

ELECTRON BEAM CONDITIONING FOR FEL APPLICATIONS*

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OUTLINE

- FEL resonance & why we need conditioning?
- Electron dynamics
 - Wiggle & betatron motion
- Examples
 - Conditioning using external fields
 - Self conditioning
- Summary

FEL RESONANCE

- FEL interaction governed by electron-ponderomotive wave phase

$$\psi(t) = \int_0^t d\tau [\omega - (k + k_w) v_z(\tau)]$$

$$(\omega \approx ck \ \& \ k_w = 2\pi/\lambda_w)$$

- Resonance is *not* smeared if

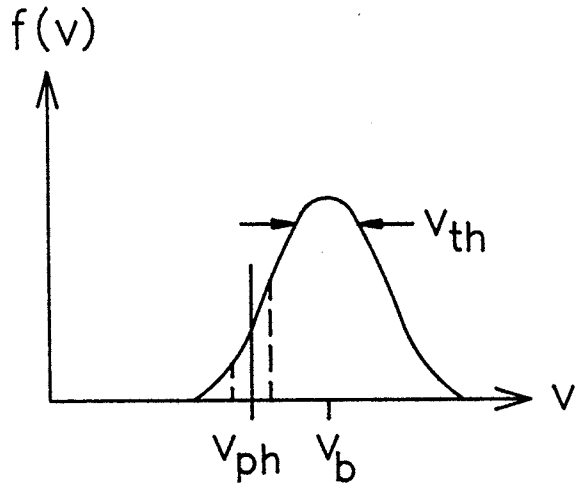
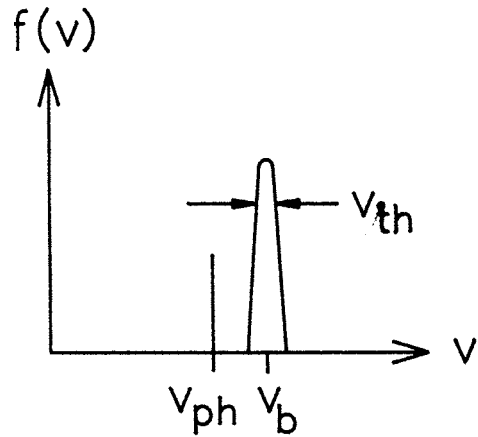
$$\frac{|\delta v_z|}{v_z} \ll \frac{\text{wavelength}}{\text{e-folding length}}$$

\Rightarrow *Bunching on optical scale*

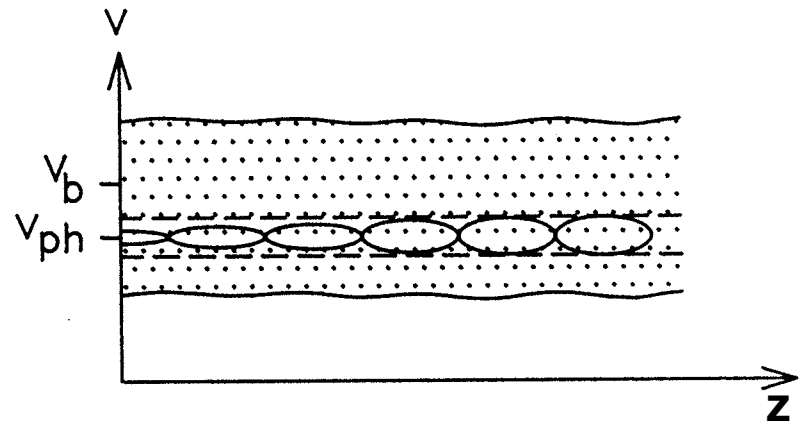
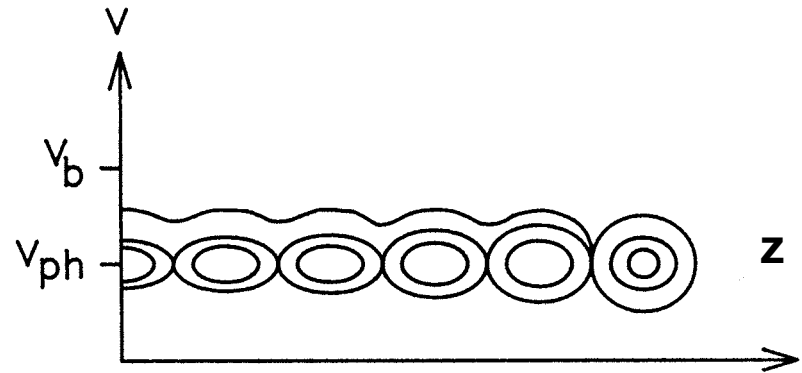
BEAM INTERACTION WITH PONDEROMOTIVE WAVE

cold beam vs. warm beam

Velocity Distribution



Phase Space



WHY WE NEED CONDITIONING?

- FEL interaction:
 - **Cold beam regime**: coupled +ve & -ve energy modes \Rightarrow exponential growth
 - **Warm beam** regime: inverse Landau damping

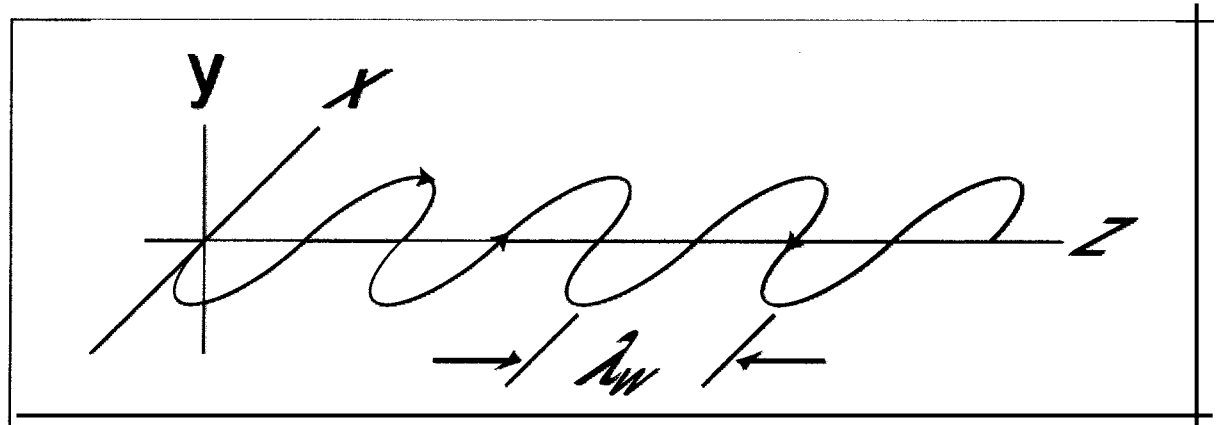
- Efficient operation limits emittance & energy spread; e.g.
 - High energy, short wavelength ($1 < \lambda[\text{\AA}] < 3000$) FELs*
 - Low voltage, compact FELs

* *Committee on free electron lasers & other advanced coherent light sources*, National Research Council, 1994

