Beam Conditioning - Effects due to Strong Focusing Lattice

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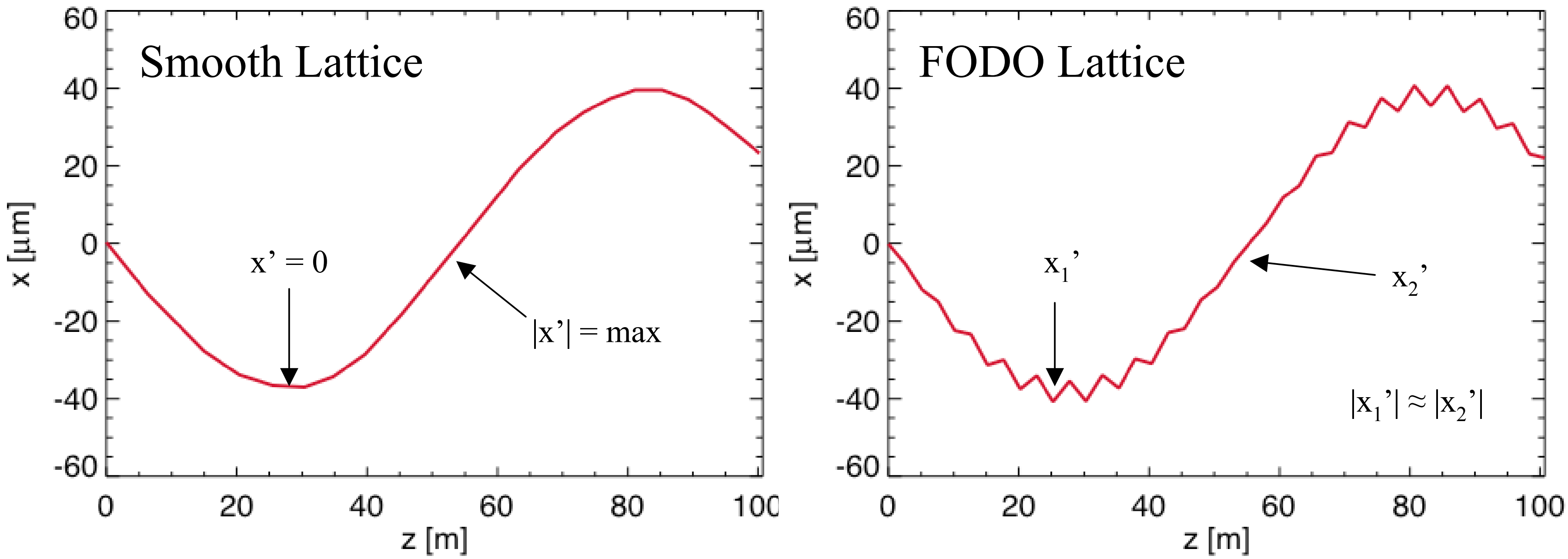
Overview + Motivation

- Motion in a Strong Focusing Lattice
- Beam conditioning
- Effect on the FEL performance
- Improving the performance

Emittance effects become important for VUV and X-ray FELs. However all these FELs depend on strong focusing to reduce the saturation length and to propagate the beam of a typical distance between 10 - 200 m. The cancellation of the betatron phase in the longitudinal velocity due to the natural focusing is no longer valid.

Smooth vs. Alternating focusing

Trajectory



The transverse momentum is most likely non-zero due to 2nd order focusing of the FODO lattice.

Transverse Momentum

FODO lattice with short quads much smaller than cell length $2L$.

1st Half ($0 < z < L$)

$$x'\big|_{F\to D} = \sqrt{\frac{I_x}{\beta_0}}\left[\cos(\phi_0) - \alpha_0 \sin(\phi_0)\right]$$

Sawtooth behaviour of alternating gradient focusing

2nd Half ($L < z < 2L$)

$$x'\big|_{D\to F} = \sqrt{\frac{I_x}{\beta_0}}\left[\left(1 + \frac{2\alpha_1 L}{\beta_1}\right)\cos(\phi_0) + \left(1 - \frac{2L}{\alpha_1\beta_1}\right)\alpha_0 \sin(\phi_0)\right]$$

