17TH ADVANCED BEAM DYNAMICS WORKSHOP ON
FUTURE LIGHT SOURCES

Different Alpha Operation with Realistic Effects of Potential-Well Distortion

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Different α Operation

~~STORAGE RING FREE ELECTRON LASERS~~

with

Realistic Effects of Potential-Well Distortion

* Low α
* Negative α
* Energy Spread

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Bunch lengthening due to potential-well distortion

Potential-well distortion

wakefield in the beam pipe
electromagnetic interaction with the impedance of environments (ring impedance)
=> Bunch lengthening
=> single-bunch instabilities (microwave instability)

Broad-Band resonator model

Beam pipe => low Q resonator

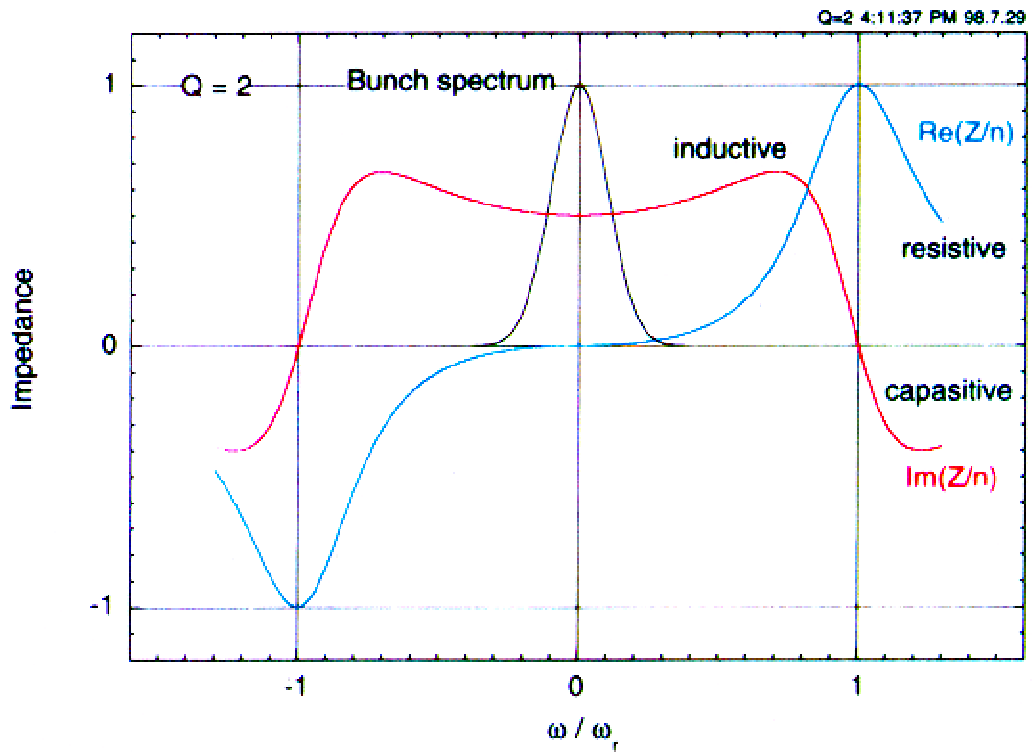
At low beam frequency regime
(long bunch length)

(inductive part)

Im $(Z/n) \approx$ ***constant***

(resistive part)

Re $(Z/n) \approx$ ***constant (may be small)***



Inductive and Resistive Impedance Model

Ring impedance $Z(\omega) = -i\,\omega\,L(\omega) + R(\omega)$

Wakefield $V_w(t) = -L\,\dfrac{d\,I(t)}{d\,t} + R\,I(t)$

Hamiltonian $H = -\dfrac{1}{2}\,\varepsilon^2 - \dfrac{1}{2}\,\tau^2 - \dfrac{e}{\alpha\,\sigma_{\varepsilon 0}{}^2\,E_0\,T_0}\displaystyle\int_0^{\tau} V_w(\bar{\tau})\,d\,\bar{\tau}$

$\varepsilon \equiv \delta e / \sigma_{\varepsilon 0}$
$\tau \equiv \delta z / \sigma_{z0}$

Vlasov equation $\dfrac{\partial\psi}{\partial t} + \dot{\tau}\dfrac{\partial\psi}{\partial\tau} + \dot{\varepsilon}\dfrac{\partial\psi}{\partial\varepsilon} = 0$ and $\dfrac{\partial\psi}{\partial t} = 0$ then $\rho(\tau) = N_p \displaystyle\int_{-\infty}^{+\infty} \psi(H)\,d\varepsilon$

Electron population $\rho(\tau) = K\,\exp\left(-\dfrac{1}{2}\,\tau^2 - a\,L\,N_p\,\rho(\tau) - a\,R\,N_p \displaystyle\int_0^{\tau} \rho(\bar{\tau})\,d\,\bar{\tau}\right)$

K ; Normalization factor
$a \equiv e^2 / \alpha\,\sigma_{\varepsilon 0}{}^2\,E_0\,T_0$

