17TH ADVANCED BEAM DYNAMICS WORKSHOP ON
FUTURE LIGHT SOURCES

PHASE - A Software Tool for the Description and Propagation of Diffraction Limited Light Sources

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PHASE - a software tool for the description and propagation of diffraction limited light sources

- J. Bahrdt. "Fourth Order Optical Aberrations and Phase Space Transformation for Reflection and Diffraction Optics", Journal of Applied Optics Vol.34 No.1 (1995) 114-127

- J. Bahrdt, U. Flechsig, F. Senf, "Beamline Optimization and Phase Space Transformation", Rev. of Scientific Instruments, Vol. 66 (3) (1995) 2719-2723

- J. Bahrdt, "Wave Front Propagation: Design Code for Synchrotron Radiation Beam Lines", Applied Optics, Vol.36, No.19 (1997)4367-4381

- SPIE 1997, J. Bahrdt, U. Flechsig, "Wave Front Propagation in Synchrotron Radiation Beamlines"

Starting point: Electric field distribution on screen downstream of ID

these data have to be transformed:

⇒ Centre of ID (for beamline designers)

⇒ sample (for user)

Different Approaches:

Fourier Optics
Physical Optics

Quasi Stationary Phase Approach (PHASE)

Propagation of Electric Field Vector

$$\vec{E}(\vec{a'}) = \int h(\vec{a'}, \vec{a}) \cdot \vec{E}(\vec{a}) \cdot d\vec{a}$$

The propagator:

$$h(\vec{a'}, \vec{a})$$

describes the optical system

Example 1: Aperture.

$$= \frac{1}{\lambda} \cdot \frac{e^{i \cdot kr}}{r} \cdot \cos(\beta)$$

$\Rightarrow$ Fresnel Kirchhoff diffraction equation

Example 2: Mirror / Grating

$$= \frac{1}{\lambda} \cdot \int_S \frac{e^{ik(r+r')}}{r \cdot r'} \cdot b(w,l) \cdot \frac{\cos(\alpha) + \cos(\beta)}{2} \cdot d\vec{s}$$

Combination of 2 optical elements

$$\tilde{h}(\vec{a''}, \vec{a}) = \int h_2(\vec{a''}, \vec{a'}) \cdot h_1(\vec{a'}, \vec{a}) \cdot d\vec{a'}$$

without approximations

CPU-time explodes

Approximation of "Quasi Stationary Phase"

$$pl(\tilde{w},\tilde{l}) = pl(\tilde{w}=0,\tilde{l}=0) + \frac{\tilde{w}^2}{a^2} + \frac{\tilde{l}^2}{b^2}$$

- expansion of the optical path length up to 2nd order
- separation of surface integral into 2 parts

$$h(\vec{a}',\vec{a}) = c \cdot \frac{1}{r_0 \cdot r_0'} \cdot e^{ik\cdot(r_0+r_0')} \cdot \int e^{ik(\tilde{w}^2/a^2)} \cdot d\tilde{w} \cdot \int e^{ik(\tilde{l}^2/b^2)} \cdot d\tilde{l}$$

$$r_0' = r'(y',z',w_0,l_0)$$

$$r_0 = r(y,z,w_0,l_0)$$

- extension of integration limits to infinity → analytical integration
- change variable from $\bar{a}=(y,z)$ to $(dy';dz')$

$$\boxed{\vec{E}(\vec{a}') = \frac{1}{\lambda} \cdot \int \vec{E}(\vec{a}) \cdot e^{ik\cdot(r+r')} \cdot T \cdot \left|\frac{\partial(y,z)}{\partial(dy',dz')}\right| \cdot d(dy') \cdot d(dz')}$$

- solve this integral numerically with "PHASE"

Scaling factor

$$T = S \cdot \frac{cos(\alpha) + cos(\beta)}{2 \cdot r \cdot r'}$$

$$S = \left(\left| \frac{\partial^2 pl}{\partial w^2} \cdot \frac{\partial^2 pl}{\partial l^2} - \left(\frac{\partial^2 pl}{\partial w \partial l} \right)^2 \right| \right)^{-1/2}$$