Small phase advance per FODO cell $\left(L << \bar{\beta} = (\beta_0 + \beta_1)/2\right)$

$$\left(x'\big|_{F\to D}\right)^2 + \left(x'\big|_{D\to F}\right)^2 = \frac{I_x}{\bar{\beta}} + O\left(\left[\frac{2L}{\bar{\beta}}\right]^2\right)$$

Longitudinal Velocity

Longitudinal velocity, averaged over one FODO cell

$$\boxed{\beta_z = 1 - \frac{1+a_u^2}{2\gamma^2} - \frac{(x')^2 + (y')^2}{2} \approx 1 - \frac{1+a_u^2}{2\gamma^2} - \frac{I_x + I_y}{2\bar{\beta}}}$$

Transverse dependence of a_u is negligible, because the average β-function is much smaller than in the case of natural focusing.

On the other hand the longitudinal velocity is not modulated on the scale of the betatron-period but the FODO cell length instead.

Compensation

If the FODO cell length is shorter than the gain length the delay of the longitudinal velocity can be regarded to be constant for the FEL process. Emittance compensation is possible with:

$$\boxed{\Delta\gamma = \frac{\lambda_u \gamma}{4\lambda\bar{\beta}}\left(I_x + I_y\right)}$$

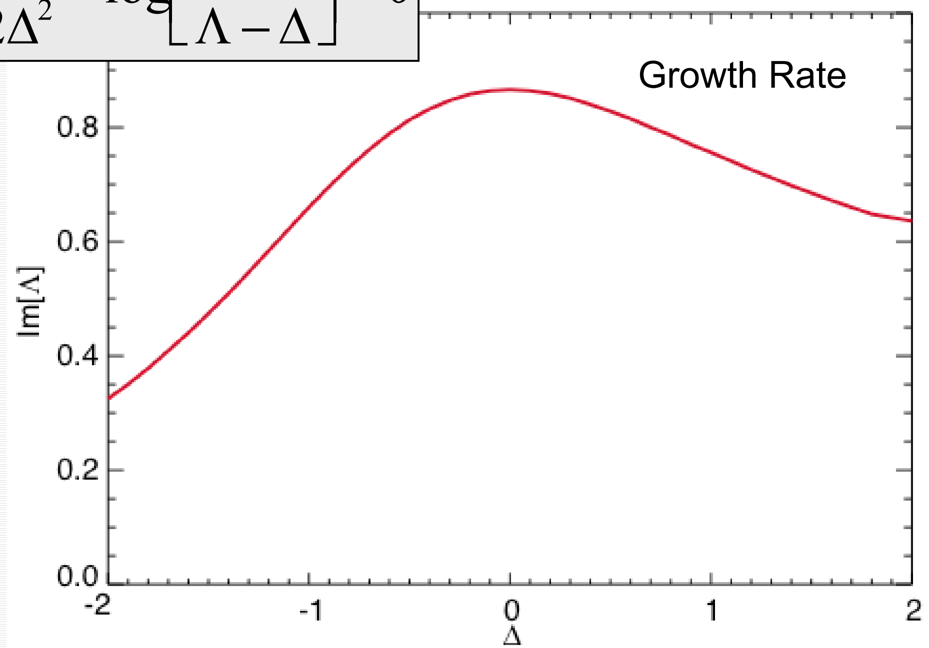
For LCLS and a particle with an amplitude $\gamma I_x = 1$ µrad, the required energy shift is 5 MeV (≈ 0.04%).

1D Model

Any degree of over- or under-compensation yield an asymmetric distribution in ‘energy’.

1D dispersion equation for water-bag beam

$$(\Lambda-\delta)\left(\Lambda^2-\Delta^2\right)+1+\frac{\Lambda}{\Delta}-\frac{\Lambda^2-\Delta^2}{2\Delta^2}\log\left[\frac{\Lambda+\Delta}{\Lambda-\Delta}\right]=0$$



