17TH ADVANCED BEAM DYNAMICS WORKSHOP ON

FUTURE LIGHT SOURCES

Experimental Studies of the Nonlinear Momentum Compaction Factor

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Experimental Studies of the Nonlinear Momentum Compaction Factor*

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- nonlinear momentum compaction factor α at different values of the horizontal chromaticity

$$\xi_x = -5.5 \ldots + 8.1$$

- stable storage of particles in the storage ring with momentum deviations in the range of

$$-4\% < \Delta p/p_0 < +8\%$$

* presented at the PAC-1999
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The momentum compaction factor α (two definitions)

- differential small variations of $dp/p \approx 10^{-3}$
  - given by the energy width of the beam
  - defines the synchrotron tune

$$\text{local definition: } \alpha_p(p) = \frac{dL}{dp} \Big/ \frac{L}{p} = -\frac{df_{rf}}{dp} \Big/ \frac{f_{rf}}{p}$$

- steps of final width, variations of $\Delta p/p_0 \approx 10^{-2}$
  - given by rf-frequency detuning
  - limited by momentum acceptance of the ring

$$\text{global definition: } \alpha(p) = \frac{\Delta L}{\Delta p} \Big/ \frac{L_0}{p_0} = \frac{\Delta f_{rf}}{\Delta p} \Big/ \frac{f_{rf,0}}{p_0}$$

From Synchrotron Tune to the Momentum Compaction Factor:

- Synchrotron Tune

$$Q_{s,p}^2 = \frac{h\eta_p e V_{cav} \cos\psi_p}{2\pi\beta_p c p} \quad \rightarrow \quad \frac{Q_{s,p}^2}{Q_{s,0}^2} = \frac{\alpha_p}{\alpha_0}\frac{p_0}{p}$$

- replacing α_p and integration yields:

$$\alpha_0 \frac{p - p_0}{p_0} = \int \frac{Q_{s,0}^2}{Q_{s,p}^2}\frac{dL}{L} = -\int \frac{Q_{s,0}^2}{Q_{s,p}^2}\frac{df_{rf}}{f_{rf}}$$

