KJK 9/23/03

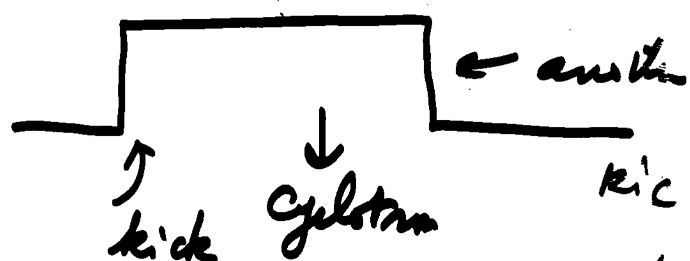
Motion in Solenoid

look in Larmor frame

$$\frac{d\phi}{dz} = \frac{qB}{p_z} = \kappa$$

$$x'' + \kappa^2 x = 0$$

Lab frame



$$x(z) = x_0 \cos\phi + \frac{x_0'}{\kappa} \sin\phi$$

$$x' = -\kappa x_0 \sin\phi + x_0' \cos\phi$$

$$\kappa\beta \gg 1 \rightarrow$$

Now $\phi = \phi_0 \left(1 - \frac{\Delta\gamma}{\gamma}\right)$, $\phi_0 = n\pi$

$$\rightarrow \alpha_f = \alpha_0$$

$$\alpha_f' = \alpha_0' + \underbrace{(\kappa \alpha_0) 2\pi \delta}_{\text{chromatic lens}}$$

$$\kappa^2 = k_0^2 + 2k_0^2 \delta$$

Eq for β function

$$\frac{1}{2}\beta'' + \kappa^2 \beta - \frac{1}{\beta}\left(1 + \frac{\beta'^2}{4}\right) = 0$$

for $\delta = 0$, $\beta = \beta_0$ with

$$\beta_0 = \frac{1}{\kappa_0}$$

$$\beta = \beta_0 + \beta_1$$

$$\frac{1}{2}\beta_1'' + \overbrace{k_0^2\left(1 + \frac{1}{k_0^2 \beta_0^2}\right)}^{2\kappa_0^2}\beta_1 = -2\kappa_0^2 \beta_0 \delta$$

solution for $\beta_1(0) = \beta_1'(0) = 0$
(all particles with same ellipse)

$$\beta_1 = -\beta_0\, \delta \,(1 - \cos 2\varkappa_0 s)$$

if $\varkappa_0 L = n\pi$ then

all particles end up with same emittance. (A. Wolski)

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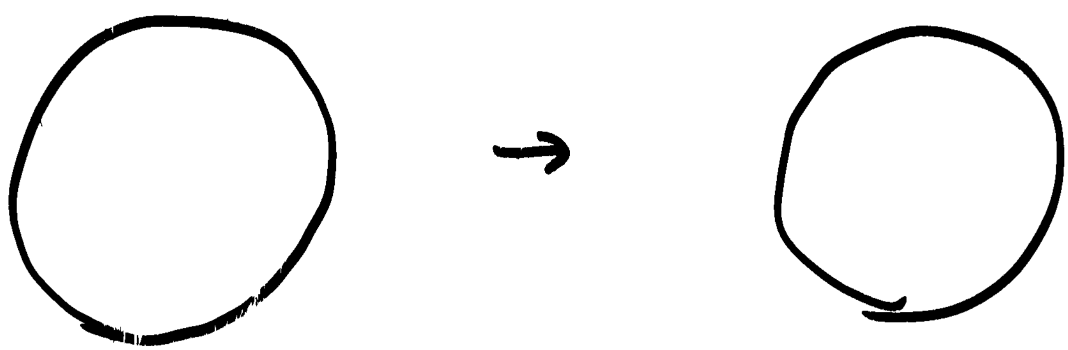
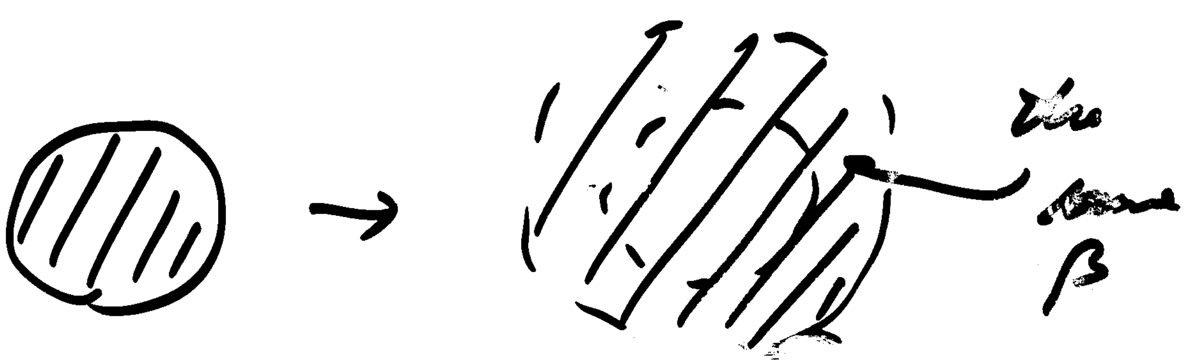
if