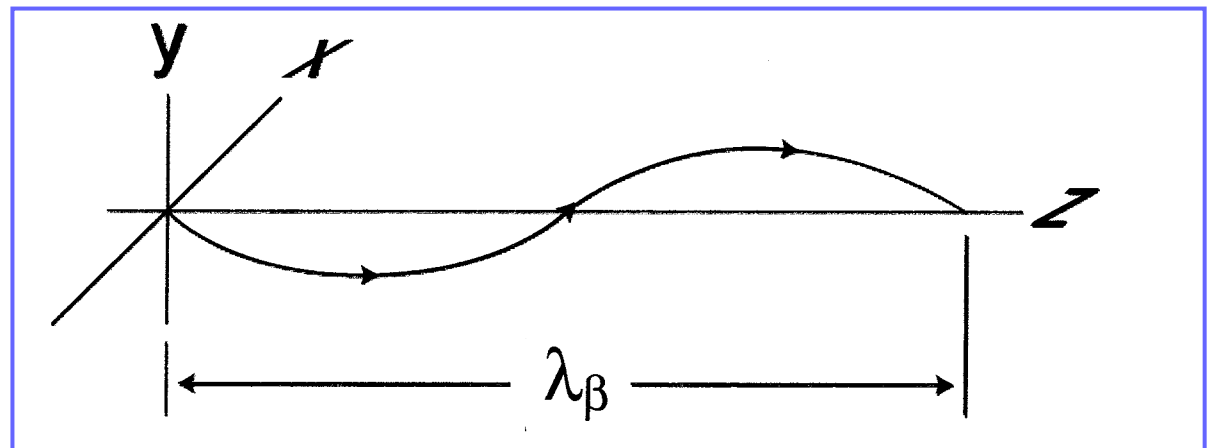
WIGGLE & BETATRON MOTION

$$\mathbf{A}_w = A_w \cosh(k_w y) \sin(k_w z) \mathbf{e}_x$$

Wiggle motion in horizontal plane



Betatron motion in vertical plane



VELOCITY SPREAD

- Axial velocity

$$\beta_z = \beta_{z0} + \delta\beta_z$$

- Common axial velocity

$$\beta_{z0} = 1 - \frac{1 + a_w^2 / 2}{2\gamma_0^2}$$

- Deviation in axial velocity

$$\delta\beta_z = \frac{1 + a_w^2 / 2}{\gamma_0^2} \frac{\delta\gamma}{\gamma_0} - \frac{1}{2} k_{\beta 0}^2 y_\beta^2 + \frac{k_{p0}^2}{2\beta_{z0}\gamma_{z0}\gamma_0^{1/2}} y_\beta^2$$

Energy
spread

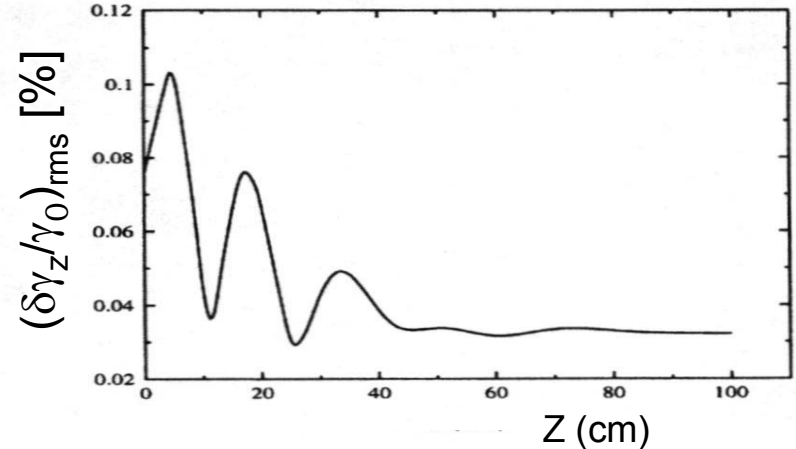
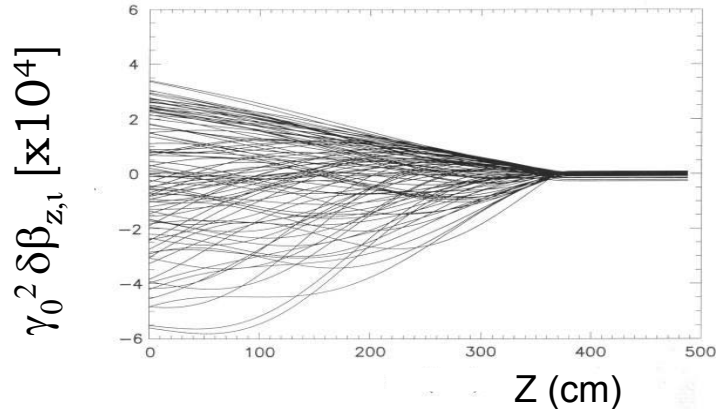
Betatron
oscillation

Space
charge

BEAM CONDITIONING WITH EXTERNAL FIELDS

- Remove axial velocity spread by introducing correlation between betatron amplitude & energy
- Conditioning with periodic array of FODO channels & *cavity* modes, *waveguide* modes or *laser beam*

	10 μm	Conditioned	3000 \AA	Conditioned	500 \AA	Conditioned
$mc^2\gamma_0$ (MeV)	54	54	483	153	1004	304
ϵ_n (m)	$8 \times 10^{-4}\pi$	$8 \times 10^{-4}\pi$	$5 \times 10^{-5}\pi$	$5 \times 10^{-5}\pi$	$2 \times 10^{-5}\pi$	$2 \times 10^{-5}\pi$
λ_β (m)	8.9	8.9	20	12	34	19
λ_w (cm)	8.0	8.0	4.8	2.8	3.7	2.0
B (T)	0.25	0.25	1.0	0.52	1.26	0.66
$L_G/2$ (m)	8.0	1.6	3.1	1.4	4.6	2.1
$mc^2\Delta\gamma_c$ (MeV)		3.6		2.0		2.1
$mc^2\gamma_c$ (MeV)		54		51		51
N		10		20		50
N_c		1		10		10
L_c (m)		10		20		50



Sessler *et al.*, Phys. Rev. Lett. **68**, 309 (1992); Sprangle *et al.*, Phys. Rev. Lett. **70**, 2896 (1993); Liu & Neil, Phys. Rev. Lett. **70**, 3557 (1993)

SELF CONDITIONING

- Betatron oscillations due to wiggler gradients

$$y'' = -\frac{1}{2} \left(\frac{a_w k_w}{\gamma_0 \beta_{z0}} \right)^2 y \equiv -k_{\beta 0}^2 y$$

- With self-electric & -magnetic fields

$$y'' = -k_{\beta 0}^2 y + \frac{k_p^2}{\gamma_0 \gamma_{z0}^2 \beta_{z0}^2} y$$

$$k_{\beta} = k_{\beta 0} (1 - SFP)^{1/2}$$

- *Self field parameter* $SFP \equiv \left(\frac{k_{p0} / k_{\beta 0}}{\beta_{z0} \gamma_{z0} \gamma_0^{1/2}} \right)^2$, cf *Brillouin flow*

BEAM ENVELOPE SIMULATION

- Betatron oscillations in vertical plane with *no* space charge



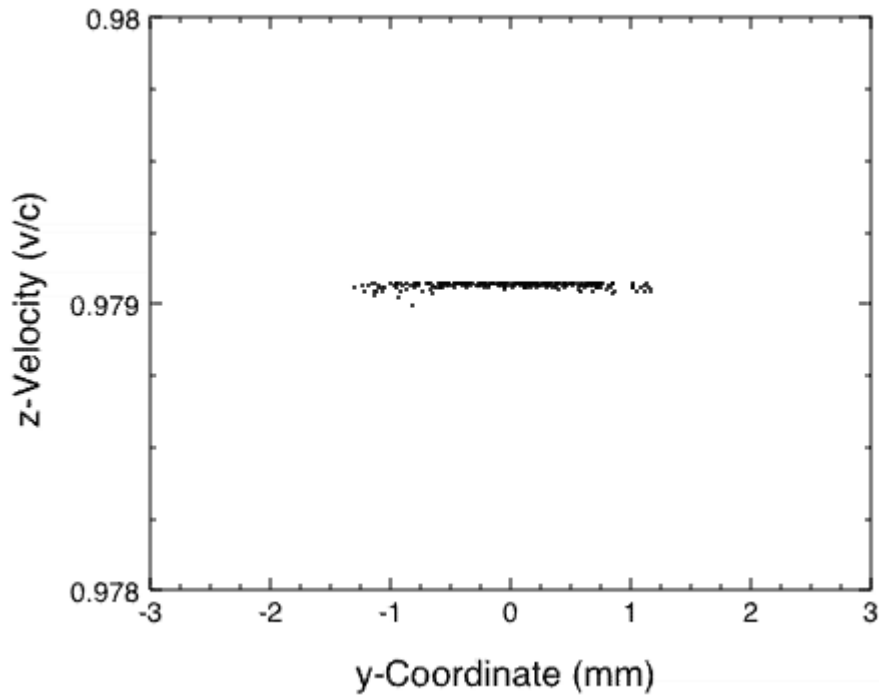
$$\delta\beta_z \approx -\frac{1}{2}k_\beta^2 y_\beta^2, \quad k_\beta = k_{\beta 0}(1-SFP)^{1/2}, \quad SFP = \left(\frac{k_p / k_{\beta 0}}{\beta_{z0} \gamma_{z0} \gamma_0^{1/2}} \right)^2$$

