Ion-Electron Beam Oscillations at Bates

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Ions created in the restgas by an oscillating electron beam are themselves oscillating about the beam thus enhancing the beam oscillations

- Undisturbed vertical motion of electrons in a storage ring:

$$\ddot{y}_e + c^2 K(z_0 + ct) \times y_e = 0$$

$z_0 =$ position of electron at $t = 0$

Solution: $y_e = a \times r_o(z_o + ct) \cos\varphi(z_0, t)$

$a =$ relative oscillation amplitude

$r_0 =$ damped beam radius $= \sqrt{\varepsilon_y \beta(z_0 + ct)}$

Periodicity in z_0 requires

$$\varphi = \phi(z_o + ct) + \frac{2\pi}{L} z_o(n - \nu) + \phi_n(z_o)$$

$$\phi_n(z_o) = \phi_n(z_o + L)$$

$$\nu = tune$$

$$L = ring\ circumference$$

$$n = integer$$

Choose special electron oscillation mode: $\phi_n(z_o)$

→ Equation for electron beam oscillation in mode n:

$$y_{beam} = y_{electron}(z_o = z - ct) = ar_o(z)\cos\varphi(z,t)$$

$$\varphi = \phi(z) + \phi_n + \Omega_n \frac{z}{c} - \Omega_n t$$

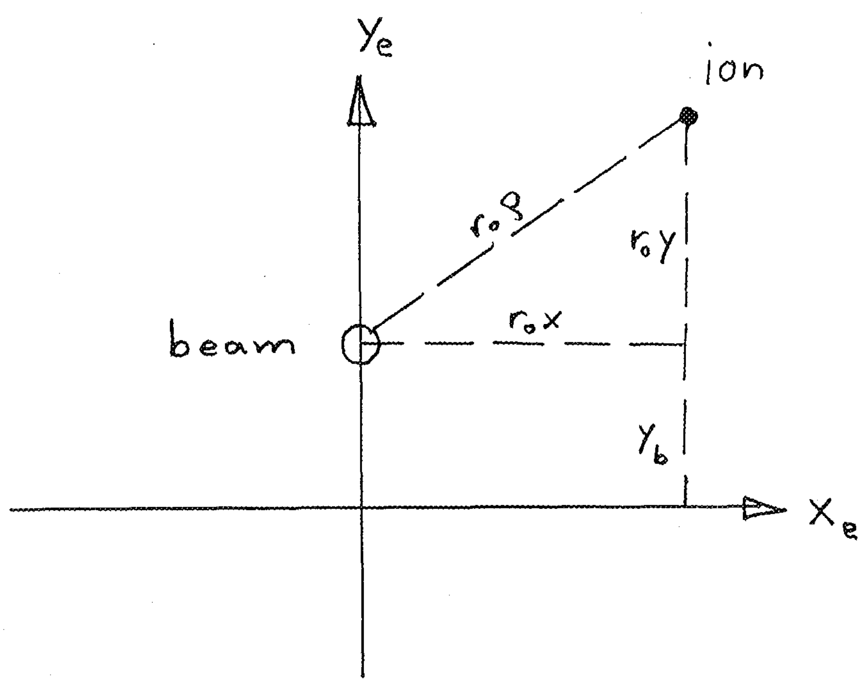
$$\Omega_n \equiv \frac{2\pi}{T}(n - \nu) \qquad (T = L/c)$$

- Qualitative mechanism of ion formation and exitation of electron beam mode n:

Fig. 1

- <u>Quantitative Treatment</u>

Ion oscillations exited by a vertically oscillating beam:



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  Ion oscillations exited by a vertically oscillating beam:

- Electric force on ions of mass m:

$$F_x = -m\omega^2 r_o g(\rho) x$$

$$F_y = -m\omega^2 r_o g(\rho) y$$

$$\omega^2 \equiv \frac{60eV}{m \times r_o^2} \frac{I_o}{(A)} = 1.174 \times 10^{-4} \frac{m_e}{m} \frac{c^2}{r_o^2} \frac{I_o}{(A)}$$

- Equ. of motion for ions created at t= t_0:

$$\frac{d^2 x}{d\theta^2} + \frac{g(\rho)}{\mu_n^2} x = 0$$

$$\frac{d^2 y}{d\theta^2} + \frac{g(\rho)}{\mu_n^2} y = a \cos\varphi$$

$$\theta \equiv |\Omega_n| (t - t_o) = \varphi - \varphi_o$$

$$\mu_n^2 \equiv \Omega_n^2 / \omega^2$$

- Solutions:

