# Effect of Tune Shift in Fast Ion Instability

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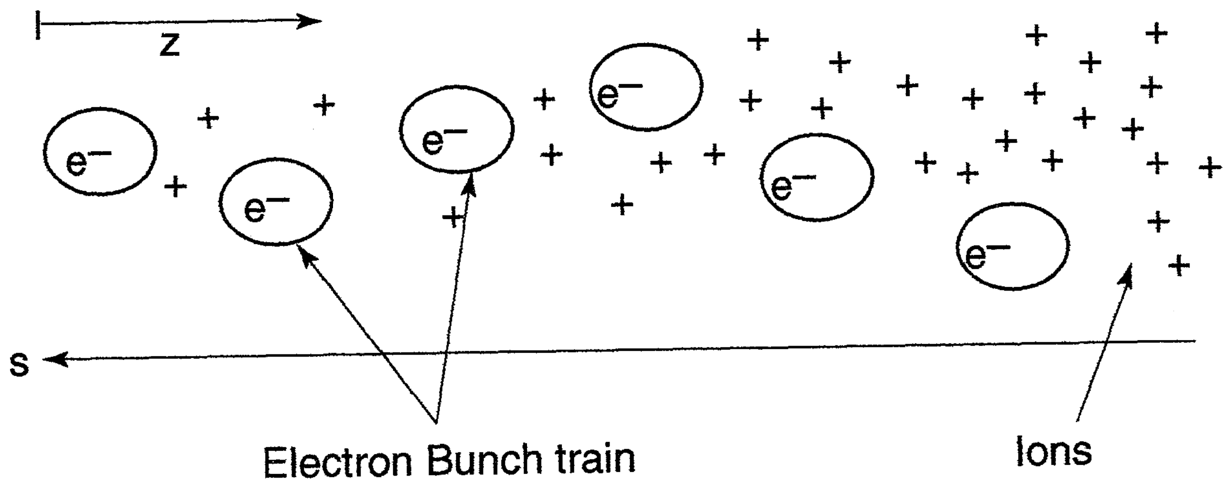
- Usually, the derivation of FII focuses on the growth rate. There is also a tune shift along the beam that can be used as a signature of the instability.
- Can the tune shift serve as a damping mechanism (BNS damping)? See paper by D. Pestrikov "Natural BNS damping of the fast ion instability" (PRST-AB, vol.2, p.044403, 1999) and my comment in PRST-AB, vol.3, p.019401, 2000.

# Fast Ion Instability

(Raubenheimer and Zimmermann. *Phys. Rev.* E, **52**, 5487,1995).



# Tune Shift

## Master equation for FII

$$\frac{\partial^2}{\partial s^2} y_b(s,z) + \frac{\omega_{\beta 0}^2}{c^2} y_b(s,z) = \kappa z \left[ y_i(s,z) - y_b(s,z) \right]$$

where

$$\kappa \equiv \frac{4\dot{\lambda}_{ion} r_e}{3\gamma c \sigma_y (\sigma_x + \sigma_y)} ,$$

$\gamma$ – relativistic factor,

$r_e$ – the classical electron radius

$\sigma_{x,y}$ – the horizontal and vertical rms-beam size,

$\dot{\lambda}_{ion}$ – the number of ions per meter generated by the beam per unit time.