- Betatron oscillations in vertical plane with space charge

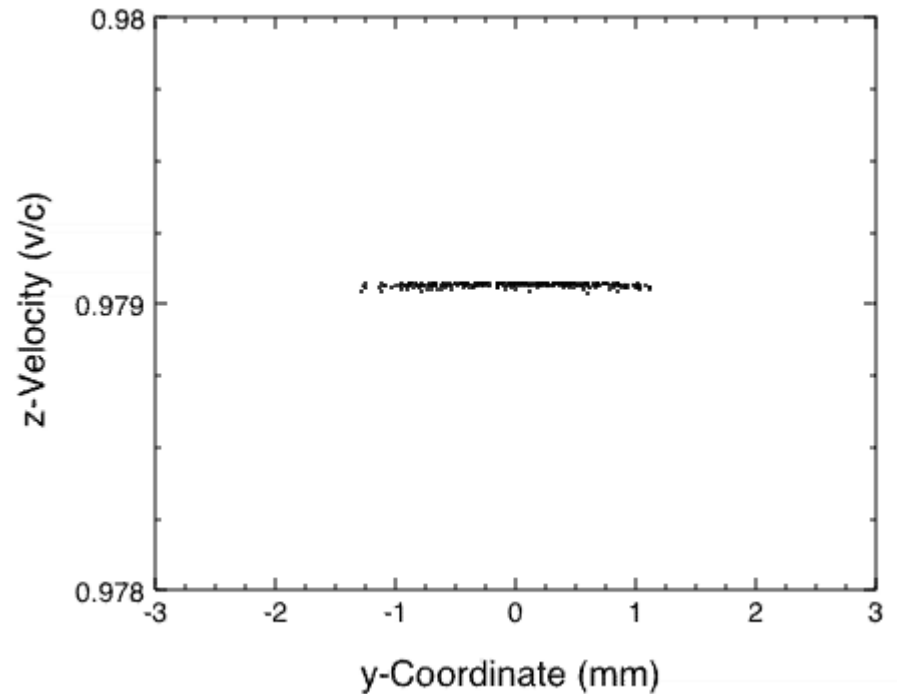


WIGGLER FLOW

•Current = 0

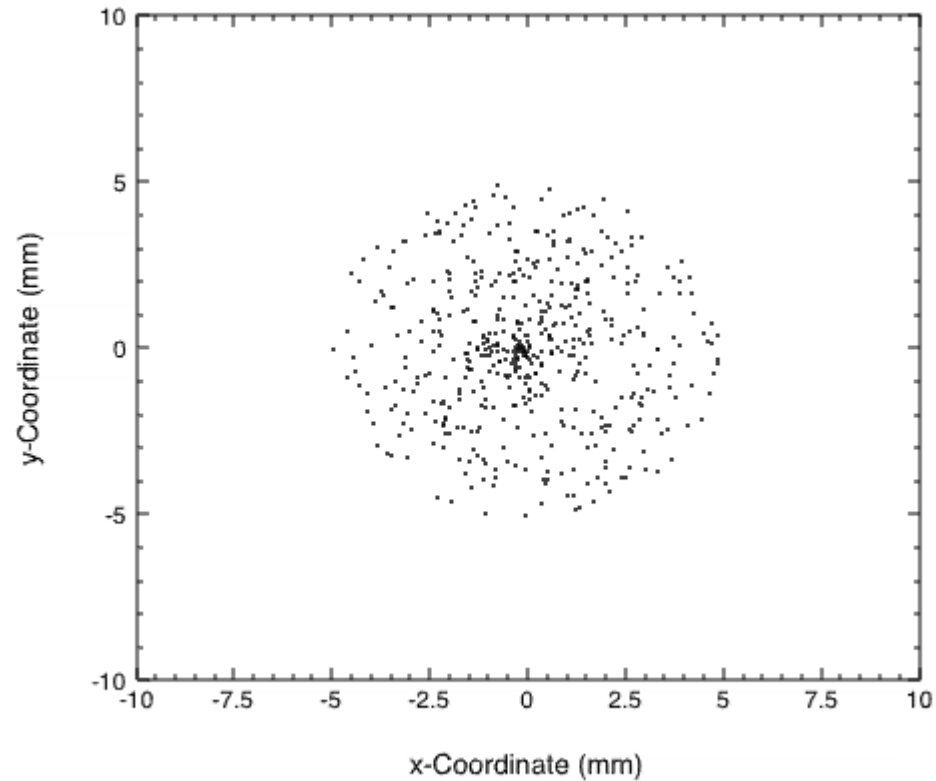


•Current = 210 A



BRILLOUIN FLOW

- Current = 120 A

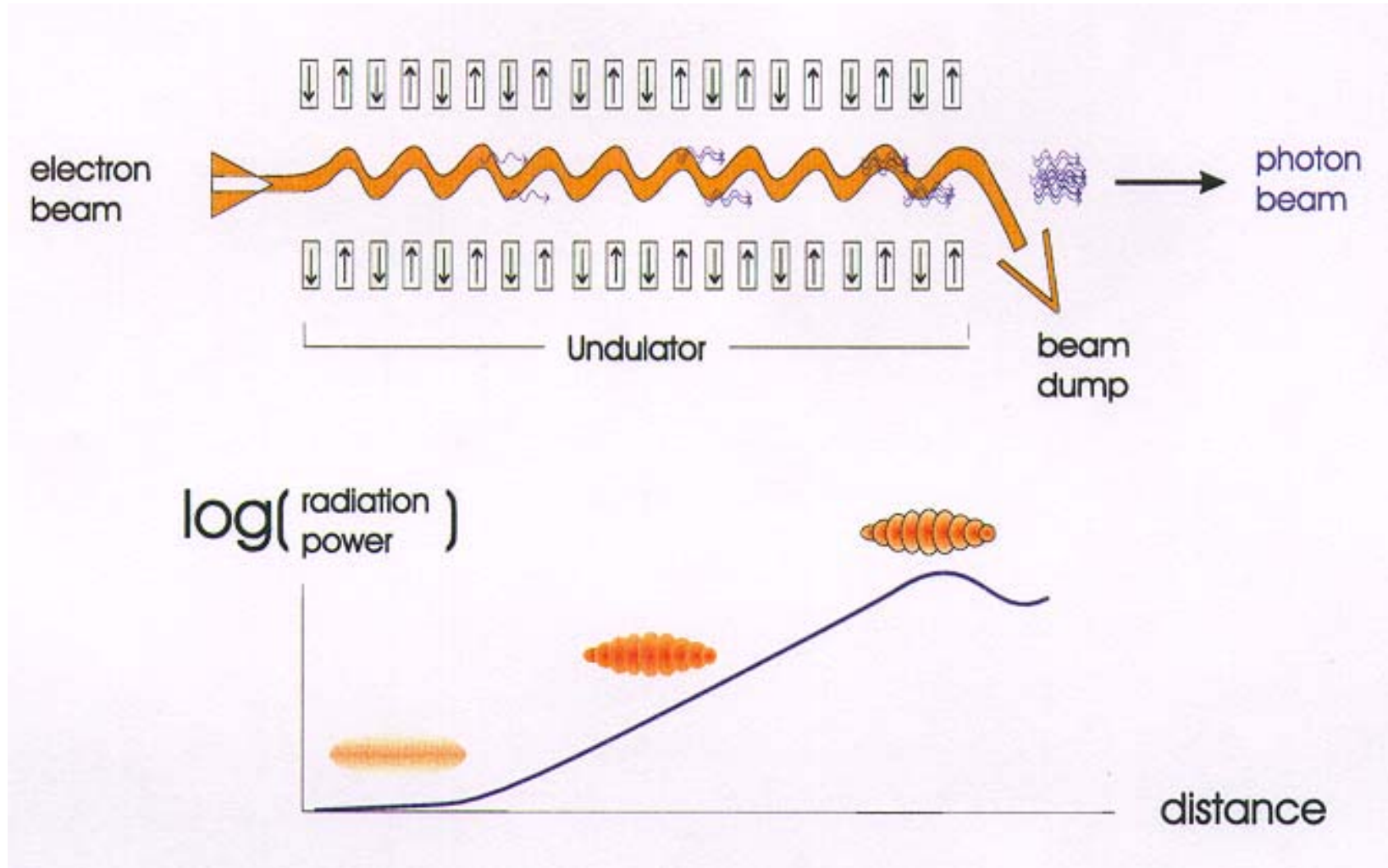


CONDITIONING: SUMMARY

- Conditioning improves electron beam quality for FELs
- For short wavelengths conditioning requires external fields
- For low-voltage, high-current FELs self-conditioning appears to be possible

