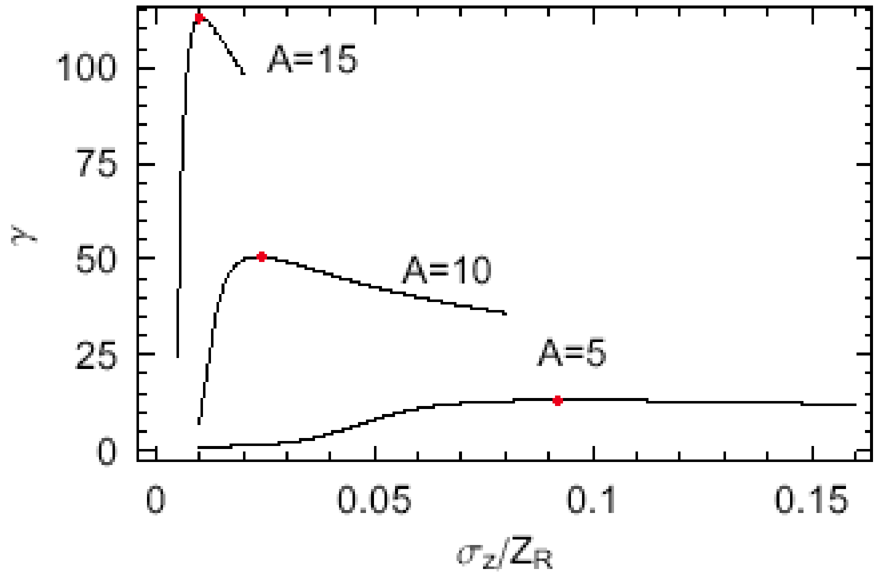
Two lowest order spatial Gaussian modes with orthogonal polarization

In plane wave

$$\gamma\beta_\perp = a$$

$$\gamma\beta_{||} = \frac{a^2}{2}$$

$$\gamma = 1 + \frac{a^2}{2}$$



$$\gamma_{opt} = 1 + \frac{a^2}{2}$$

$$Z_{Ropt} = \sigma_z\, a^2/2$$

Maximum acceleration is achieved at optimal z_R for fixed laser pulse length τ and laser energy a