In the following we use another scaling factor, which gives similar numbers but is easier to handle for optics with several optical elements:

$$\tilde{T} = \left(\left| \frac{\partial(y', z')}{\partial(dy, dz)} \right| \right)^{-1/2}$$

Brightness Definition acc. to K.-J. Kim

$$B_0(\vec{x},\vec{\Phi}) = c \cdot \int d^2\vec{\xi} \cdot A(\vec{x},\vec{\xi}) \cdot exp(i \cdot \frac{2\pi}{\lambda} \cdot \vec{\Phi} \cdot \vec{\xi})$$

$$A(\vec{x},\vec{\xi}) = \vec{E}_y^*(\vec{x}+\vec{\xi}/2) \cdot \vec{E}_y(\vec{x}-\vec{\xi}/2) + \vec{E}_z^*(\vec{x}+\vec{\xi}/2) \cdot \vec{E}_z(\vec{x}-\vec{\xi}/2)$$

Convolution of brightness with electron beam emittance

$$B(\vec{x},\vec{\Phi}) = N_e \cdot \int d^2\vec{x}_e \cdot d^2\vec{\Phi}_e \cdot B_0(\vec{x}-\vec{x}_e, \vec{\Phi}-\vec{\Phi}_e) \cdot f(\vec{x}_e,\vec{\Phi}_e)$$

Diffraction at apertures are described via a convolution of the brightness with the slit function G:

$$G(\vec{x},\vec{\Phi}) = \frac{1}{\lambda^2} \cdot \int d^2\vec{\xi} \cdot S^*(\vec{x}+\vec{\xi}/2) \cdot S(\vec{x}-\vec{\xi}/2) \cdot exp(i \cdot \frac{2\pi}{\lambda} \cdot \vec{\Phi} \cdot \vec{x})$$

where S = transmittance of the aperture

Introduction of Stokes parameters in the brightness formalism

$$\vec{B} = (B_{S_0}, B_{S_1}, B_{S_2}, B_{S_3})$$

$$= c \cdot \int d^2\vec{\xi} \cdot \vec{A}(\vec{x}, \vec{\xi}) \cdot exp(i \cdot \frac{2\pi}{\lambda} \cdot \vec{\Phi} \cdot \vec{\xi})$$

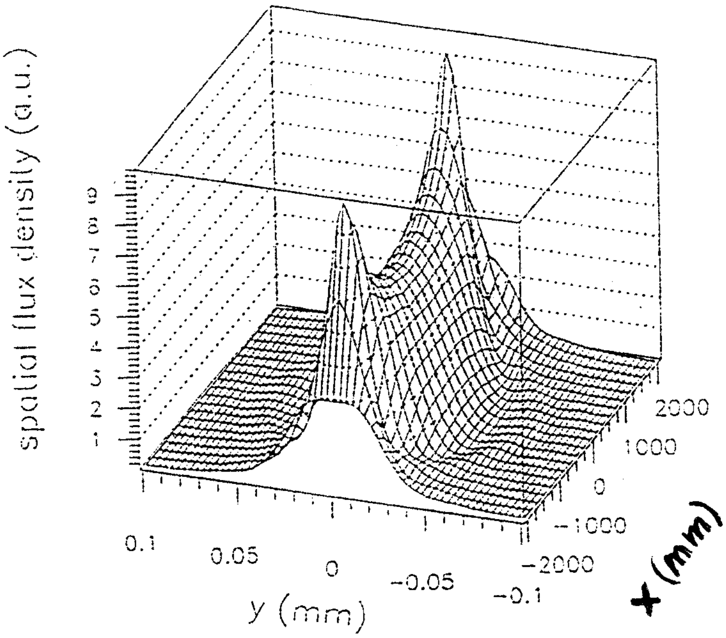
$$A_{S_0} = \vec{E}_y^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_y(\vec{x} - \vec{\xi}/2) + \vec{E}_z^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_z(\vec{x} - \vec{\xi}/2)$$