Back-up View Graphs

SELF AMPLIFIED SPONTANEOUS EMISSION*



ELECTRON DYNAMICS

- Axial velocity

$$\beta_z \approx 1 - \frac{1}{2\gamma^2} - \frac{\beta_\perp^2}{2}$$

- Wiggler field

$$\mathbf{A}_w = A_w \cosh(k_w y) \sin(k_w z) \mathbf{e}_x$$

- Transverse velocity

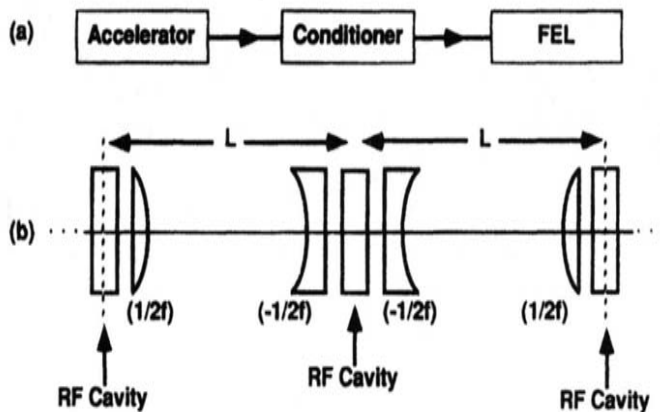
$$\boldsymbol{\beta}_\perp = \frac{a_w}{\gamma} (1 + k_w^2 y^2 / 2) \sin(k_w z) \mathbf{e}_x + \frac{dy}{dz} \mathbf{e}_y$$

BEAM CONDITIONING WITH CAVITIES

- Remove axial velocity spread by introducing correlation between betatron amplitude & energy
- Conditioning with periodic array of FODO channels & *cavities* excited in TM_{210} mode (~ 5 GHz)

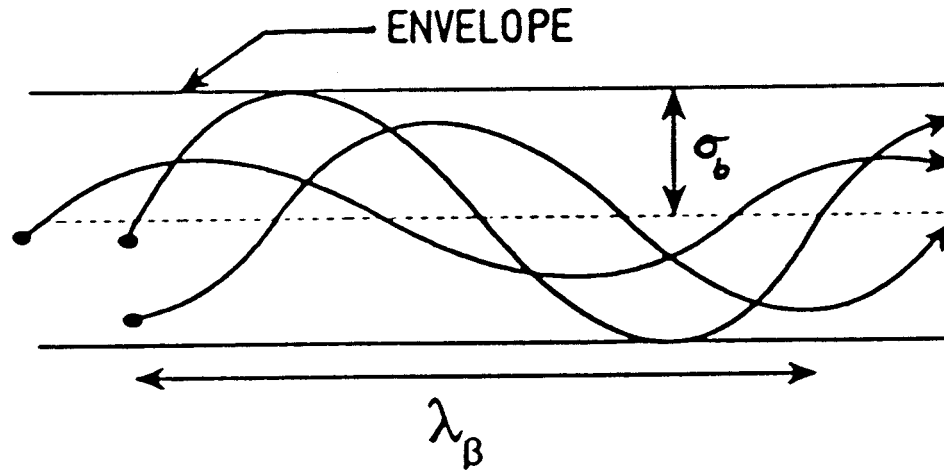
TABLE I. Parameters for several example FEL designs, with and without a conditioned beam. Current is fixed at $I \sim 300$ A, with energy spread $\sigma/\gamma \sim 4.4 \times 10^{-4}$. In each case k_w was varied to minimize L_G .

	10 μm	Conditioned	3000 \AA	Conditioned	500 \AA	Conditioned
$mc^2\gamma_0$ (MeV)	54	54	483	153	1004	304
ε_n (m)	$8 \times 10^{-4}\pi$	$8 \times 10^{-4}\pi$	$5 \times 10^{-5}\pi$	$5 \times 10^{-5}\pi$	$2 \times 10^{-5}\pi$	$2 \times 10^{-5}\pi$
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$L_G/2$ (m)	8.0	1.6	3.1	1.4	4.6	2.1
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N		10		20		50
N_c		1		10		10
L_c (m)		10		20		50



¹Sessler *et al.*, Phys. Rev. Lett. **68**, 309 (1992)

ELCTRON BEAM ENVELOPE



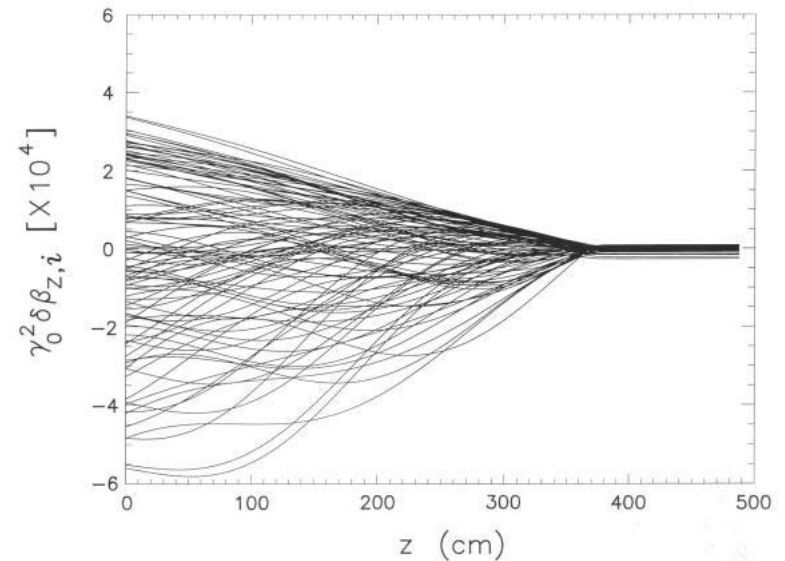
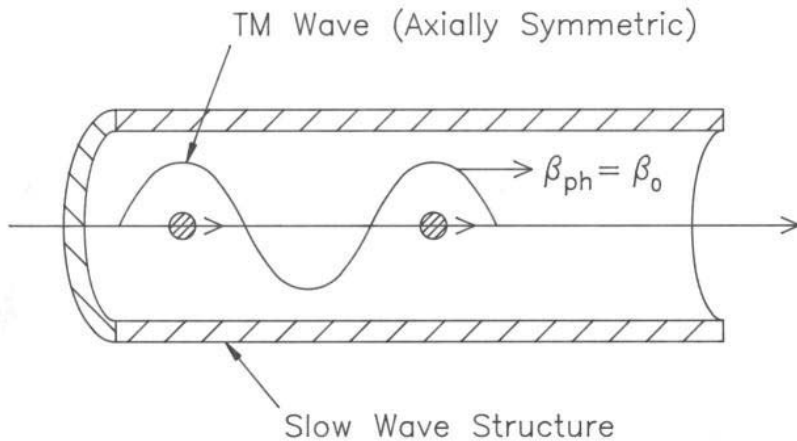
- Spread $\delta\beta_z \approx -\frac{1}{2}k_\beta^2 y_\beta^2$
- Matched beam $k_\beta \sigma_b^2 = \varepsilon$