$x(\theta, \varphi)$ and $y(\theta, \varphi)$ periodic in φ

for x,y <<1 g(ρ) $\cong 1 \rightarrow$ harmonic oscillation

for g $(\rho) = \left(1 - e^{-\rho^2}\right) / \rho^2$ Gaussian beam density

compute numerical solutions x and y

- Force of a trace of ions formed within dt_0 on beam electrons:

$$dF_x = 0 \qquad \text{(symmetry)}$$

$$dF_y = m\omega^2 r_o g(\rho)\, y \times \frac{d\varepsilon}{dt} \times dt_0$$

$\frac{d\varepsilon}{dt} dt_0 =$ fraction of beam charge contained in the ion trace

- Force of all ions:

$$F_y = \int_{t_0=-\infty}^{t} dF_y \equiv m_e \gamma k f(\varphi)$$

$$f(\varphi) = \int_0^{\infty} d\theta e^{-\tilde{\gamma}\theta} \; g\left[\rho(\theta,\varphi)\right] y(\theta,\varphi)$$

$$k = \frac{1.87\times10^{-5}}{\gamma} \frac{I_0}{(A)} \frac{c^2 T}{r_0(\nu - n)} \left(\frac{d\varepsilon}{dt}\right)$$

$\tilde{\gamma}$ = diffusion factor

Since y, x, ρ periodic in φ→ f (φ) periodic in φ

$$\rightarrow f(\varphi) \equiv \sum_m a_m \cos(m\varphi) + b_m \sin(m\varphi)$$

- Total acceleration of beam electrons by all ions

$$\Delta \ddot{y}_e = F_y / m_e \gamma = k \; f(\varphi)$$

- Modified equ. of motion of beam electrons

$$\ddot{y}_e + c^2 K(z)\, y_e = k \; f(\varphi) = k \sum_m a_m \cos(m\varphi) + b_m \sin(m\varphi)$$

Since the "free" oscillations are of the form cos φ, sin φ, only the m=1 components of f (φ) contribute significantly to the oscillation (resonance):

$$y_e \cong a\, r_0 \cos\varphi - \frac{k\beta\overline{\beta}}{2c^2}\varphi\,\overline{b}_1 \cos\varphi + \frac{k\beta\overline{\beta}}{2c^2}\varphi\,\overline{a}_1 \sin\varphi$$

- Ion induced growth in beam oscill. amplitude:

$$\left.\frac{da}{dt}\right/_{ion} = -\frac{k\beta\overline{\beta}}{2c^2 r_0}\overline{b}_1 \frac{d\varphi}{dt} = +\frac{0.964\times10^{-5}}{\gamma}\frac{I_0}{(A)}\left(\frac{d\varepsilon}{dt}\right)\frac{L\overline{b}_1}{\varepsilon_\gamma (n-\nu)}$$

- Condition for starting up beam oscillations:

  Assume: Only competing process considered is exponential synchrotron damping:

$$\left.\frac{da}{dt}\right/_{damp} = -\frac{a}{t_d}$$

Oscillations start if

$$\left.\frac{da}{dt}\right/_{ion} > \frac{d}{t_d}$$

at small amplitudes (harmonic oscillations) where

$$\frac{\bar{b}_1}{a} = \frac{\bar{\mu}_n^2\left(1+\bar{\mu}_n^2\right)}{\left(1-\bar{\mu}_n^2\right)^2}$$

→ Start up condition:

$$S_0 \equiv 0.93\times10^{-5}\,\frac{\mu_n^2\left(1+\mu_n^2\right)}{\left(1-\mu_n^2\right)^2}\,\frac{t_d}{\gamma(n-\nu)}\,\frac{I_0}{(A)}\left(\frac{d\varepsilon}{dt}\right)\frac{L}{\varepsilon_y}>1$$

- Maximum amplitude is reached when b_1 has decreased (due to smaller ion-electron coupling at large oscillation amplitudes) to a value b_1 (a_{max}) where

$$S = 0.93\times10^{-5}\left(\frac{b_1}{a}\right)_{a_{\max}}\frac{t_d}{\gamma(n-\nu)}\frac{I_0}{(A)}\left(\frac{d\varepsilon}{dt}\right)\frac{L}{\varepsilon_y}=1$$

Numerical results of b_1/a for Gaussian beam densities: (Fig 2)