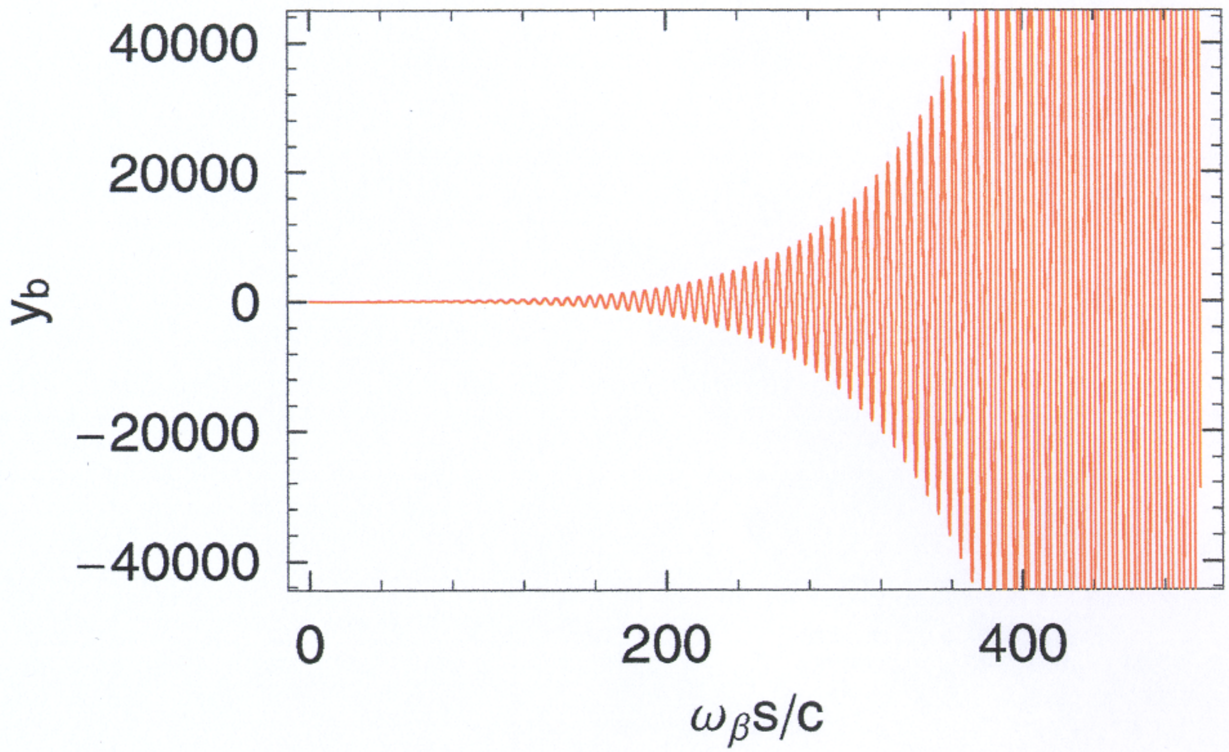
Naively,

$$\delta\omega_\beta = \frac{\kappa z c^2}{2\omega_{\beta 0}}$$

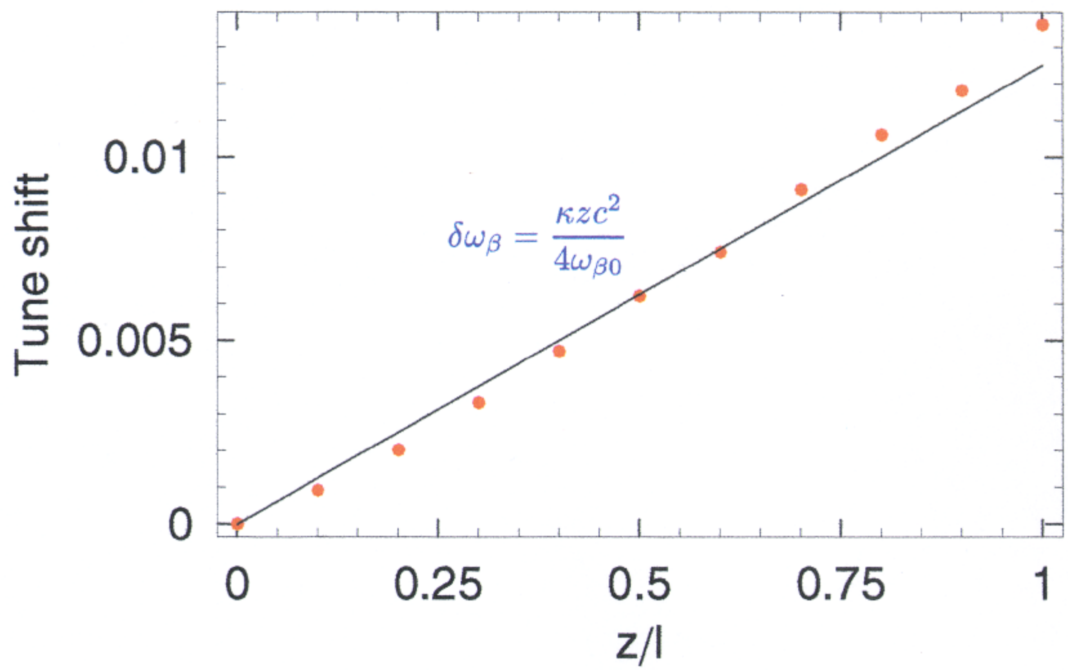
however, after some transformation ($y = y_b$)