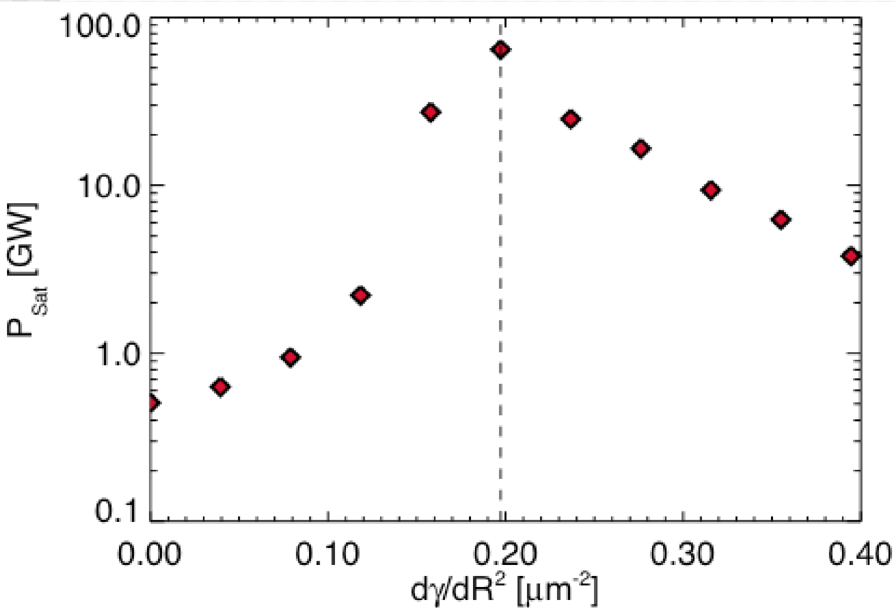
δ = normalized detuning

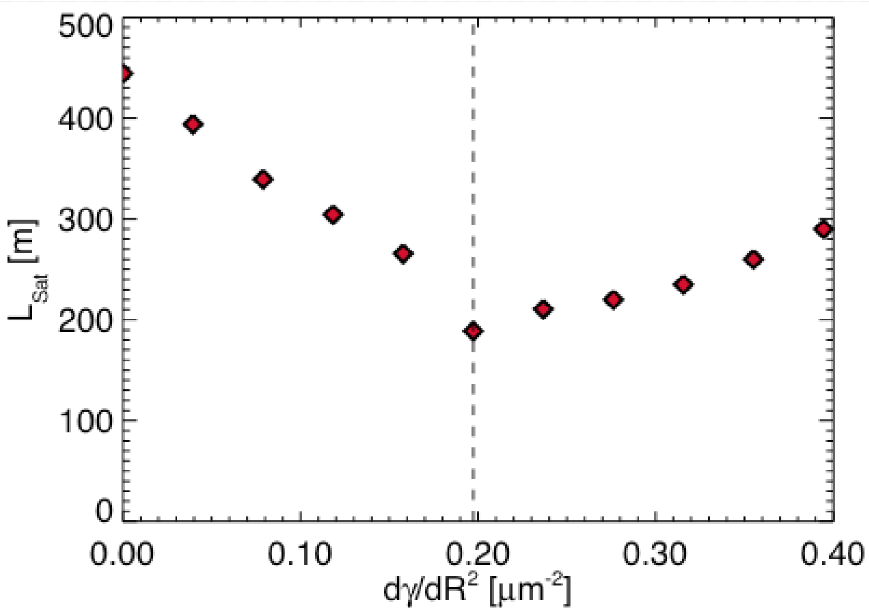
Δ = width of normalized energy distribution, including compensation (optimum compensation for Δ = 0)

3D Simulations

Example:
TESLA-FEL with ε_n = 9 mm·mrad.

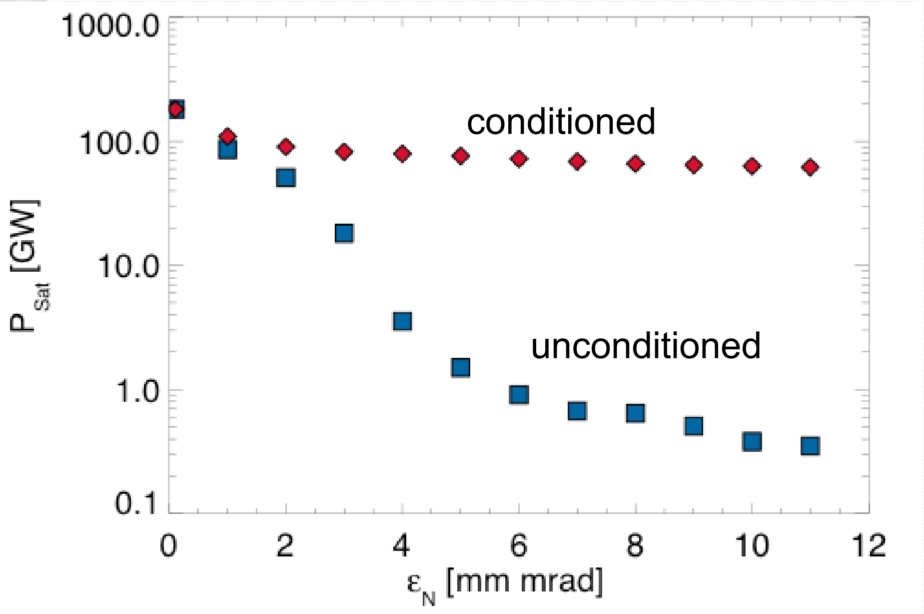
Optimum conditioning is $d\gamma/dR^2 = 0.197\ \mu m^{-2}$, where R^2 is the square sum of the amplitudes in x and y within the undulator.