Find $b_1 / a \cong 3.5\, a^{-1.85}\mu_n^{-0.2}$

Using $d\varepsilon / dt \cong 0.088(Hz)\times M\times p / nTorr$

Find $r_0 a_{\max} = A_{\max} \cong 2.7(mm)R^* G^* I^* T^* \sqrt{\beta/(m)}$

$$R^* = (t_d/s)^{0.54}\,\gamma^{-0.54}\,(L/m)^{0.6}\,(\varepsilon_y/nm)^{-0.1}$$

$$G^* = M^{0.5}\,(p/nTorr)^{0.54}$$

$$I^* = (I_0/A)^{0.6}$$

$$T^* = \nu^{0.05}\,(n-\nu)^{-0.65}$$

- Results from first SHR measurements, January 1999

  Qualitative result from BPM changing ring tune from

  $\nu = 7.83 \rightarrow 8.05$: Fig 3

SHR Parameters

$$\left.\begin{aligned} E_0 &= 0.66\ \text{GeV} \\ \gamma &= 1292 \\ t_d &= 0.47\,s \\ L &= 190\,m \\ \varepsilon_y &= 7.3\,nm \\ \nu &= 7.83(8.05) \end{aligned}\right\} R^* = 0.265$$

For N_2 or CO: M=28 p=3nTorr — For H_2 : M=2 $p \cong 20$nTorr

$G^* = 9.6$ — $G^* = 7.1$

- Start-up limits S_0=1 for N_2 and H_2 ion induced oscillations: Find from Fig.4

3nTorr N_2 or CO, $I_0 \gtrsim$ 10mA, — 20nTorr H_2, $I_0 \cong 100$mA

$n\text{-}\nu > 0.12$ — $n\text{-}\nu \gtrsim 3$

- Max amplitudes at SHR internal target $\left(\beta_y = 2m\right)$:

Find from Fig. 5:

for 3nTorr N_2

$$\left.\begin{array}{l} \nu = 7.83 \quad n = 8 \\ I_0 = 60mA \end{array}\right\} A_{max} = 5.9mm$$

$$\left.\begin{array}{l} \nu = 8.05 \quad n = 9 \\ I_0 = 120mA \end{array}\right\} A_{max} = 3.1mm$$

for 20nTorr H_2

$$\left.\begin{array}{l} \nu = 7.83 \quad n = 11 \\ I_0 = 60mA \end{array}\right\} A_{max} < 0.7mm$$