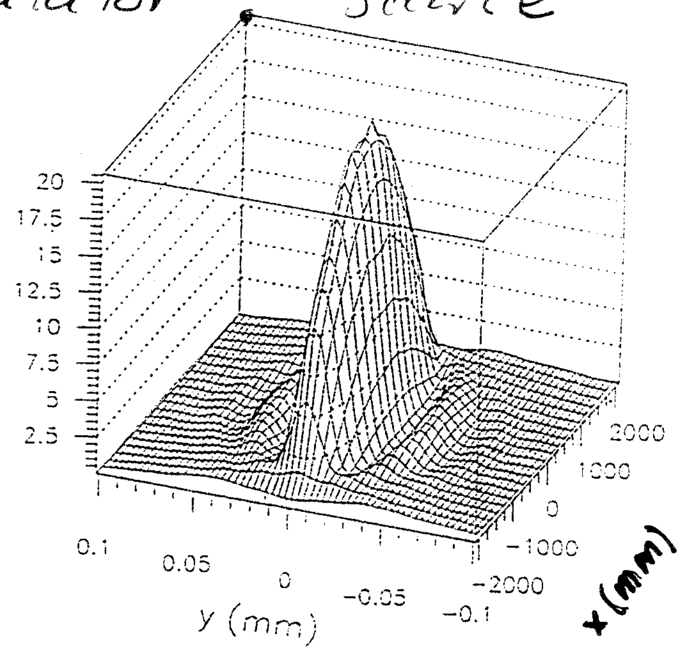
$$A_{S_1} = -\vec{E}_y^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_y(\vec{x} - \vec{\xi}/2) + \vec{E}_z^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_z(\vec{x} - \vec{\xi}/2)$$

$$A_{S_2} = \vec{E}_y^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_z(\vec{x} - \vec{\xi}/2) + \vec{E}_z^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_y(\vec{x} - \vec{\xi}/2)$$

$$A_{S_3} = i \cdot (\vec{E}_y^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_z(\vec{x} - \vec{\xi}/2) - \vec{E}_z^*(\vec{x} + \vec{\xi}/2) \cdot \vec{E}_y(\vec{x} - \vec{\xi}/2)$$

BESSY II U41 Undulator : Source

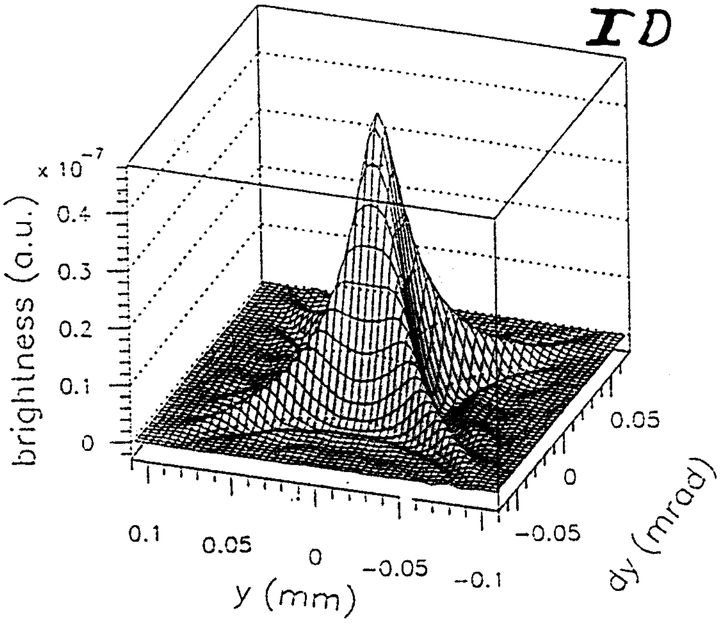


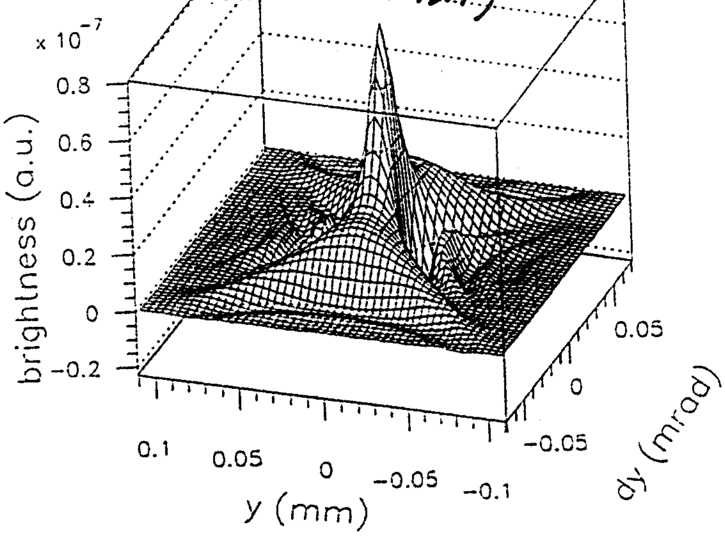


on resonance (500eV)