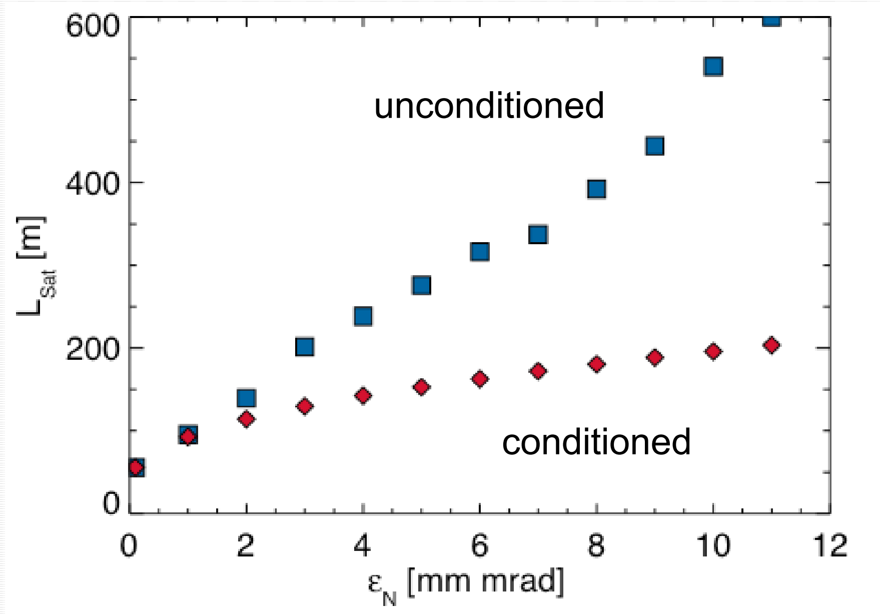




Efficiency

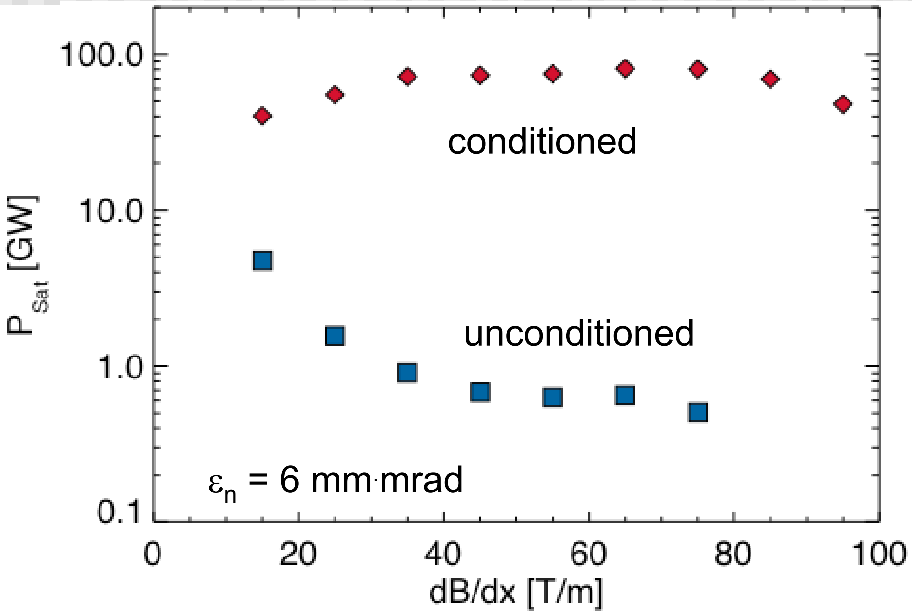
Dependence of saturation power and length on emittance is $\varepsilon_n^{-1/3}$ and $\varepsilon_n^{1/3}$, respectively, agreeing with the scaling of the electron density in the FEL parameter ρ. Phase spread effects due to emittance are completely removed.