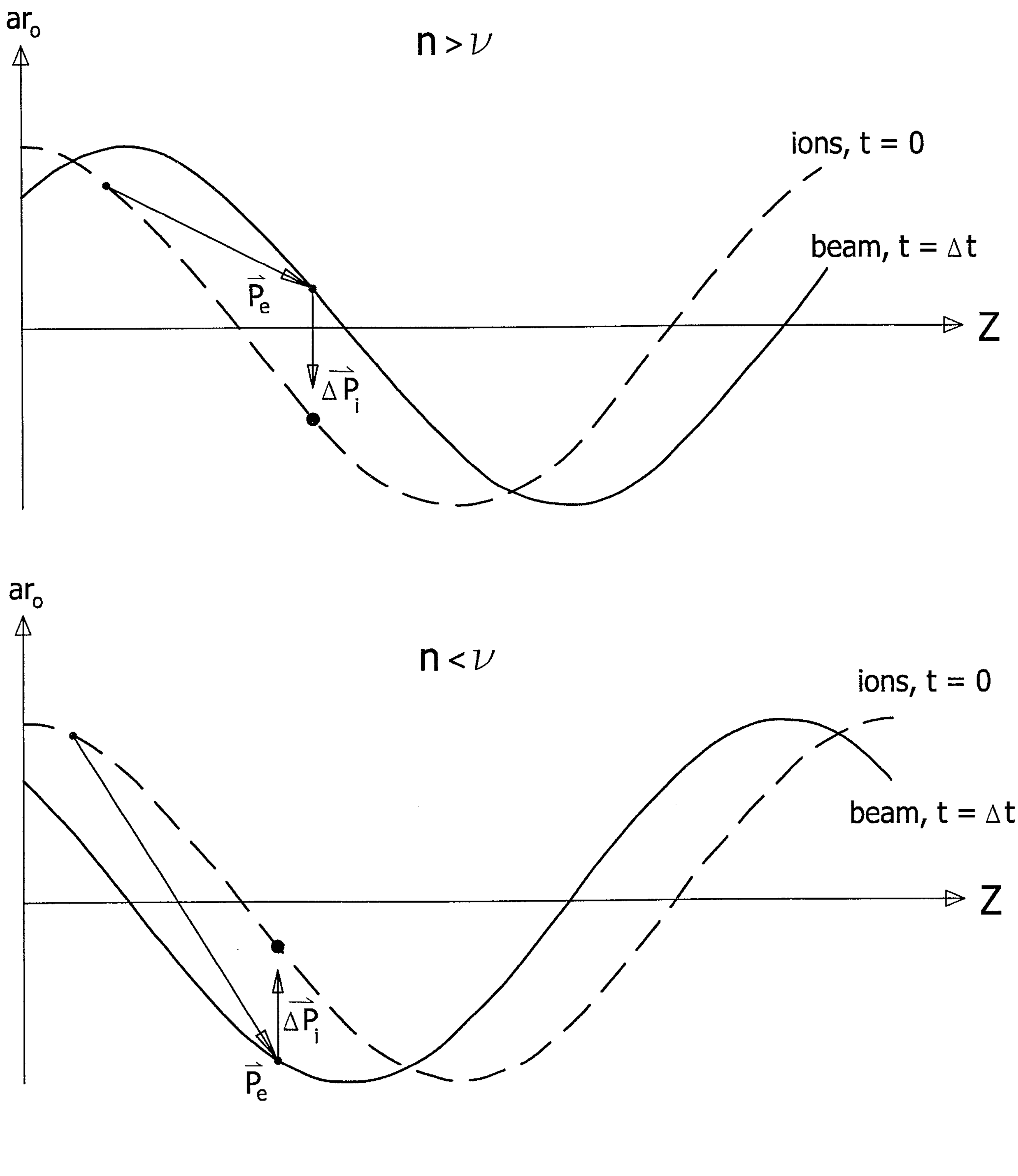
$$\left.\begin{array}{l} \nu = 8.05 \quad n = 11 \\ I_0 = 120mA \end{array}\right\} A_{max} < 0.3mm$$

Good agreement with vertical beam clearance in 15 mm diameter target tube

- Conclusion
  - Ion exitations of beam oscillations are real
  - Single mode calculations agree reasonably with beam width measurements
  - Effect of tune on oscillation amplitude confirmed
  - Effect of gas pressure needs testing
  - Effect of current needs testing
  - Effect of emittance is small
  - Effect of beam energy needs testing

Prevention of oscillations:

- tune above nearest integer
- Gas pressure (N_2, CO) low
- Fast ion clearing by beam gap?
  (slow clearing by electrodes not effective)



ar_o
$n > \nu$
ions, t = 0
beam, t = Δt
$\vec{P}_e$
$\overrightarrow{\Delta P_i}$
Z
ar_o
$n < \nu$
ions, t = 0
beam, t = Δt
$\overrightarrow{\Delta P_i}$
$\vec{P}_e$
Z