$$\frac{\partial^2}{\partial s^2} y(s,z) + \frac{\omega_\beta^2}{c^2} y(s,z) = -\kappa \int_0^z z' \frac{\partial y(s,z')}{\partial z'} \cos\omega_i (z - z')\, dz' .$$

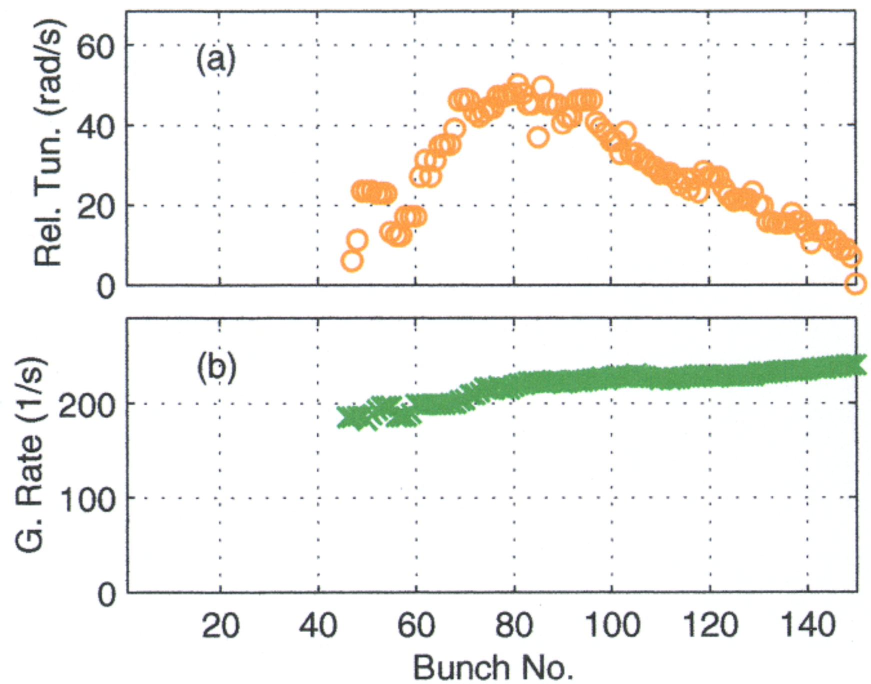
A computer code was written that solves this equation.




Oscillations of the last bunch, $y_b(z = l, s)$, for $lc^2\kappa/\omega_\beta^2 = 0.05$, $l\omega_i/c = 20$, and initial condition $y_b(z, s = 0) = \cos(\omega_i z/c)$ ($l$ is the length of the train).




Theoretical considerations also show the scaling $\delta\omega_\beta = \frac{\kappa z c^2}{4\omega_{\beta 0}}$.




Measurement of tune shift and growth rate of the instability in PEP-II (from S. Prabhakar et. al., PRST-AB, vol 2. p. 084401, 1999). The betatron frequency is $\omega_\beta = 2 \cdot 10^7$ rad/s.

# Does the tune shift suppress the instability (BNS damping)?

Recently D. Pestrikov ("Natural BNS damping of the fast ion instability", PRST-AB, vol.2, p.044403, 1999) found a new solution for the FII.

