17TH ADVANCED BEAM DYNAMICS WORKSHOP ON

FUTURE LIGHT SOURCES

Kick and Phase Errors in SASE Performance

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phase error

$$\phi = k \int \frac{dz}{v_0} \left[v_z - v_0 \right]$$

$$\sigma_\phi^2 = \langle \phi^2 \rangle$$

variable: $\tau = 2 \rho k_u z = \frac{1}{\sqrt{3}} \frac{N}{N_G}$

N_G : # of periods in one power Gain length (ideal)

- Effects of magnet errors on radiation performance have been studied by many authors. In particular,

A) <u>phase errors</u> on spontaneous radiation

(B. Kincaid)

$$R = e^{-\sigma_\phi^2} \approx 1 - \sigma_\phi^2$$

B) <u>Kick errors in</u> High-Gain FEL (Yu, Krinsky, Gluckstern, van Zeijts)

$$R = 1 - \frac{4}{9}\sigma_\phi^2$$

- We extend these ~~results~~ calculations to other cases:

Reduction due to magnet errors

	Random Kick model	Random Phase Model
Spontaneous rad.	$1 - \frac{\sigma_\phi^2}{3}$	$1 - \sigma_\phi^2$ *
High-Gain FEL	$1 - \frac{4\sigma_\phi^2}{9}$ **	$1 - \frac{\sigma_\phi^2}{3}\frac{N}{N_G}$

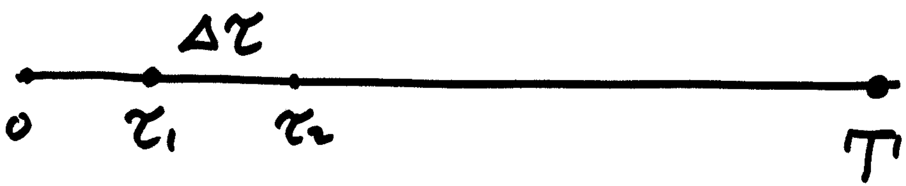
* B. Kincaid

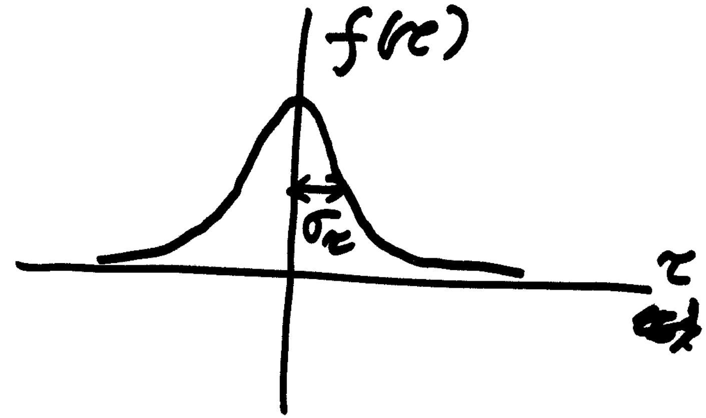
** Yu, Krinsky, Gluckstern, van Zeijts

Magnet Errors : Random Kick Model

Phase "variation" is a sequence of δ-like kicks.

- $\dfrac{d\phi}{d\tau} \equiv \Delta(\tau) = \sum_j \epsilon_j f(\tau - \tau_j)$





- $\int f(\tau)\, d\tau = 1$,

- $\sigma_\tau \lesssim \Delta\tau \ll 1$

- $\langle \epsilon_i \epsilon_j \rangle_{\text{ensemble}} = \epsilon^2 \delta_{ij}$

properties of RKM

$$\phi(\tau) = \int_0^{\tau} \Delta(\tau)\, d\tau \doteq$$

$$\phi_n = \sum_{j=1}^{n} \epsilon_j \; ; \; \tau_n < \tau < \tau_{n+1}$$

$$\sigma_\phi^2 = \frac{1}{N} \left\langle \sum_{n=1}^{N} \phi_n^2 \right\rangle$$

$$\langle \phi_n^2 \rangle = n \epsilon^2$$

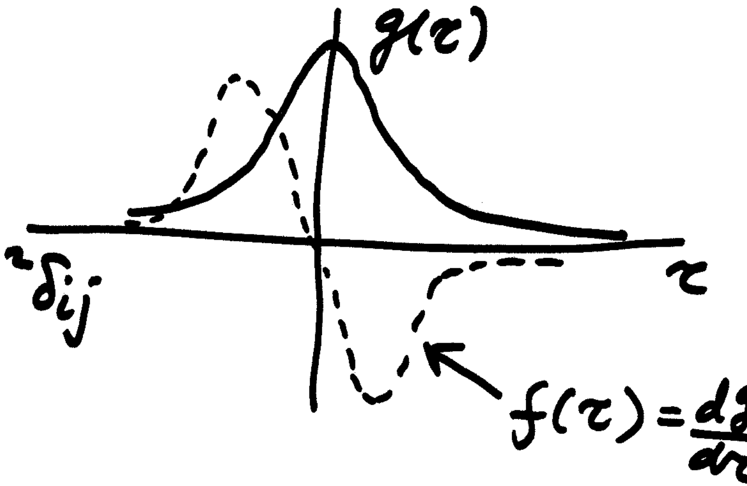
$$\sigma_\phi^2 = \frac{N(N+1)}{2N} \epsilon^2 \approx \frac{N}{2} \epsilon^2$$