Fig. 1

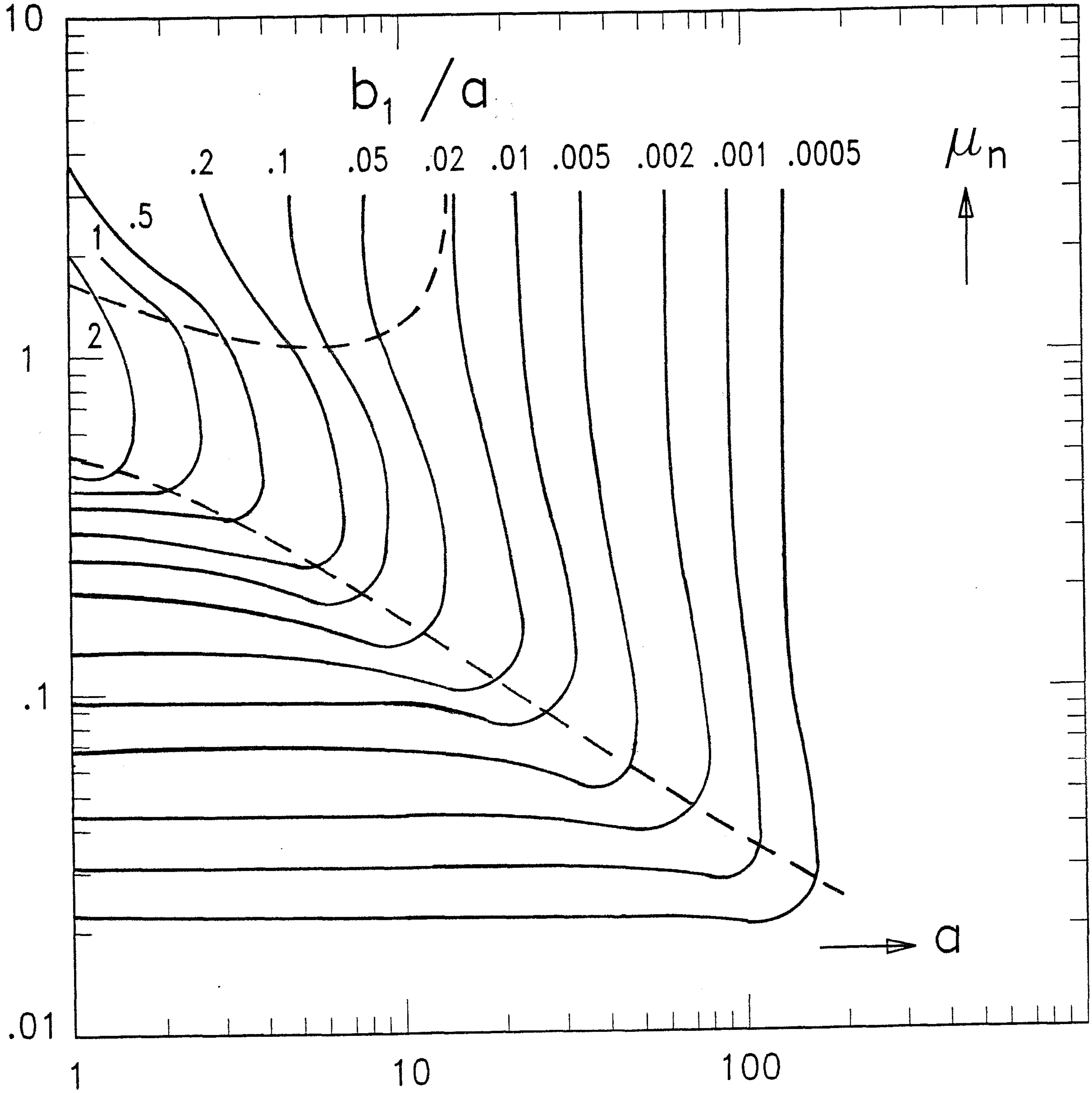


Fig 2

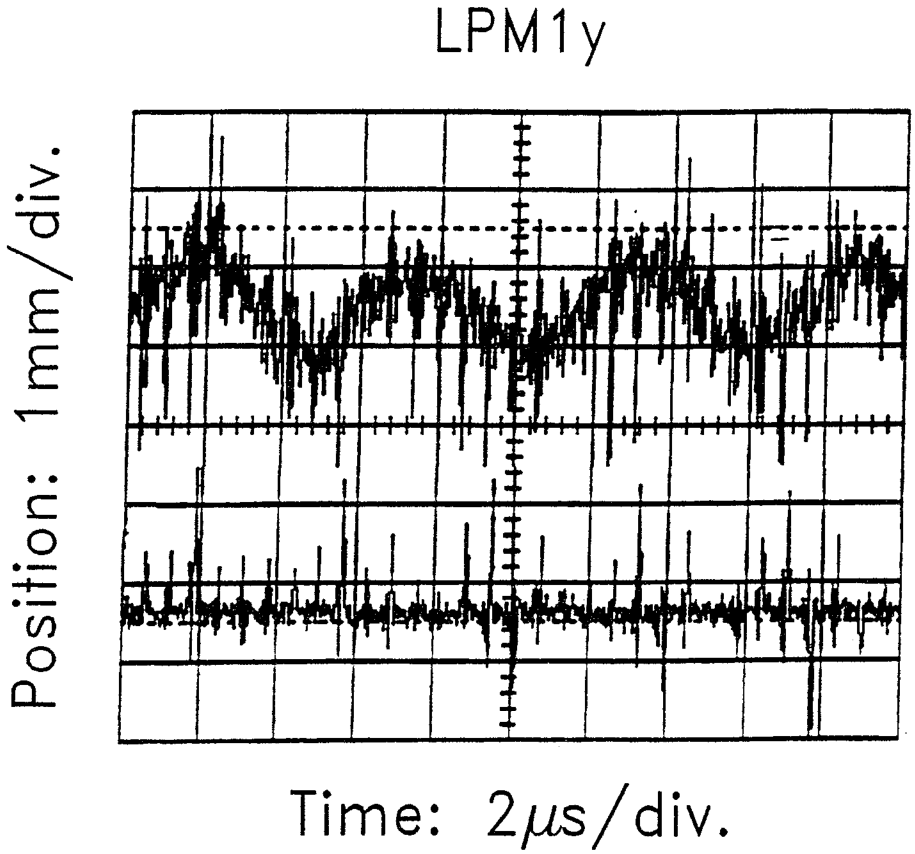


Fig. 3

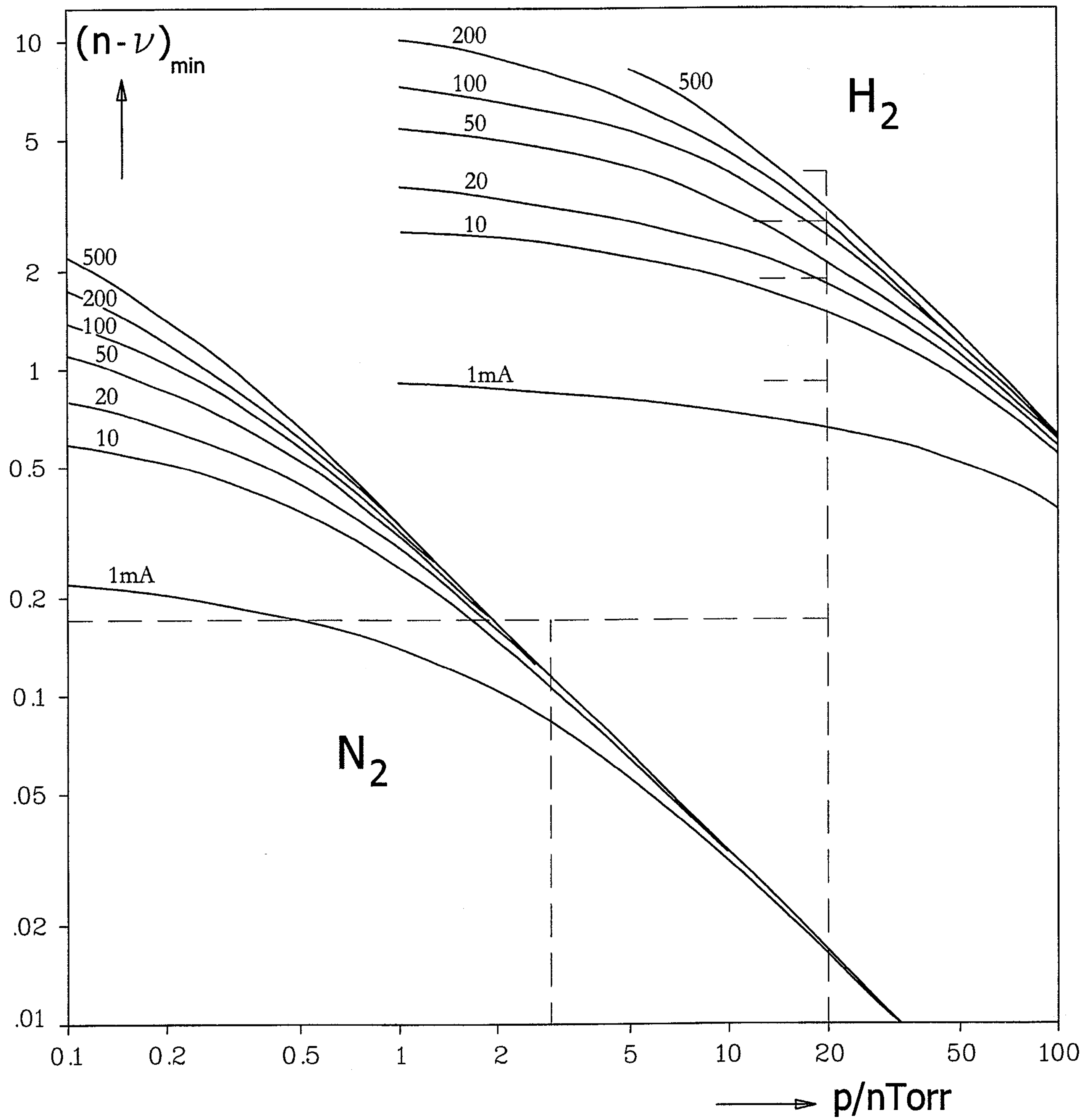

$(n-\nu)_{min}$
H_2
N_2
200
500
100
50
20
10
1mA
500
200
100
50
20
10
1mA
10
5
2
1
0.5
0.2
0.1
.05
.02
.01
0.1
0.2
0.5
1
2
5
10
20
50
100
p/nTorr