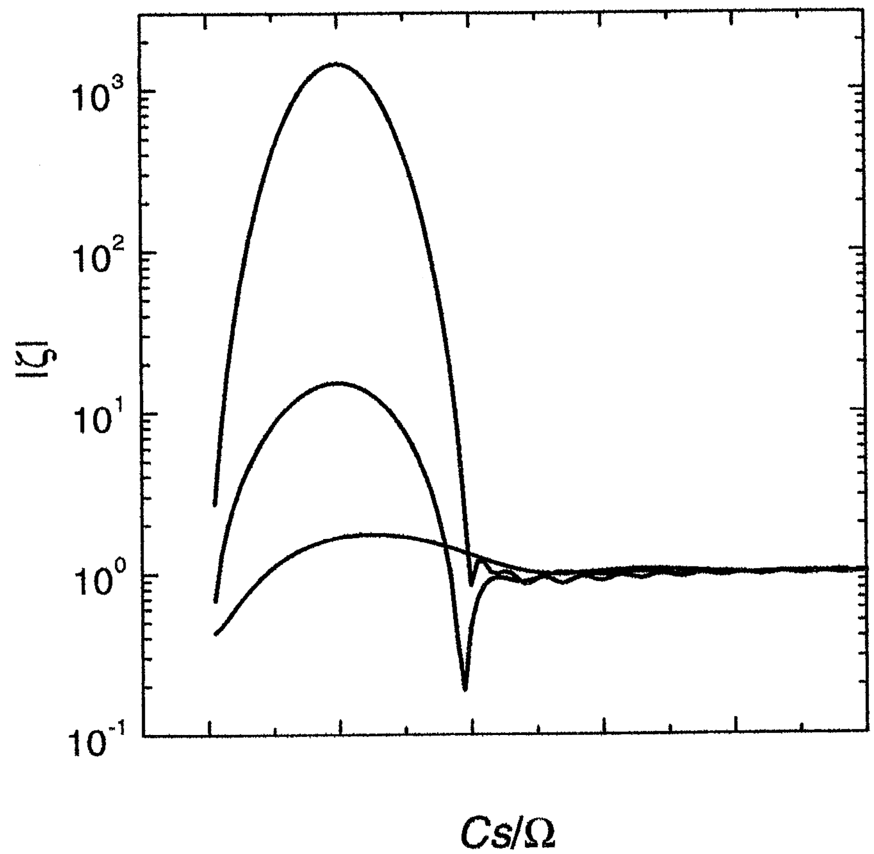



FIG. 7. Dependences of the (dimensionless) amplitude $|\zeta|$ on the time ($Cs/\Omega$). From top to bottom at $Cs/\Omega$ - 2, x - 10, 5, and 2.

He claims that "the prediction containing the obtained solutions do not agree with the results of previous calculations...".

Approximations made in the derivation of FII (see Stupakov, Raubenheimer and Zimmermann, *Phys. Rev.* E, **52**, 5499,1995).