But



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Further Further Remark 9/23/03 KJK

Continue with

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos\phi , & \frac{\sin\phi}{\varkappa} \\ -\varkappa\sin\phi , & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

$$\phi = \varkappa z$$

$$\phi_f = \varkappa L = 2n\pi(1-\delta) , \quad \delta = \frac{\delta\gamma}{\gamma}$$

$$= 2n\pi - \Delta \quad , \Delta = 2n\pi\delta$$

$$\Delta \ll 1$$

$$\Rightarrow \quad x_f = \left(1 - \frac{\Delta^2}{2}\right) x_0 - \frac{\Delta}{\varkappa} x_0'$$

$$x_f' = \varkappa\Delta x_0 + \left(1 - \frac{\Delta^2}{2}\right) x_0'$$

Assume $\langle x_0 x_0' \rangle = \langle \Delta \rangle = 0 \Rightarrow \langle x_f x_f' \rangle = 0$

$$\langle x_f^2 \rangle = \left(1 - \langle \Delta^2 \rangle\right) \langle x_0^2 \rangle + \frac{\langle \Delta^2 \rangle}{\varkappa^2} \langle x_0'^2 \rangle$$

$$\langle x_f'^2 \rangle = \langle \Delta^2 \rangle \varkappa^2 \langle x_0^2 \rangle + \left(1 - \Delta^2\right) \langle x_0'^2 \rangle$$

Let $\varkappa\beta = 1$ (Matching) then $\varkappa \langle x_0^2 \rangle = \frac{\langle x_0'^2 \rangle}{\varkappa} = \varepsilon_0$

$$\langle x_f^2 \rangle = \frac{\varepsilon_0}{\varkappa} , \quad \langle x_f'^2 \rangle = \varkappa \varepsilon_0$$

$\therefore \quad \varepsilon_f = \varepsilon_0$ (Perfect Condition)

OR even simpler, do not make Δ small assumption. but only

$$\langle \sin 2\phi_f \rangle = 0$$

then

$$\langle x_f^2 \rangle = \left\langle x_0^2 \cos^2\phi + \frac{\sin^2\phi}{\chi^2} x_0'^2 \right\rangle$$

For $\chi\beta = 1$

$$\cancel{\langle x_f^2 \rangle} = x_0^2 = \chi \varepsilon_0$$

$$\langle x_f'^2 \rangle = \frac{\varepsilon_0}{\chi}$$

Delay

$$\int_0^L \frac{\langle x'\rangle^2}{2}\, dz = \int \left(\varkappa^2 \langle x_0^2 \rangle \sin^2\phi + \langle x_0'^2 \rangle \cos^2\phi \right) dz$$

$$= \frac{1}{2}\left(\frac{\varkappa L \mathcal{E}_0}{2} + \frac{\varkappa L \mathcal{E}_0}{2} \right) = \frac{\varkappa L \mathcal{E}_0}{2}$$

→ conditioning

solenoid

$(\Delta z, \delta)$

$$\boxed{H = \varkappa_0 (1-\delta) J + \frac{(\Delta z)^2}{2}\left[\delta(z - z_1) + \delta(z - z_2) \right]}$$

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Thus $\dfrac{\partial H}{\partial t} = \dot{\iota}$

must

$$\frac{dz}{dz} = -\varkappa_0 J \quad \text{delay}$$

$\langle (1-\delta) J \rangle = \langle J \rangle$ conserved