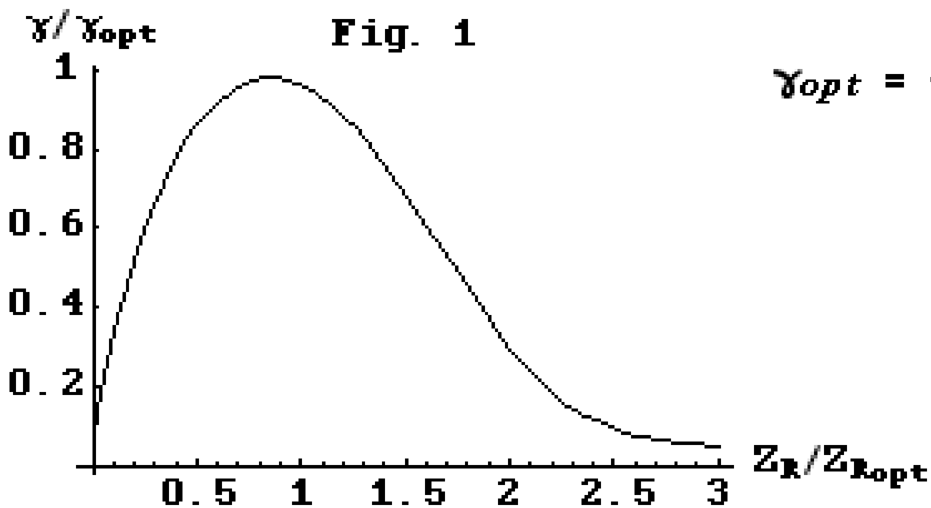


Fig. 1

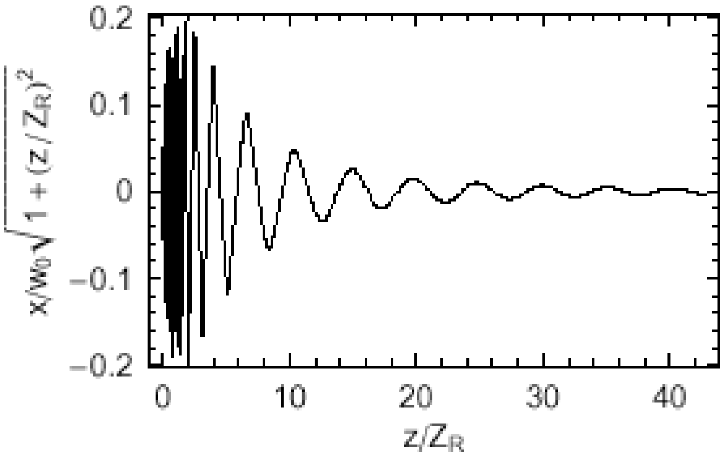
$$\gamma_{opt} = \frac{300}{\tau\,(fs)} \sqrt{E\,(J)\,\lambda\,(\mu)}$$

For constant laser energy, optimal regime is

$Z_R = \sigma_z\, a^2/2$

Transverse movement

$$\frac{x}{W} \;\ldots\; \sqrt{\frac{4\,\ldots}{\ldots}}$$

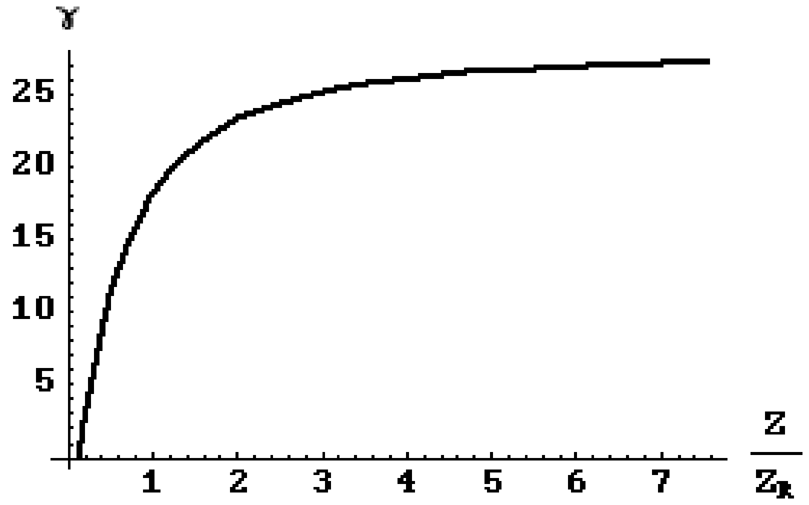


For estimates we will use

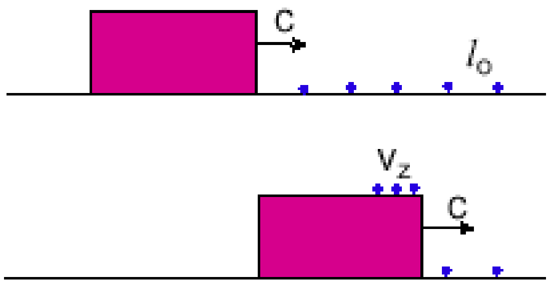
E_L=1 J, λ = 800 nm, σ_z/c = 8.5 fs (20 fs FWHM)