$$|\delta\beta_z| < \frac{1}{2}k_\beta \varepsilon \quad : \text{surprise}$$

$$k_\beta = \frac{a_w k_w}{\sqrt{2} \beta_{z0} \gamma_0} (1 - SFP)^{1/2}$$

BEAM CONDITIONING IN WAVEGUIDE

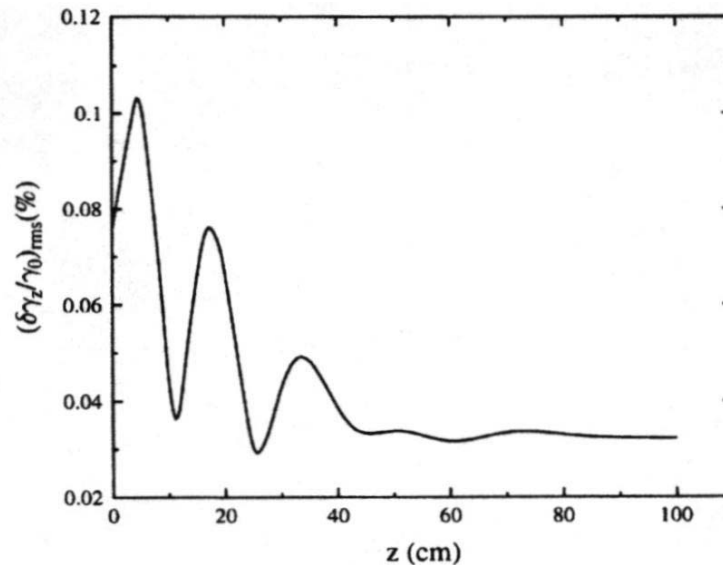
- Introduce correlation between betatron amplitude & energy
- Conditioning with TM *waveguide* mode (~ 15 GHz)



¹⁾Sprangle *et al.*, Phys. Rev. Lett. **70**, 2896 (1993)

BEAM CONDITIONING WITH LASER BEAM

- Introduce correlation between betatron amplitude & energy
- Conditioning with TEM₁₀ Gaussian *optical* beam (~1 or 10 μm laser)



¹⁾Liu & Neil, Phys. Rev. Lett. **70**, 3557 (1993)

BEAM QUALITY

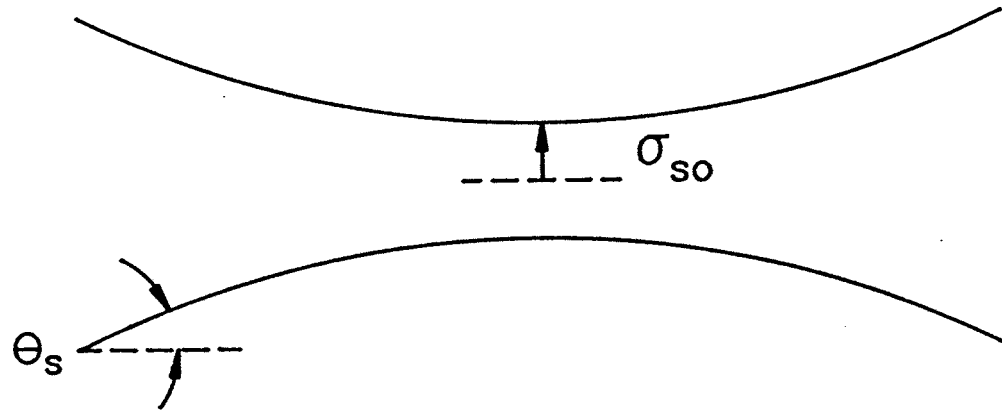
- FEL interaction dominated by **axial** velocity
- Adopt scaled thermal velocity S is measure of beam quality

$$S \equiv \frac{v_{th,z}}{\langle v_z \rangle - v_{ph}}$$

- v_{ph} depends on FEL parameters, including electron **density**
 - Warm beam regime: $S \geq 1$
 - Cold beam regime: $S \ll 1$

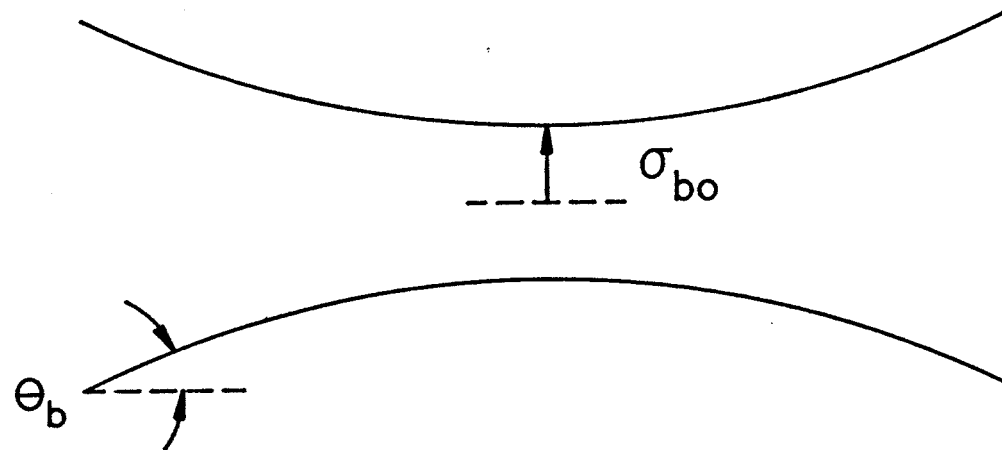
EMITTANCE & DIFFRACTION

Optical beam expands
due to *diffraction*
 $\Delta k \Delta x \approx 1$



• wave

Electron beam expands
due to *emittance*
 $\Delta \beta \Delta x \approx \varepsilon$



VLASOV-MAXWELL MODEL

- Wiggler & optical fields

$$\mathbf{A}_w = A_w \cosh(k_w y) \sin(k_w z) \mathbf{e}_x$$

$$\mathbf{A}_s = \frac{1}{2} A_s (y, z) \exp[i(kz - \omega t)] \mathbf{e}_x + c.c.$$

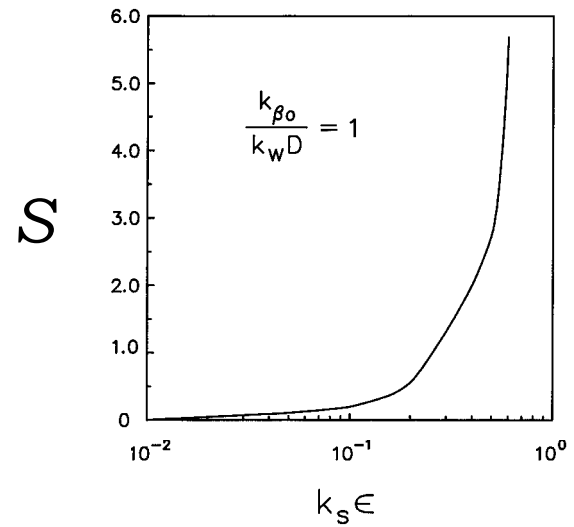
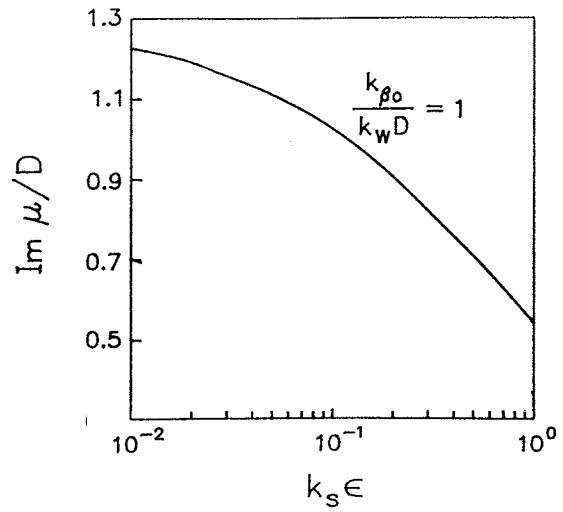
- Electron distribution function

$$F(E, P_x, J) = \frac{I_b / (\beta_{z0} I_0) \exp[-(E - E_0)^2 / (\sigma_\gamma mc^2)^2]}{2\pi m c r_e \epsilon_N \sqrt{\pi} \sigma_\gamma mc^2} \\ \times \delta(P_x) \exp(-k_{\beta 0} J / mc \epsilon_N k_\beta)$$