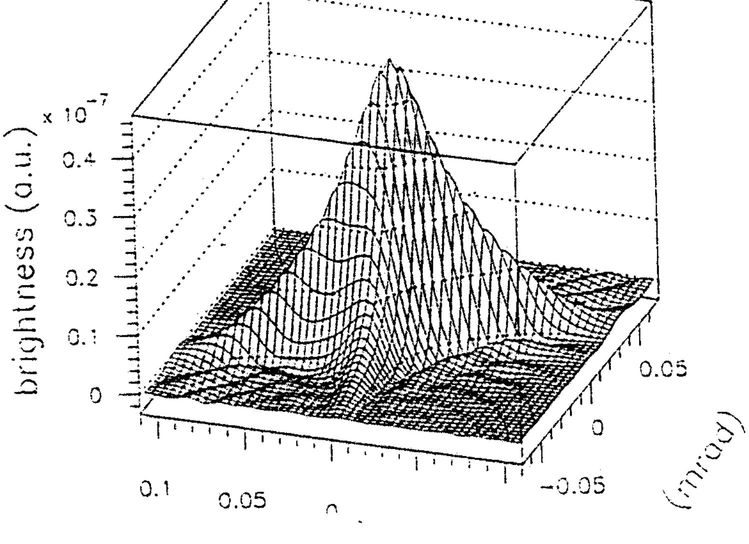
off resonance (493.8eV)

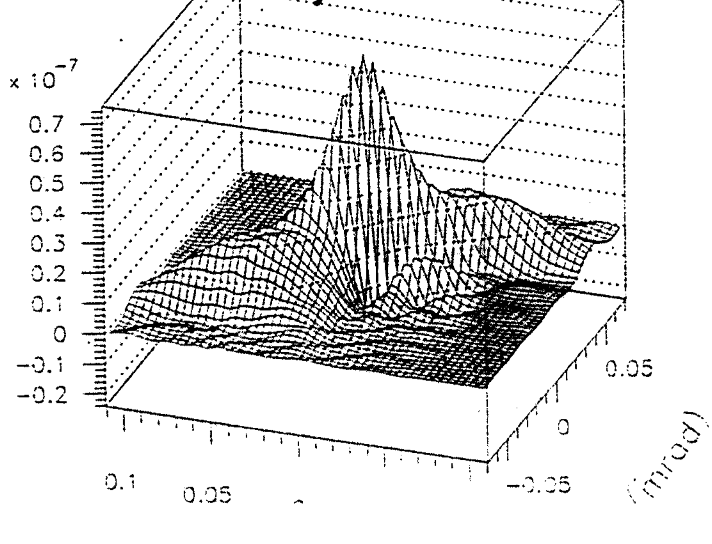
ID centre (x = 0 mm)





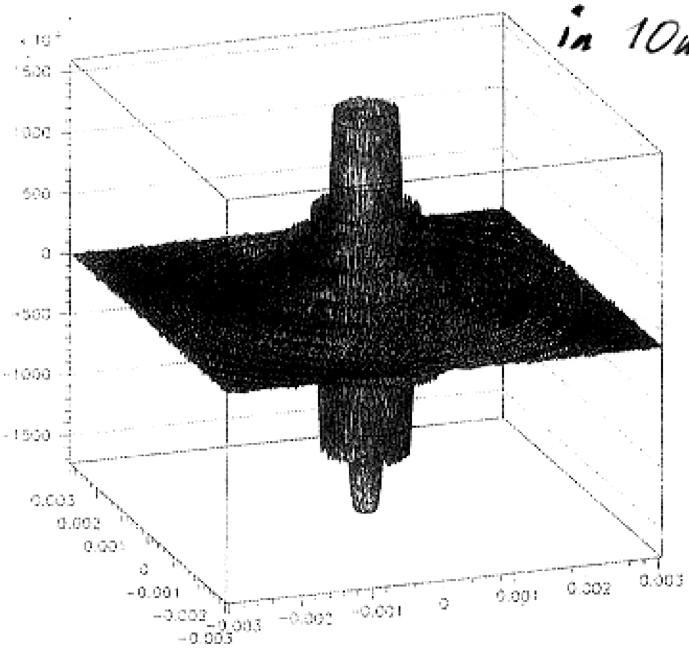
ID end (1800mm)