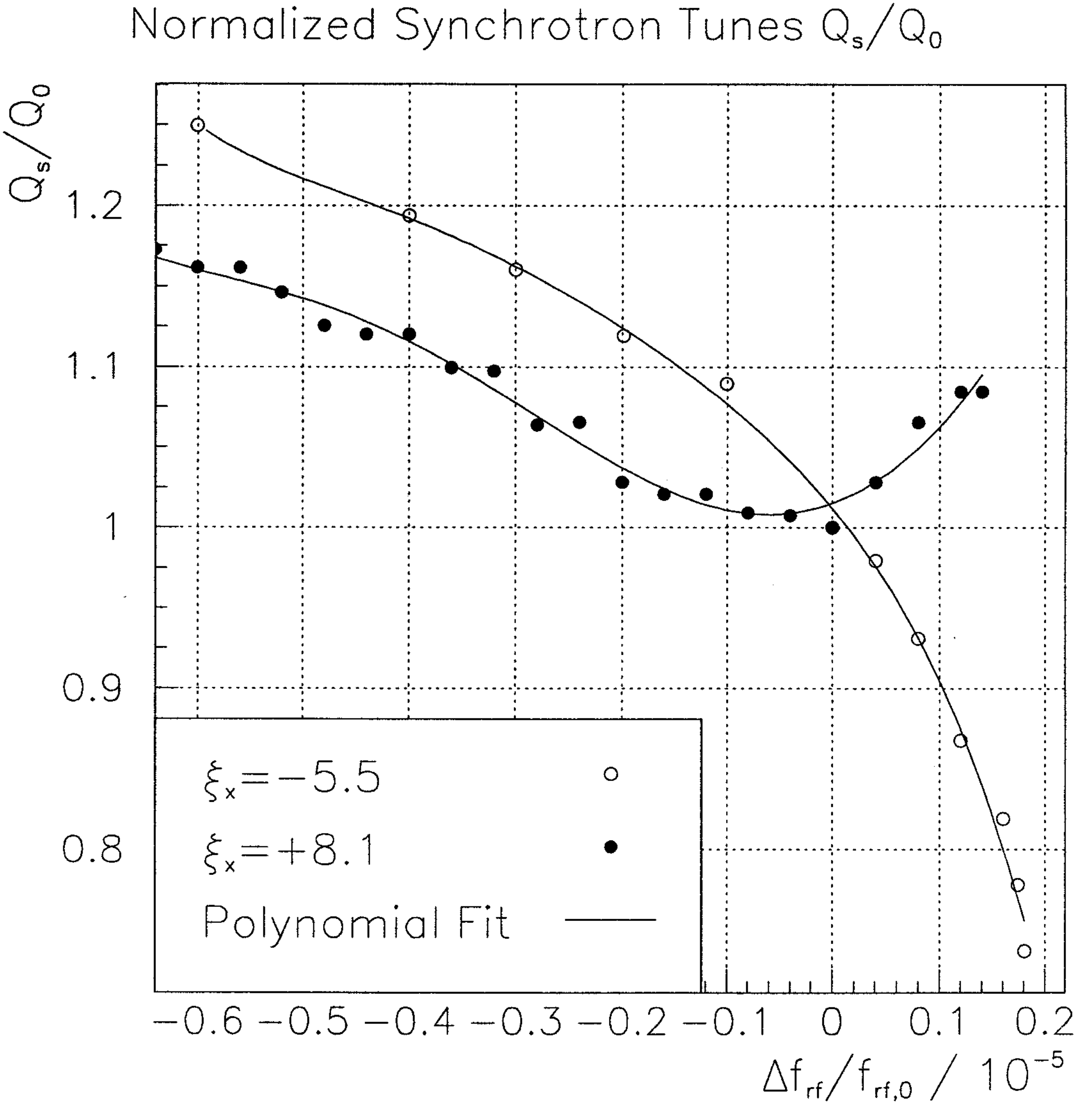
- the integrant is composed by the measured data and only a function