1. Weak interaction between the beam and the ions,

$$c^2\kappa l \ll \omega_i^2, \ \omega_\beta^2 \ ,$$

2. The most unstable solution is represented as a wave with a slowly varying amplitude and phase,

$$y\,(s,z) = \mathrm{Re}A\,(s,z)\,e^{-i\omega_\beta s/c + i\omega_i z/c} \ ,$$

3. $A$ is a 'slow' function of its variables,

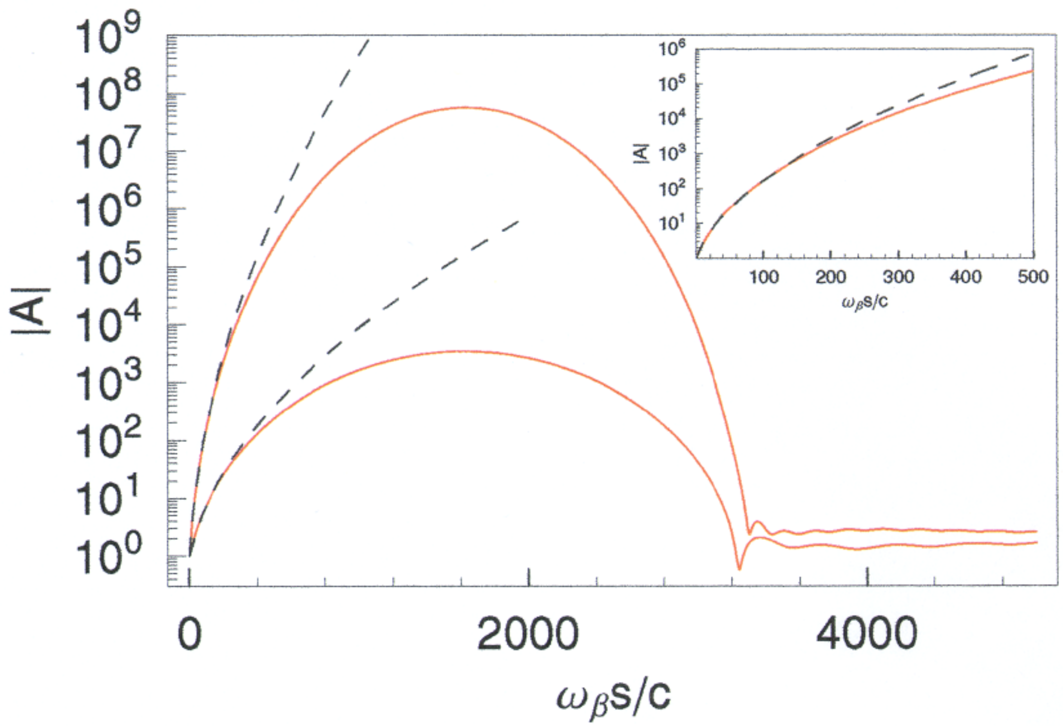
$$\left|\frac{\partial \ln A}{\partial s}\right| \ll \frac{\omega_\beta}{c} \ , \quad \left|\frac{\partial \ln A}{\partial z}\right| \ll \frac{\omega_i}{c}$$

Using an averaging method, for initial condition $A(s=0,z)=1$, one can find the solution: $A(s,z)=I_0(z\sqrt{\kappa\omega_i s/2\omega_\beta})$. For $s \gg \omega_\beta/\kappa\omega_i l^2$,

$$A(s,z) \approx \left(2\pi z\sqrt{\frac{\kappa\omega_i s}{2\omega_\beta}}\right)^{-1/2} \exp\left(z\sqrt{\frac{\kappa\omega_i s}{2\omega_\beta}}\right)$$

The applicability condition

$$s \ll s_{\max} \equiv \frac{2\omega_\beta\omega_i}{\kappa c^2}$$




A numerical solution for $lc^2\kappa/\omega_\beta^2 = 0.05$, $l\omega_i/c = 20$ and initial condition $A(s = 0, z) = 1$. For these parameters, $s_{\max}\omega_\beta/c = 400$.

The maximum growth allowed by the asymptotic solution, $s = \alpha s_{\max}$. This gives the maximum amplification factor in the asymptotic regime as

$$|A|_{\max} = \frac{1}{(2\pi\alpha z\omega_i/c)^{1/2}} e^{\alpha z \omega_i/c}.$$

We calculated $|A|_{\max}$ assuming $\alpha = 0.1$ for the FII in different accelerators using the values for the bunch train $l$ and the ion frequency $\omega_i$. The result is shown in Table.1.

Table 1: Maximum amplification for the asymptotic solution

| Accelerator | PEP-II HER | NLC PL | NLC ML |
|---|---|---|---|
| $l$ [m] | 2000 | 38 | 38 |
| $\omega_i/2\pi$ $[MHz]$ | 4 | 104 | 200 |
| $A_{\max}$ | $5.8 \times 10^{21}$ | $1.8 \times 10^{10}$ | $5.4 \times 10^{20}$ |
| Accelerator | ALS | HERA e- | CESR |
| $l$ [m] | 200 | 6048 | 670 |
| $\omega_i/2\pi$ $[MHz]$ | 25 | 0.8 | 0.6 |
| $A_{\max}$ | $1.7 \times 10^{13}$ | $5.9 \times 10^{12}$ | 3.5 |

For most accelerators the amplification lies between 10 and 20 orders of magnitude.

## Conclusions

- Tune shift caused by ion in FII is two times smaller that predicted by a "naive" approach
- Due to the tune shift, after initial growth, in linear theory, the instability saturates at some level, and the amplitude goes down (as predicted by Pestrikov).
- However, well before the saturation occurs the amplitude of the oscillations would become comparable to the size of the transverse beam, the linear theory would break down and the instability would proceed into nonlinear regime.
- For most practical cases, a simplified description, obtained in first papers, adequately describes the growth of the oscillations in the beam until the nonlinear effects become important.