Fig. 4

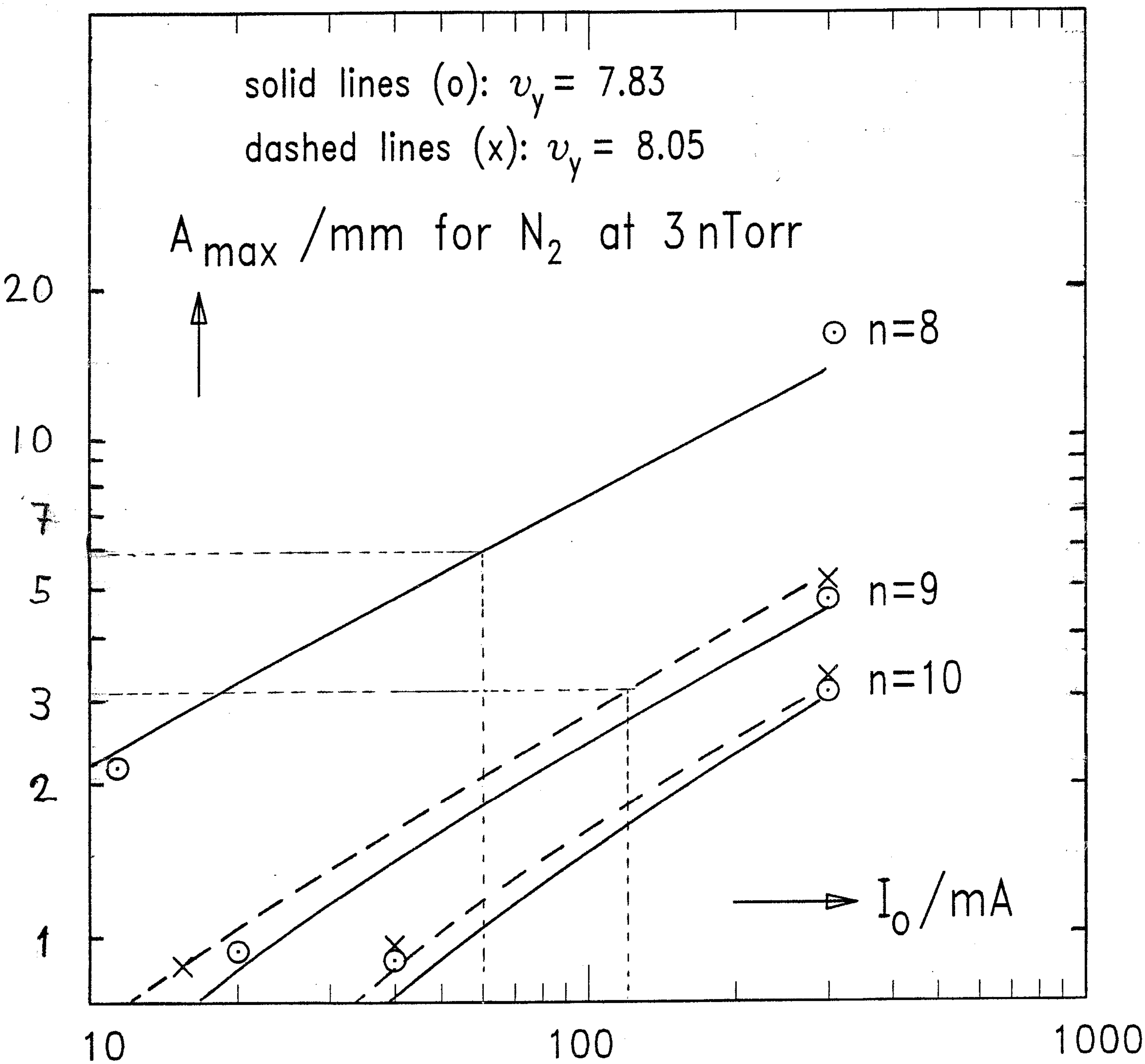
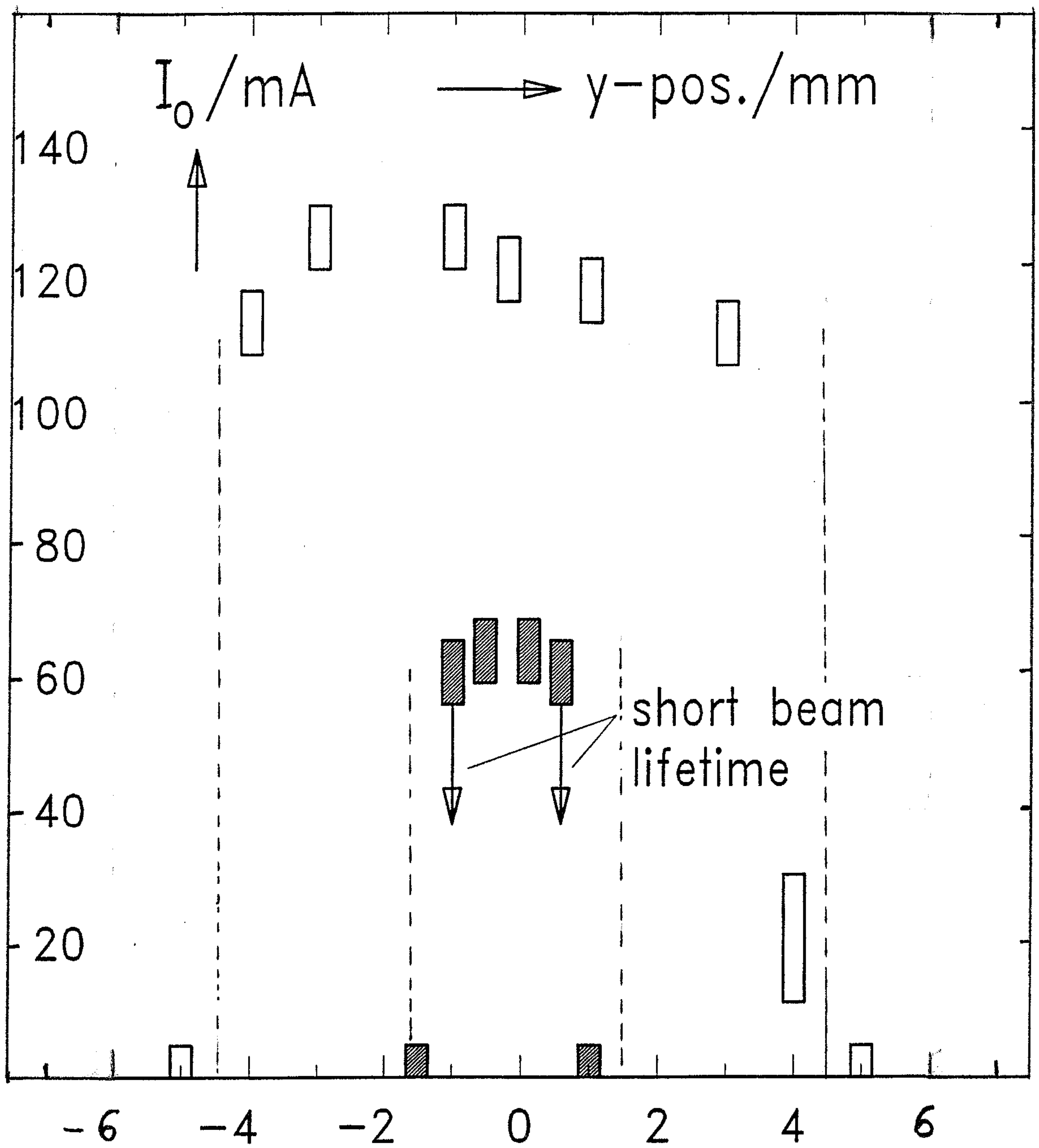

solid lines (o): v_y = 7.83
dashed lines (x): v_y = 8.05
A_{max} /mm for N_2 at 3 nTorr
20
10
7
5
3
2
1
n=8
n=9
n=10
I_0/mA
10
100
1000

Fig. 5



I_0/mA
y-pos./mm
140
120
100
80
60
40
20
short beam
lifetime
-6
-4
-2
0
2
4
6

Fig. 6