of the rf-frequency

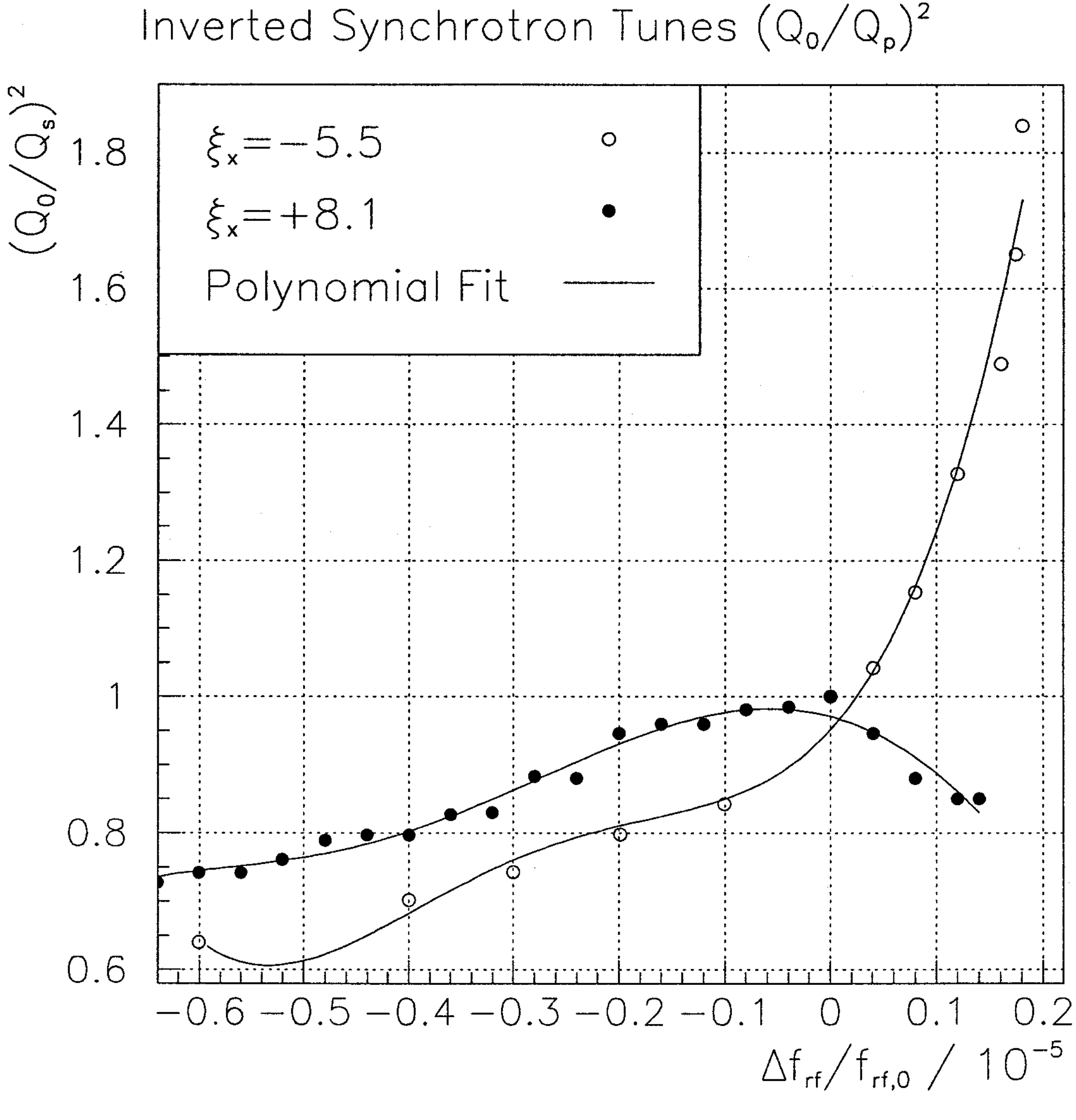
  - Polynomial fit and integration yields:

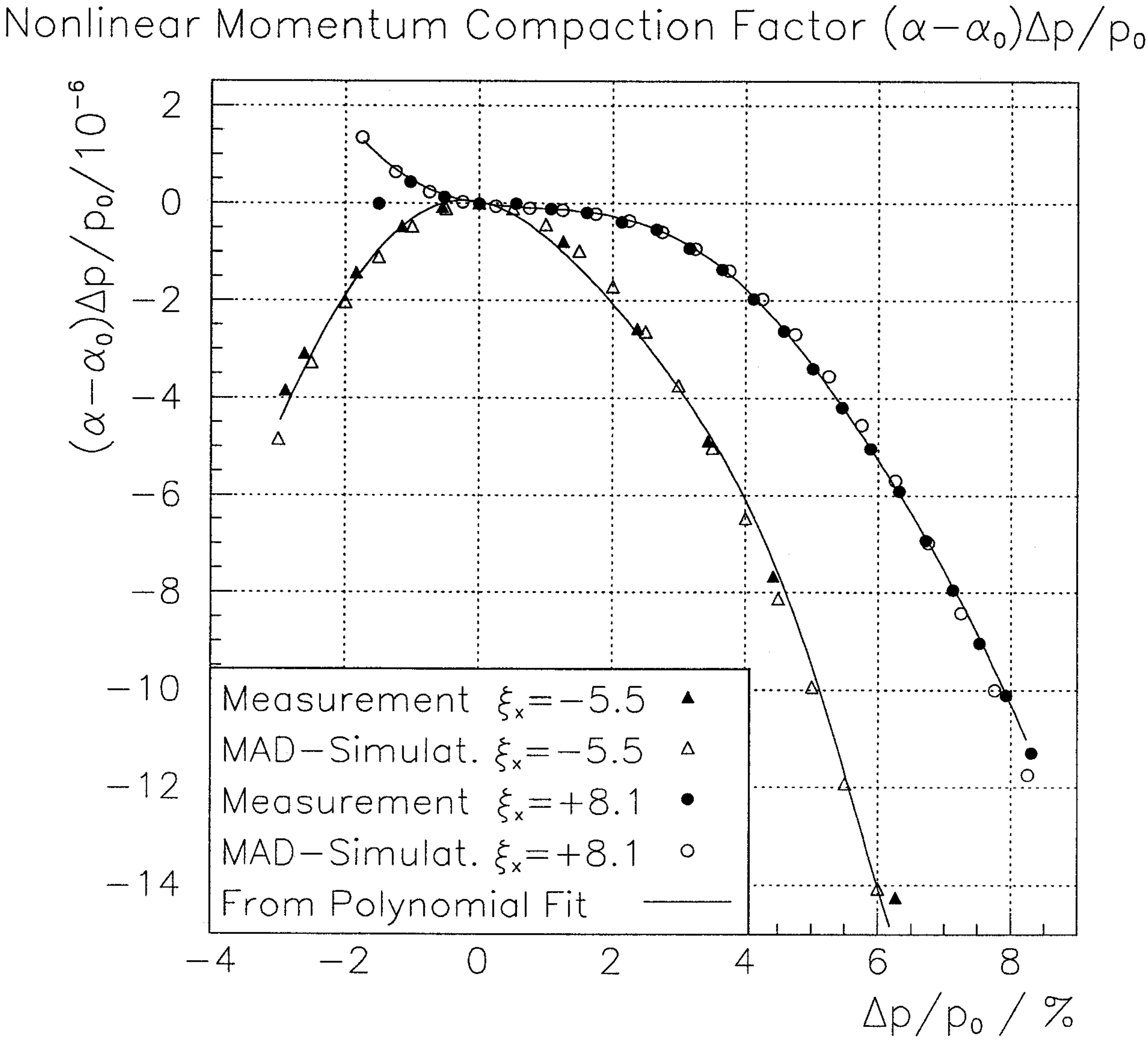
$$-\alpha_0 \frac{\Delta p}{p_0} = \frac{\Delta f_{rf}}{f_{rf,0}} + \frac{1}{2}\, a_2 \left(\frac{\Delta f_{rf}}{f_{rf,0}}\right)^2 + \frac{1}{3}\, a_3 \left(\frac{\Delta f_{rf}}{f_{rf,0}}\right)^3 + \ldots$$