If we subtract the linear part out

$$\sigma_{\Delta\phi}^2 = \left\langle \frac{1}{N} \sum_n \underbrace{(\phi_n - \xi n)^2}_{\xi \text{ determined to minimize}} \right\rangle = \frac{N\epsilon^2}{10}$$

Magnet Errors: Random Phase Model

phase error itself is a sequence of δ-like contributions.



- $\phi(\tau) = \sum_j \epsilon_j \, g(\tau - \tau_j)$

- $\int g^2(\tau)\, d\tau = 1$, $\langle \epsilon_i \epsilon_j \rangle = \epsilon^2 \delta_{ij}$

- $\Delta(\tau) \equiv \frac{d\phi}{d\tau} = \sum_j \epsilon_j \, f(\tau - \tau_j)$

  Note $\int f(\tau)\, d\tau = 0$

- $\sigma_\phi^2 = \frac{1}{T} \int \phi^2(\tau)\, d\tau = \frac{N\epsilon^2}{T} = \frac{\epsilon^2}{\Delta\tau}$

- $\int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\tau'} d\tau'' \, f(\tau') f(\tau'') \left[1, \tau'', \tau' + \tau'' \right] = \left[0, \frac{1}{2}, 0 \right]$

Derivation [Follows YKGvZ closely]

- $$\frac{d^3 a}{d\tau^3} = i a + i \frac{d^2}{d\tau^2}\left[\Delta(\tau)\, a(\tau)\right]$$

$$\Delta(\tau) = \frac{d\phi}{d\tau} \quad (\sim \text{local detuning})$$

- For $\Delta = 0$, $e^{i\lambda_j \tau}$ $\qquad \lambda_j = \left[\frac{1 \mp \sqrt{3}\, i}{2}, -1\right]$

$$\sum_j \lambda_j = 0, \quad \sum_j \lambda_j^2 = 0 = \sum_j \frac{1}{\lambda_j}$$

- SASE :

$$a_0(\tau) = \frac{\theta_0}{3} \sum_j \frac{e^{i\lambda_j \tau}}{\lambda_j}$$

- Undulator regime $(\tau \ll 1)$

$$= i\theta_0 \tau + O(\tau^4)$$

- HG

$$= \frac{\theta_0}{3\lambda_1} e^{i\lambda_1 \tau} \quad : \quad \lambda_1 : \text{leading root}$$

For $\Delta \neq 0$

- $$a(\tau) = a_0(\tau) + i\int_0^\tau \Phi(\tau-\tau')\,\Delta(\tau')\,a(\tau')\,d\tau'$$

$$\Phi(\tau) = \frac{1}{3}\sum_j e^{i\lambda_j \tau} \quad : \quad \Phi(\tau) \approx 1 + O(\tau^3),\ \tau \ll 1.$$

- To second order in Δ

$$a(\tau) = a_0(\tau) + i\int_0^\tau \Phi(\tau-\tau')\,\Delta(\tau')\,a_0(\tau')\,d\tau'$$

$$- \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' \;\Phi(\tau-\tau')\,\Phi(\tau'-\tau'')\,\Delta(\tau')\,\Delta(\tau'')\,a_0(\tau'')$$

$$|a(\tau)|^2 = |a_0(\tau)|^2 R$$

$$\boxed{R = 1 + |F_1(\tau)|^2 - (F_2 + F_2^*)}$$

$$F_1(\tau) = \frac{1}{a_0(\tau)}\int_0^\tau \Phi(\tau-\tau')\,\Delta(\tau')\,a_0(\tau')\,d\tau'$$

$$F_2(\tau) = \frac{1}{a_0(\tau)}\int_0^\tau d\tau' \int_0^{\tau'} d\tau'' \;\Phi(\tau-\tau')\,\Phi(\tau'-\tau'')\,\Delta(\tau')\,\Delta(\tau'')\,a_0(\tau'')$$

For random errors (RKM & RPM)

$$F_1(\tau) = \frac{1}{a_0(\tau)} \sum_j \epsilon_j \int_{-\infty}^{\infty} \Phi(\tau-\tau'-\tau_j)\, f(\tau')\, a_0(\tau'+\tau_j)\, d\tau'$$

$$\langle |F_1(\tau)|^2 \rangle = \epsilon^2 \sum_j \left| \int_{-\infty}^{\infty} \Phi(\tau-\tau'-\tau_j)\, f(\tau')\, \frac{a_0(\tau'+\tau_j)}{a_0(\tau)}\, d\tau' \right|$$

$$\langle F_2(\tau) \rangle = \epsilon^2 \sum_j \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\tau'} d\tau''\, \Phi(\tau-\tau'-\tau_j)\, \Phi(\tau'-\tau'')\, f(\tau')\, f(\tau'')\, a_0(\tau_j+\tau'')$$

- τ' & τ'' can be considered small.