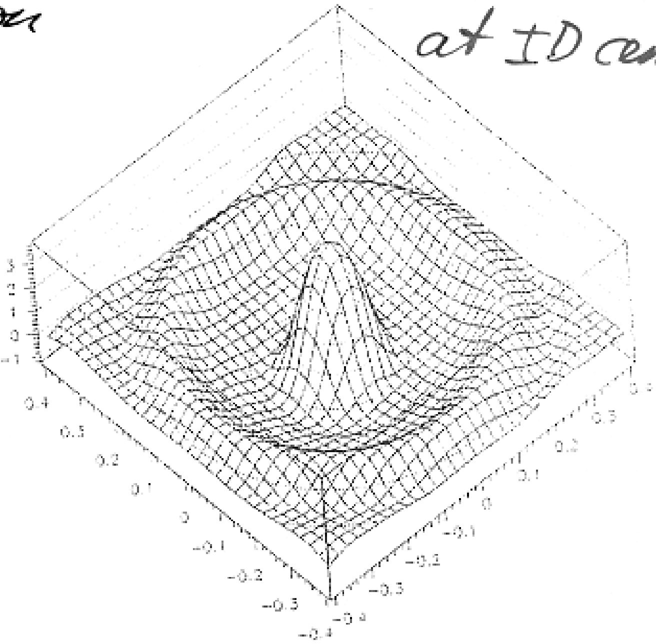


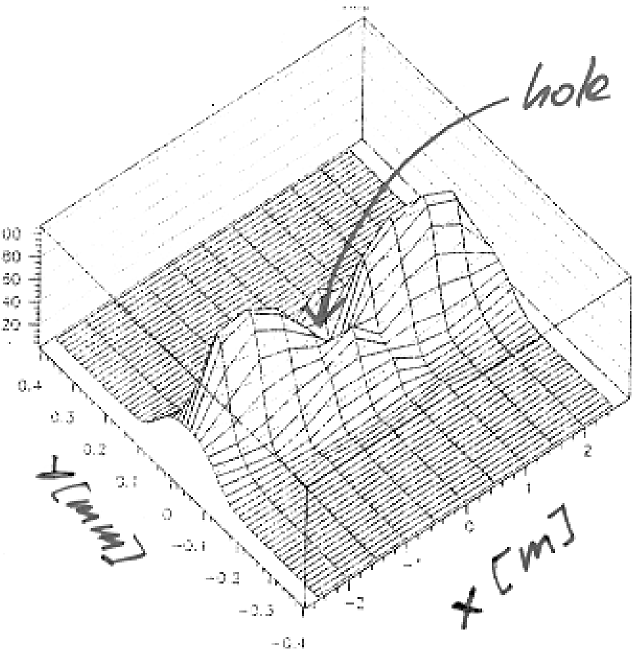
BESSY II UE56 double undulator with modulator