Optimal parameters

a = 8, Z_R = 42 μm, w_0 =3.2 μm



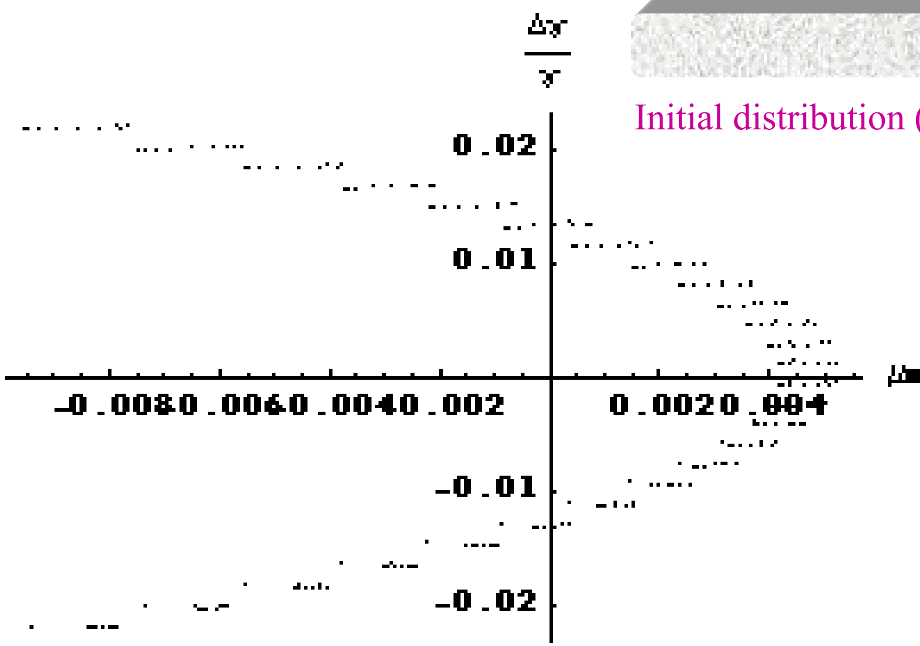
Bunch Length Compression



$$l_1 = l_0 \frac{c - v_z}{c} \approx l_0 \frac{1}{2\gamma^2}$$

In reality, the pulse is not rectangular, and its amplitude decreases during the interaction.

Calculated longitudinal phase space after acceleration



Initial distribution (particles at rest) 1x1x5 μm^3
where z_R=42 μm

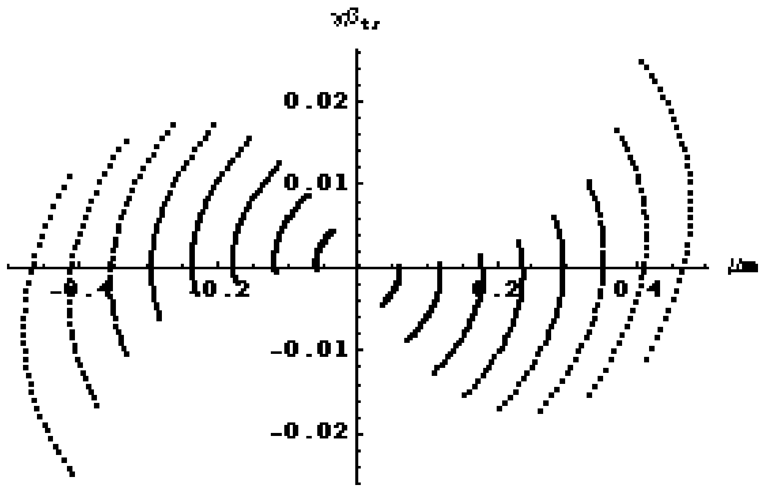
$$\varepsilon_{invLong} = \Delta\gamma\, \sigma_{long} = 1.5\ 10^{-3}\ \mu m$$

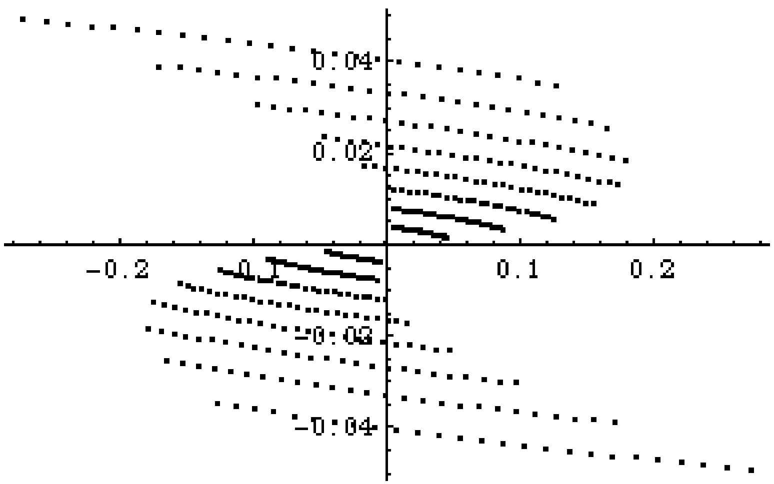
Calculated bunch length ~ 50 attoseconds

Calculated transverse phase space after acceleration

Initial distribution (particles at rest) 1x1x5 μm^3
where z_R=42 μm

Comparison of transverse phase space for two different sets of initial conditions





$$\varepsilon_{invTrans} = \gamma\, \sigma_r\, \sigma_\theta = 2\ 10^{-3}\ \textbf{mm mrad}$$

Estimate of the Number of Particles in the Bunch

If $\sigma_{\parallel} < \sigma_{\perp}/\gamma$,

$$E_{\perp} \sim \frac{eN}{\sigma_{\perp}^{2}}\gamma$$

$$F_{\perp} \sim \frac{eE_{\perp}}{\gamma^{2}} \sim \frac{e^{2}N}{\gamma\sigma_{\perp}^{2}}$$