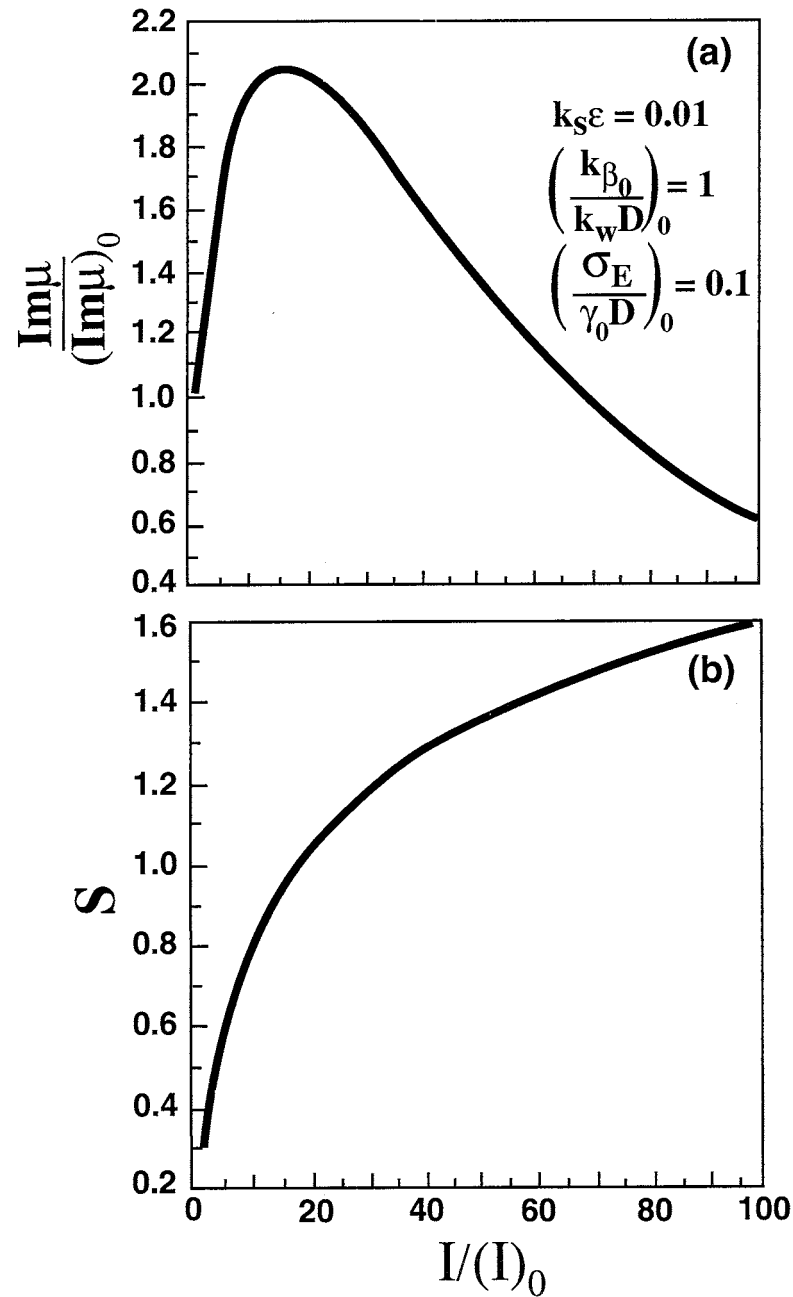
$J = \iint dy dp_y / 2\pi$: \perp phase space area

- Integrate over unperturbed orbits

EFFECT OF EMITTANCE



BEAM COMPRESSION (cont.)



COMPACT FELs

- One approach to compact X-Ray FELs is to use a **low-voltage** (10's MV) beam with an **ultra-intense laser** as wiggler; $\lambda \approx \lambda_w / (4\gamma^2)$
- Space charge effects can be limiting factor in this approach

COMPACT FEL (cont.)

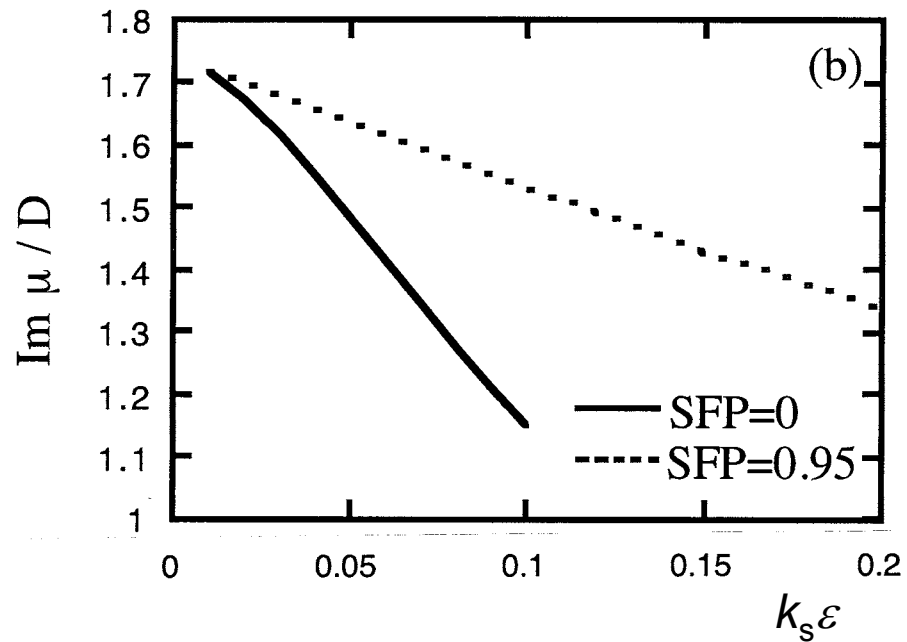
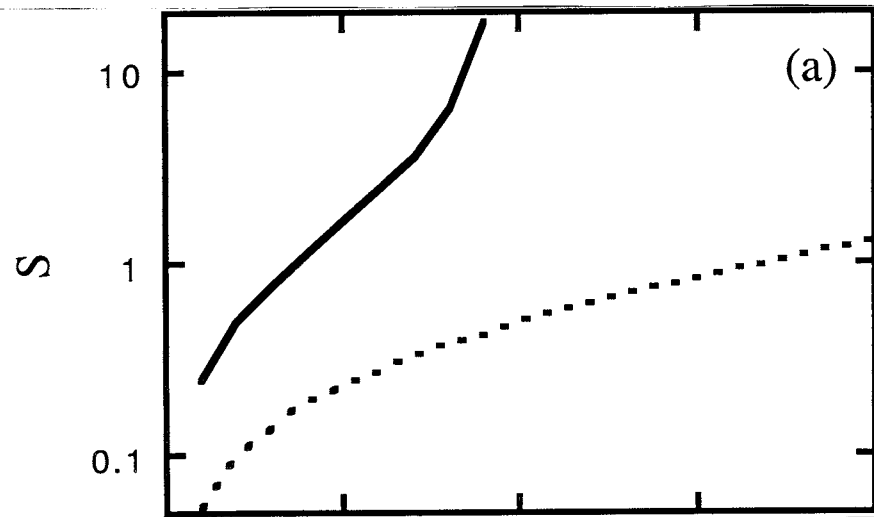
- Scaled thermal velocity including space charge

$$S = \frac{\left\{ 2 \left(\frac{\sigma_\gamma}{\gamma_0} \right)^2 + \left[\frac{k_\beta}{k_w} k_s \varepsilon \right]^2 \right\}^{1/2}}{\eta}$$

- η : 'cold' beam efficiency
- Emittance & energy spread add up in quadrature
- Self fields reduce effect of emittance

$$k_\beta \rightarrow 0, SFP \rightarrow 1$$

COMPACT FEL (cont.)