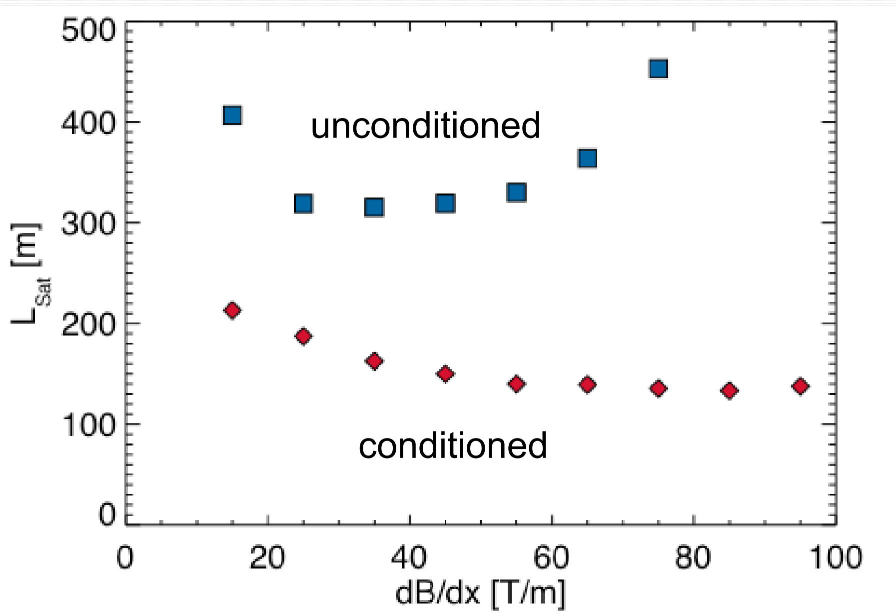




Break-down

Strong focusing is optimized to balance electron density and emittance effects in the resulting FEL amplification. With the latter removed by compensation, focusing can be increased. Break-down of emittance compensation, when betatron-phase advance per FODO-cell becomes large.

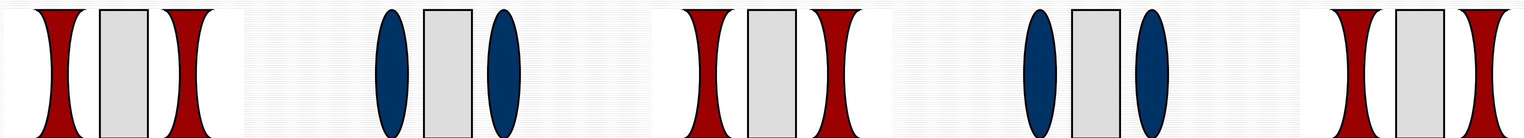




Conditioning

Same beam conditioning methods can be used, although the required strength is larger.

Efficiency of TM_{210} conditioning line is low



TM_{210} cavity with field $E(x,y)=E_0(x^2-y^2)$ and length L_E

$$\boxed{\Delta\gamma = \frac{eNL_E E_0}{mc^2}\frac{\beta_{\max}-\beta_{\min}}{2}\left(I_x + I_y\right)}$$

Example: LCLS (conditioned at 5 GeV)

$\Delta\gamma=10$, $\gamma I_x=1$ μrad, $\beta_{max}=100$ m, $\beta_{min}=1$ m, $E_0=10$ MV/m, $L_E=1$ m

Conditioning line > 2km

Conclusion

- Emittance compensation can be applied for strong focusing as long as
  - Phase advance per cell is small,
  - Cell length is smaller as or comparable to FEL gain length.
- Typical energy correlation of about 0.1% per 1mm·mrad.
- Standard conditioning lines as long as main linac.