Expansion time due to the finite emittance

$$t \sim \frac{\sigma_{\perp}}{v_{\perp}} \sim \frac{m\gamma\sigma_{\perp}}{p_{\perp}}$$

Transverse momentum due to space charge (SC)

$$\Delta p_{\perp}^{(SC)} \sim F_{\perp}t \sim mc\frac{r_{e}N}{\sigma_{\perp}}\frac{mc}{p_{\perp}}$$

We require

$$\Delta p_{\perp}^{(SC)} \sim p_{\perp}$$

$$N \sim \frac{\sigma_{\perp}}{r_{e}}\left(\frac{p_{\perp}}{mc}\right)^{2}$$

For our parameters we get $N \sim 2 \cdot 10^{5}$

Matching phase space

For SASE FEL operation it is necessary to optimize phase space beta function and energy spread. This cannot be done with conventional methods due to the unreasonable field required for these matching elements. Matching elements based on laser ponderomotive forces and plasma based focusing elements must be considered. This requires future work including simulations.

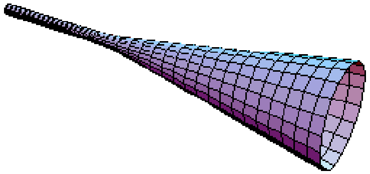
Summary

• With few joules of laser energy, ponderomotive laser acceleration and focusing might, with considerable probability of success, produce a 15 MeV electron beam with Fermi brightness of 2 10^{-6}.

• If the 15 MeV electron beam can be manipulated in phase space to be matched to the wiggler, then 10 keV X-ray SASE FEL will operate with 0.4 μm running wiggler wavelength

• If both the brightness and matching conditions are achieved, a transversally and longitudinally coherent X-ray source with 10^7 photons/pulse and pulse length of up to 50 attoseconds will be produced.

FEL small signal and small gain



If in the far field region one has fields from an external source and fields from spontaneous emission from particles

$$\vec{E}(\vec{r},t) = \vec{E}_L(\vec{r},t) + \sum \vec{E}_S(\vec{r},t-\tau_i)$$

$$\Delta \text{☞}_e = 2\int \vec{E}_L(\vec{r},t)\vec{E}_S(\vec{r},t-\tau_i)\,dtds \Rightarrow 2\int \vec{E}_L(\vec{r},\omega')\vec{E}^{*}s(\vec{r},\omega'-\omega_e)e^{i\omega'\tau_i}\,d\omega' ds$$

$$\vec{E}(\vec{r},\omega) = \vec{E}_L(\vec{r},\omega) + \{\vec{E}_S(\vec{r},\omega-\omega_e) + \frac{\partial \vec{E}_S(\vec{r},\omega-\omega_e)}{\partial \omega_e}\frac{\partial \omega_e}{\partial \text{☞}_e}\Delta \text{☞}_e\}\sum e^{-i\omega\tau_i} =$$

$$\vec{E}_L(\vec{r},\omega) + 2\frac{\partial \vec{E}_S(\vec{r},\omega-\omega_e)}{\partial \omega_e}\frac{\partial \omega_e}{\partial \text{☞}_e}\dot{N}_e\int e^{-i(\omega-\omega')\tau}d\tau\int \vec{E}_L(\vec{r},\omega')\vec{E}^{*}s(\vec{r},\omega'-\omega_e)\,d\omega' ds$$

$$\vec{E}(\vec{r},\omega) = \vec{E}_L(\vec{r},\omega)\{1 + 2\pi\dot{N}_e\frac{\partial \omega_e}{\partial \text{☞}_e}\frac{\partial}{\partial \omega_e}\int \left|\vec{E}_S(\vec{r},\omega-\omega_e)\right|^2 ds\}$$

$$g_o(\omega) = 2\pi\dot{N}_e\frac{\partial \omega_e}{\partial \text{☞}_e}\frac{\partial W_s(\omega-\omega_e)}{\partial \omega_e}; \qquad \int_{-\infty}^{\infty} W_s(\omega-\omega_e)\,d\omega = W_s$$

$$g_o(\omega) = 2\pi\dot{N}_e\frac{\partial W_s[\omega-\omega_e(\text{☞}_e)]}{\partial \text{☞}_e}$$