SUGGESTED READING

- T.M. O’Neil & J.H. Malmberg, *Transition of dispersion roots from beam-type to Landau-type solutions*, Phys. Fluids **11**, 1754 (1968). **Change in topology of dispersion relation with S**
- C.W. Roberson & P. Sprangle, *A review of free electron lasers*, Phys. Fluids B **1**, 3 (1989). **Review of FELs – the first 10 years**
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- B. Hafizi & C.W. Roberson, *Role of beam quality in free-electron lasers*, Phys. Plasmas **3**, 2156 (1996). **Kinetic effects, including space charge**
- D.F. Gordon, et al., *Requirements for a laser pumped FEL operating in the X-Ray regime*, Nucl. Instrum. Methods Phys. Res. A, (2001). **X-Ray FEL with electromagnetic wiggler**

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- C.W. Roberson, K.W. Gentle & P. Neilsen, Phys. Rev. Lett. **26**, 226 (1971)
- P. Sprangle, C.M. Tang & W. M. Manheimer, Phys. Rev. Lett. **43**, 1932 (1979)
- N.M. Kroll, P. Morton & M.N. Rosenbluth, IEEE J. Quantum Electron. **QE-17**, 1436 (1981)
- J.R. Thompson, Phys. Fluids **14**, 1532 (1971)
- T.J. Orzechowski, *et al.*, Phys. Rev. Lett. **57**, 17 (1986)

SCALED THERMAL VELOCITY

- Scaled thermal velocity including space charge

$$S = \frac{\left\{ 2 \left(\frac{\sigma_\gamma}{\gamma_0 D} \right)^2 + \left[\frac{k_{\beta 0}}{k_w D} k_s \mathcal{E}(k_\beta / k_{\beta 0}) \right]^2 \right\}^{1/2}}{\beta_{z0} \left[-\frac{\Re \mu}{D} + \frac{1 - \omega / \omega_s}{D} \right] - \frac{k_{\beta 0}}{k_w D} k_s \mathcal{E}(k_\beta / k_{\beta 0})}$$

- $k_\beta = k_{\beta 0} (1 - SFP)^{1/2}$

- $SFP = \left(\frac{k_p / k_{\beta 0}}{\beta_{z0} \gamma_{z0} \gamma_0^{1/2}} \right)^2$

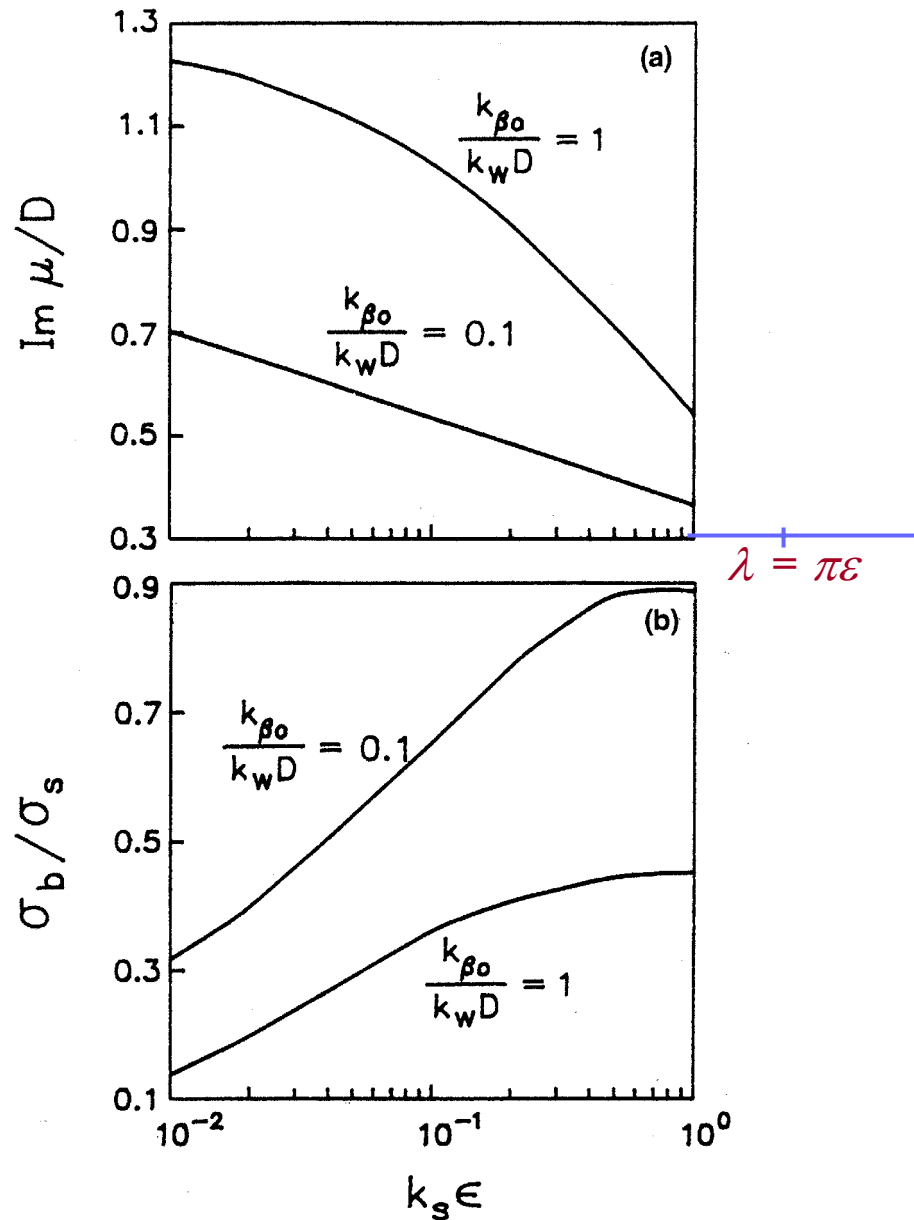
SCALED PARAMETERS

- $k_{\beta 0}/k_w D$: wiggler parameter
- μ/D : growth rate & wavenumber shift
- $(1-\omega/\omega_s)/D$: detuning
- $k_{\beta 0}\zeta_R$: Rayleigh range
- $k_s \varepsilon$: emittance; $k_s \approx 2\gamma^2 k_w$
- $\sigma_\gamma/\gamma_0 D$: energy spread

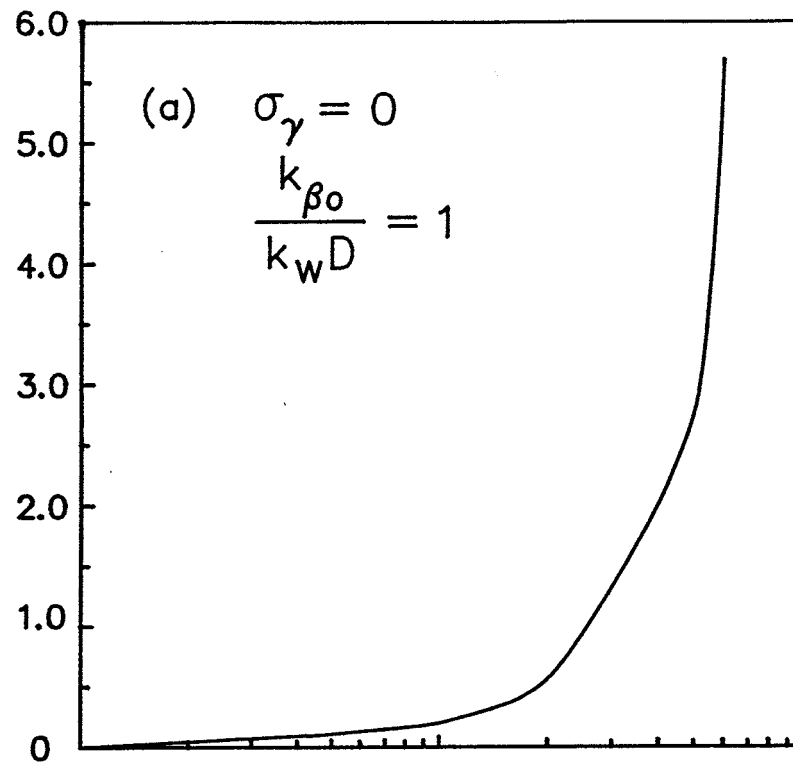
$$D = \left[\left(\frac{2\pi}{k_{\beta 0} k_s} \right)^{1/2} \frac{I_b}{I_A} \left(\frac{a_w k_s f_B}{2k_w \gamma_0^2 \beta_{z0}^2} \right)^2 (1 + a_w^2 / 2) \right]^{1/2} : 3\text{-D Pierce Parameter}$$