electric field distribution in 10m

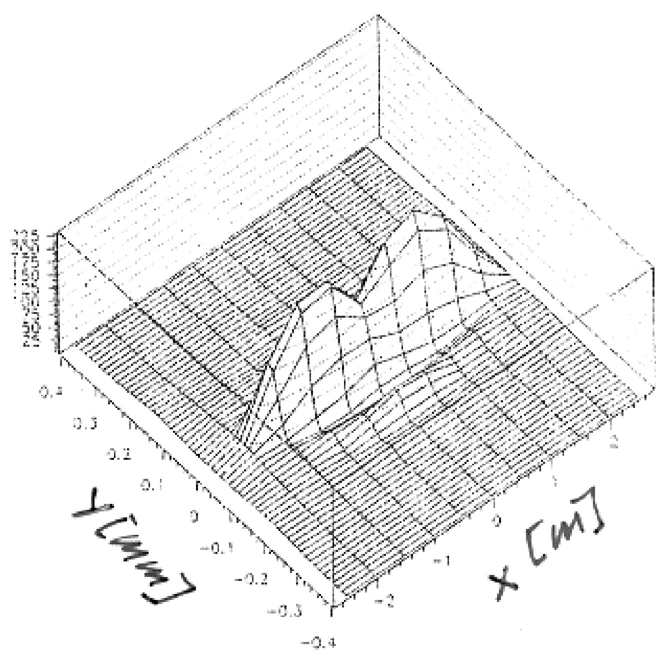


electric field dis[tribution] at ID centre



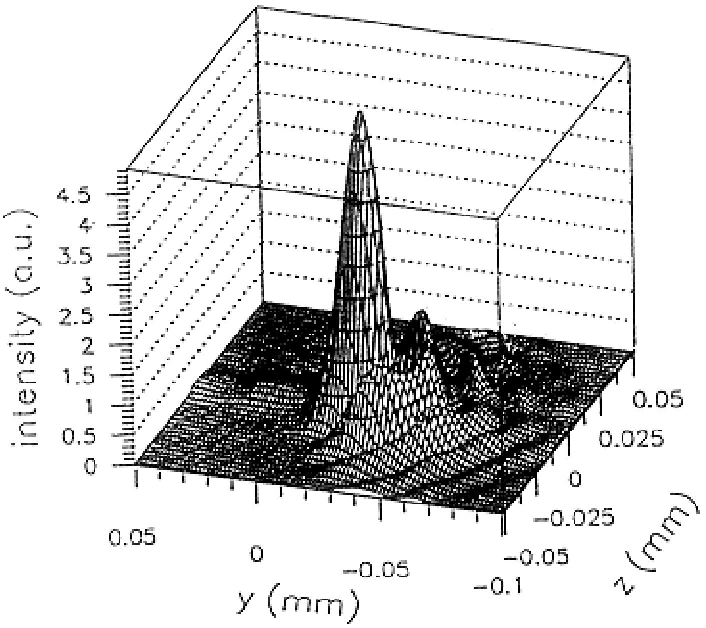


spatial flux density on resonance

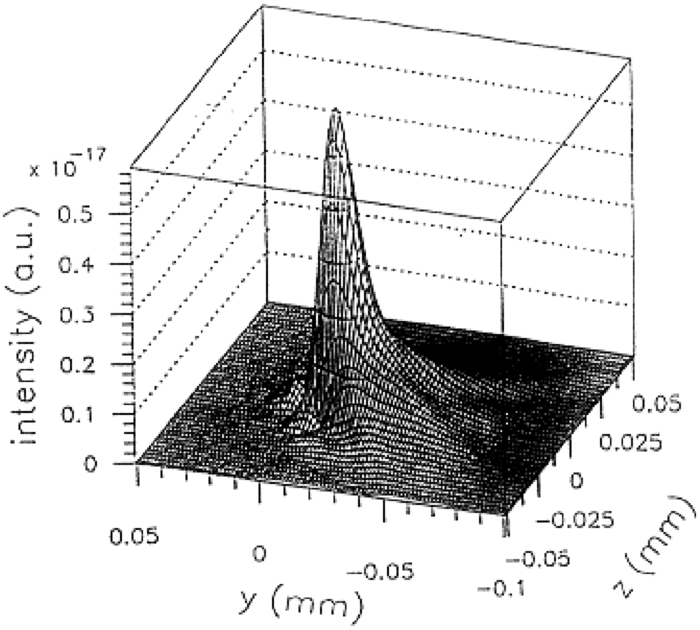


spatial flux density off resonance

Demagnification of a Dipole Source M = 20:1 , 10 eV



PHASE transformation



Brightness transformatio
(some information is lost)

emittance dominated wings

⇓

ray tracing codes
based on geometric optics

diffraction limited wings

⇓

Fourier optics
stationary phase
approach

intermediate range of
partially coherent light
is difficult to be modelled
(at least in terms of CPU time)