  - and for the inverted expansion:

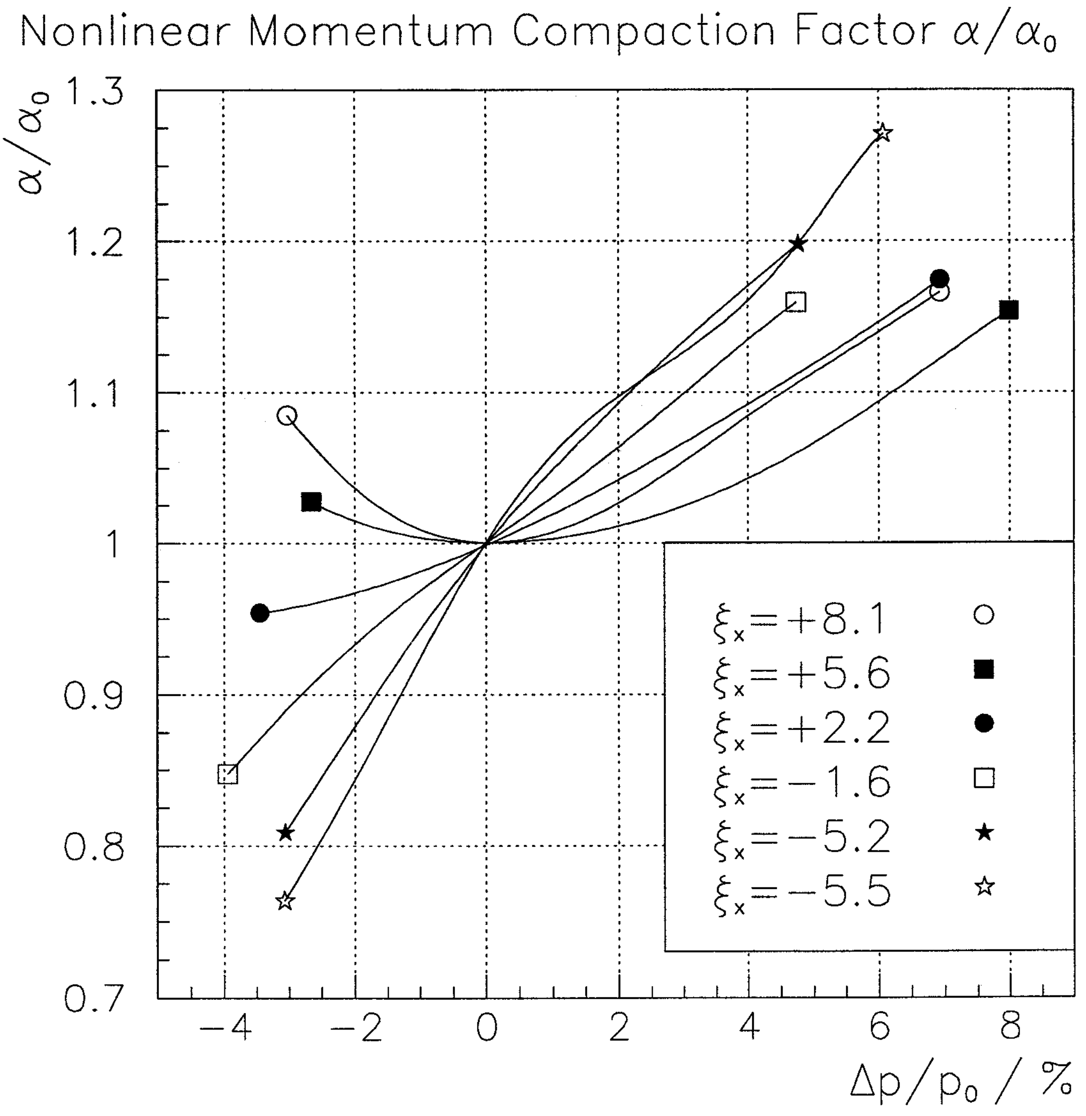
$$-\frac{\Delta f_{rf}}{f_{rf,0}} = \frac{\Delta L}{L_0} = \frac{\Delta p}{p_0}\left(\alpha_0 + \alpha_1 \left(\frac{\Delta p}{p_0}\right) + \alpha_2 \left(\frac{\Delta p}{p_0}\right)^2 + \ldots\right)$$


Normalized Synchrotron Tunes Q_s/Q_0
Q_s/Q_0
1.2
1.1
1
0.9
0.8
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
$\Delta f_{rf}/f_{rf,0}$ / 10^{-5}
$\xi_x=-5.5$
$\xi_x=+8.1$
Polynomial Fit


Inverted Synchrotron Tunes $(Q_0/Q_p)^2$
$(Q_0/Q_s)^2$
1.8
1.6
1.4
1.2
1
0.8
0.6
$\xi_x=-5.5$
$\xi_x=+8.1$
Polynomial Fit
−0.6 −0.5 −0.4 −0.3 −0.2 −0.1 0 0.1 0.2
$\Delta f_{rf}/f_{rf,0}$ / 10^{-5}



Nonlinear Momentum Compaction Factor $(\alpha-\alpha_0)\Delta p/p_0$
$(\alpha-\alpha_0)\Delta p/p_0/10^{-6}$
2
0
−2
−4
−6
−8
−10
−12
−14
Measurement $\xi_x=-5.5$
MAD−Simulat. $\xi_x=-5.5$
Measurement $\xi_x=+8.1$
MAD−Simulat. $\xi_x=+8.1$
From Polynomial Fit
−4
−2
0
2
4
6
8
$\Delta p/p_0$ / %


Nonlinear Momentum Compaction Factor α/α_0
α/α_0
1.3
1.2
1.1
1
0.9
0.8
0.7
−4
−2
0
2
4
6
8
$\Delta p/p_0$ / %
$\xi_x=+8.1$
$\xi_x=+5.6$
$\xi_x=+2.2$
$\xi_x=-1.6$
$\xi_x=-5.2$
$\xi_x=-5.5$