I_b : current, I_A : Alfvén current

EFFECT OF EMITTANCE



EFFECT OF EMITTANCE



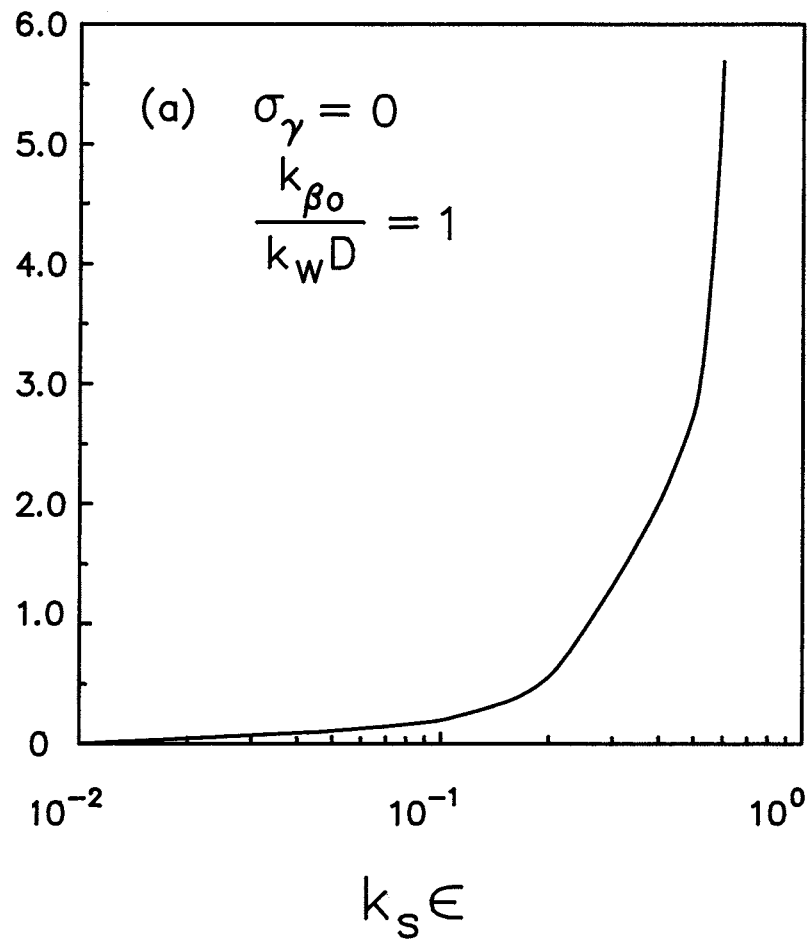
SCALED PARAMETERS

- $k_{\beta 0} / k_w D$: wiggler parameter
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- $\sigma_\gamma / \gamma_0 D$: energy spread

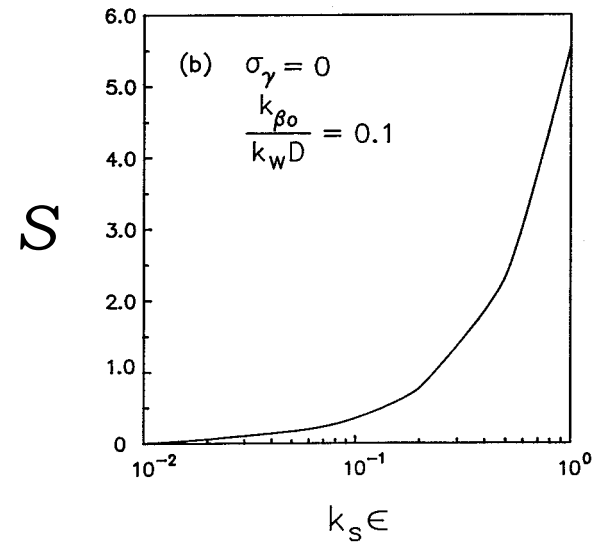
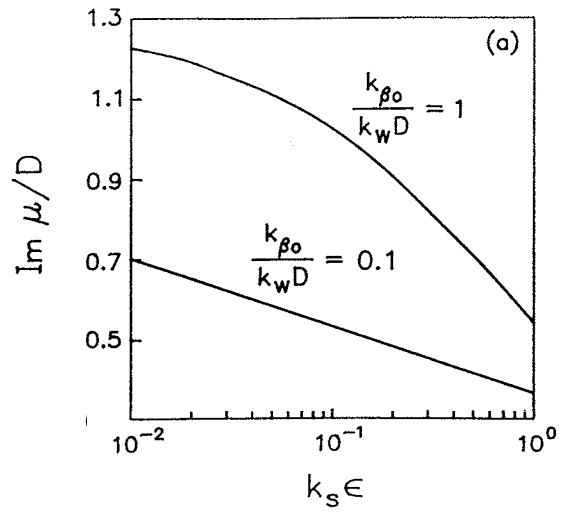
$$k_s \approx 2\gamma^2 k_w$$

$$D = \left[\left(\frac{2\pi}{k_{\beta 0} k_s} \right)^{1/2} \frac{I_b}{I_A} \left(\frac{a_w k_s f_B}{2k_w \gamma_0^2 \beta_{z0}^2} \right)^2 (1 + a_w^2 / 2) \right]^{1/2} : \text{3-D Pierce Parameter}$$

I_b : current, I_A : Alfvén current



EFFECT OF EMITTANCE



BACKGROUND

- 2-stream instability is **collective** process driven by free energy of streaming particles
 - **Cold beam**: coupled +ve & -ve energy modes in plasma \Rightarrow exponential growth
 - **Warm beam** regime: inverse Landau damping \Rightarrow growth; real part of DR not modified
- O'Neil & Malmberg found **scaled thermal velocity** $S \equiv (v_{th}/v_b)(2n_0/n_b)^{1/3} = v_{th}/(v_b - v_{ph})$ to be a useful measure of transition between these regimes
- We adopt S as a measure of beam quality in high gain FELs

BEAM QUALITY (cont.)

- $\beta_{\text{th},z}$ comes from spread, $\delta\beta_z$, of individual electrons
- $\delta\beta_z$ is due to
 - Betatron oscillations & emittance
 - Intrinsic energy spread
 - Space charge effects

VLASOV-MAXWELL MODEL

- Wiggler & optical fields

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$$\mathbf{A}_s = \frac{1}{2} A_s (y, z) \exp[i(kz - \omega t)] \mathbf{e}_x + c.c.$$

- Electron distribution function

$$F(E, P_x, J) = \frac{I_b / (\beta_{z0} I_0) \exp[-(E - E_0)^2 / (\sigma_\gamma mc^2)^2]}{2\pi m c r_e \epsilon_N \sqrt{\pi} \sigma_\gamma mc^2} \\ \times \delta(P_x) \exp(-k_{\beta 0} J / mc \epsilon_N k_\beta)$$

$J = \iint dy dp_y / 2\pi$: \perp phase space area

- Lasing wavelength $\lambda_s = \frac{\lambda_w (1 + a_w^2 / 2)}{2\gamma_0^2}$

- Integrate over unperturbed orbits