→ can expand in τ' & τ'' appropriately.

I. Undulator regime

$\tau, \tau_j \ll 1$

$\Phi \to 1 \qquad a_0(\tau) \to \tau$

$$R = 1 + \epsilon^2 \sum_j \Bigg[\left| \int d\tau' f(\tau') \frac{(\tau' + \tau_j)}{\tau} \right|^2 - 2 \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\tau'} d\tau'' f(\tau') f(\tau'') \frac{(\tau_j + \tau'')}{\tau} \Bigg]$$

$$\boxed{\text{A) RKM} : \quad R = 1 + \epsilon^2 \sum_j \left[\left(\frac{\tau_j}{\tau}\right)^2 - 2 \frac{1}{2} \frac{\tau_j}{\tau} \right] = 1 - \frac{\epsilon^2 N}{6} = 1 - \frac{\sigma_\phi^2}{3}}$$

$$\boxed{\text{B) RPM} : \quad R = 1 + \epsilon^2 \sum_j \left[0 - 2 \frac{1}{2\tau} \right] = 1 - \frac{\epsilon^2 N}{\tau} = 1 - \sigma_\phi^2 \approx e^{-\sigma_\phi^2}}$$

II. In the exponential gain regime

$$a_0(\tau) \Rightarrow \frac{\theta_0}{3\lambda_1} e^{i\lambda_1\tau}$$

$$\Phi \Rightarrow \frac{1}{3} e^{i\lambda_1\tau}$$

- F_1 is easy:

$$\langle |F_1(\tau)|^2 \rangle = \frac{N\epsilon^2}{9} \int f(\tau')\,d\tau'$$

$$= \frac{N\epsilon^2}{9} \qquad \text{RKM}$$

$$= 0 \qquad \text{RPM}$$

- F_2 part ~~takes~~ a little calculation:

$$\langle F_2(\tau)\rangle = \epsilon^2 \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\tau'} d\tau'' \, \underbrace{\Phi(\tau-\tau'-\tau_j)}_{\frac{1}{3} e^{i\lambda_1(\tau-\tau'-\tau_j)}} \, \Phi(\tau'-\tau'') f(\tau') f(\tau'')$$

$$\times \, a_0(\tau''+\tau_j)/a_0(\tau) .$$

$$a_0(\tau) \to e^{i\lambda_1 \tau}$$

For this term
non leading exponents
also contribute

$$\langle F_2(\tau)\rangle = N\epsilon^2 \sum_{n=1}^{3} \frac{1}{9} \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\tau'} d\tau'' \, e^{i\lambda_1[\tau''-\tau']} e^{i\lambda_n[\tau'-\tau'']} f(\tau') f(\tau'')$$

$$= \frac{N\epsilon^2}{9} \sum_n \int d\tau' \int d\tau'' \, \left[1 + i\underbrace{\lambda_n(\tau'-\tau'')\big]\big[1 + i\lambda_1(\tau''-\tau')}_{1 + i(\lambda_n-\lambda_1)(\tau'-\tau'')}\right] f f$$

For RKM

$$= \frac{N\epsilon^2}{3}$$

$$\mathcal{R} = 1 + \frac{N\epsilon^2}{9} - \frac{N\epsilon^2}{3} = 1 - \frac{2}{9} N\epsilon^2$$

$$= 1 - \frac{4}{9}\sigma_\phi^2$$

For RPM, must take the second term.

Can show
$$\int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\tau'} d\tau'' \, (\tau' - \tau'') f(\tau') f(\tau'') = 1$$

$$\therefore \langle F_2 \rangle = \frac{iN\epsilon^2}{9} \sum_n (\lambda_1 - \lambda_n) = \frac{N\epsilon^2}{9} i \, [\underbrace{\lambda_1 - \overbrace{\lambda_2 + \lambda_1 - \lambda_3}^{\lambda_1}}_{3\lambda_1}] .$$

$$F_2 + F_2^* = \frac{N\epsilon^2}{3} \, 2 \, \frac{\sqrt{3}}{2} = \frac{N\epsilon^2}{\sqrt{3}}$$

$$R = 1 + 0 - \frac{N\epsilon^2}{\sqrt{3}} = 1 - \frac{\sigma_\phi^2}{\sqrt{3}} T$$

$$= 1 - \frac{\sigma_\phi^2}{3} \frac{N}{N_G}$$