Solution in a differential equation (Haissinski equation)

$$\frac{df}{dt} = -\frac{t\,f \pm \xi f^2}{1 \pm f} \qquad \left(\begin{matrix} + \\ - \end{matrix}\ for\ \begin{matrix} \alpha > 0 \\ \alpha < 0 \end{matrix}\ \ and\ \ t \equiv \tau \right)$$

Normalized distribution function: $f \equiv a\,L\,N_p\,\rho(t)$

Relative resistive impedance: $\xi \equiv R\,\sigma_{\tau 0} / L$

Universal normalization factor $a \equiv e^2 / \alpha\,\sigma_{\varepsilon 0}{}^2\,E_0\,T_0$

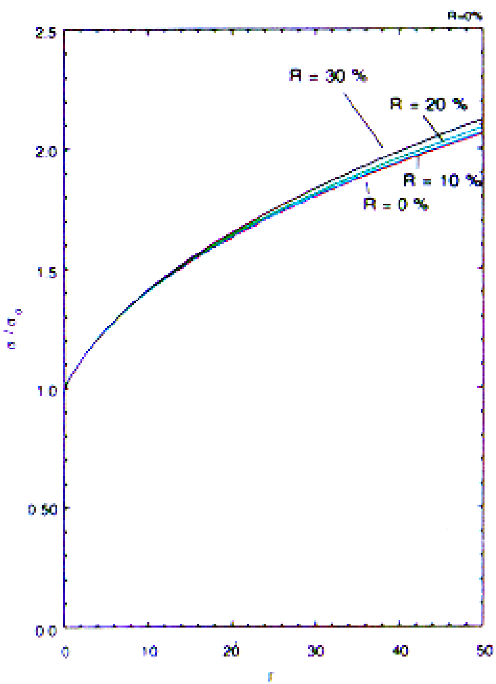
Equation can be generalized by using a normalized charge. Γ

$$\Gamma \equiv \int_{-\infty}^{+\infty} f\,dt = \frac{a\,Z\,N_p}{\sigma_{\tau 0}} = \frac{\tilde{I}\,Z}{|\alpha|\left(E_0/e\right)\sigma_{\varepsilon 0}{}^2\sigma_{\tau 0}} = \frac{\tilde{I}\,Z\,h}{2\,\pi f_{RF}{}^2\,\hat{V}_{RF}\,\sigma_{\tau 0}{}^3} \qquad Z \equiv L + \sigma_{\tau 0}\,R$$

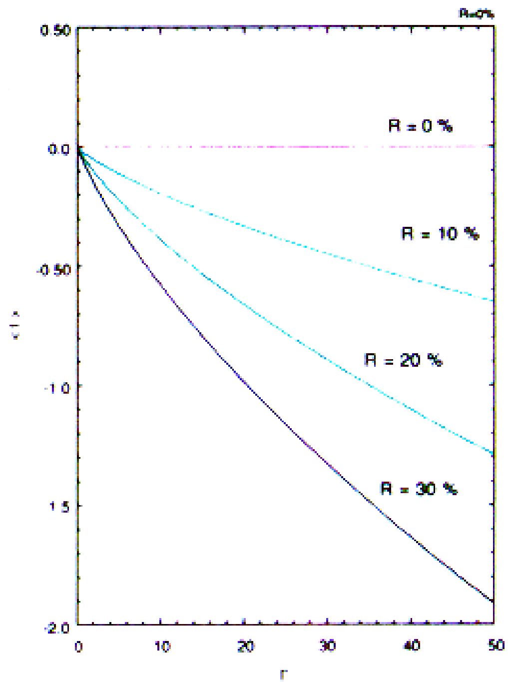
Normalized impedance $\mathbf{Im}\,|Z/n| + \mathbf{Re}\,|Z/n| \equiv \frac{f_{rf}}{h} Z$, then $\Gamma = \left(\mathbf{Im}\,|Z/n| + \mathbf{Re}\,|Z/n|\right) \frac{\tilde{I}\,h^2}{2\,\pi f_{RF}{}^3\,\hat{V}_{RF}\,\sigma_{\tau 0}{}^3}$

(The formulae are valid below the threshold for microwave instability)

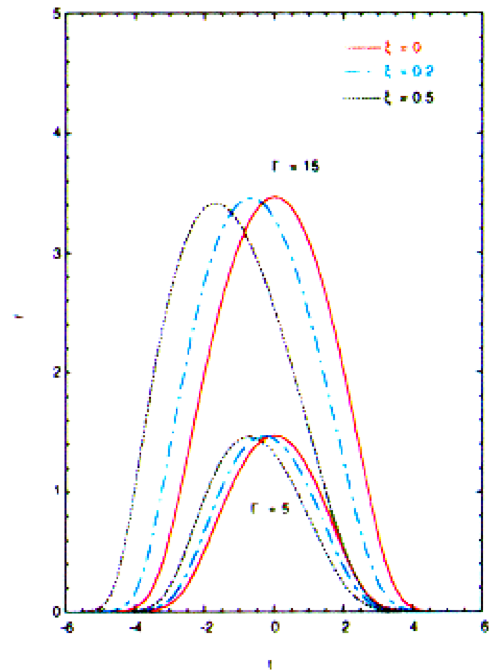
Bunch Length



Centroid Location



Bunch Shape

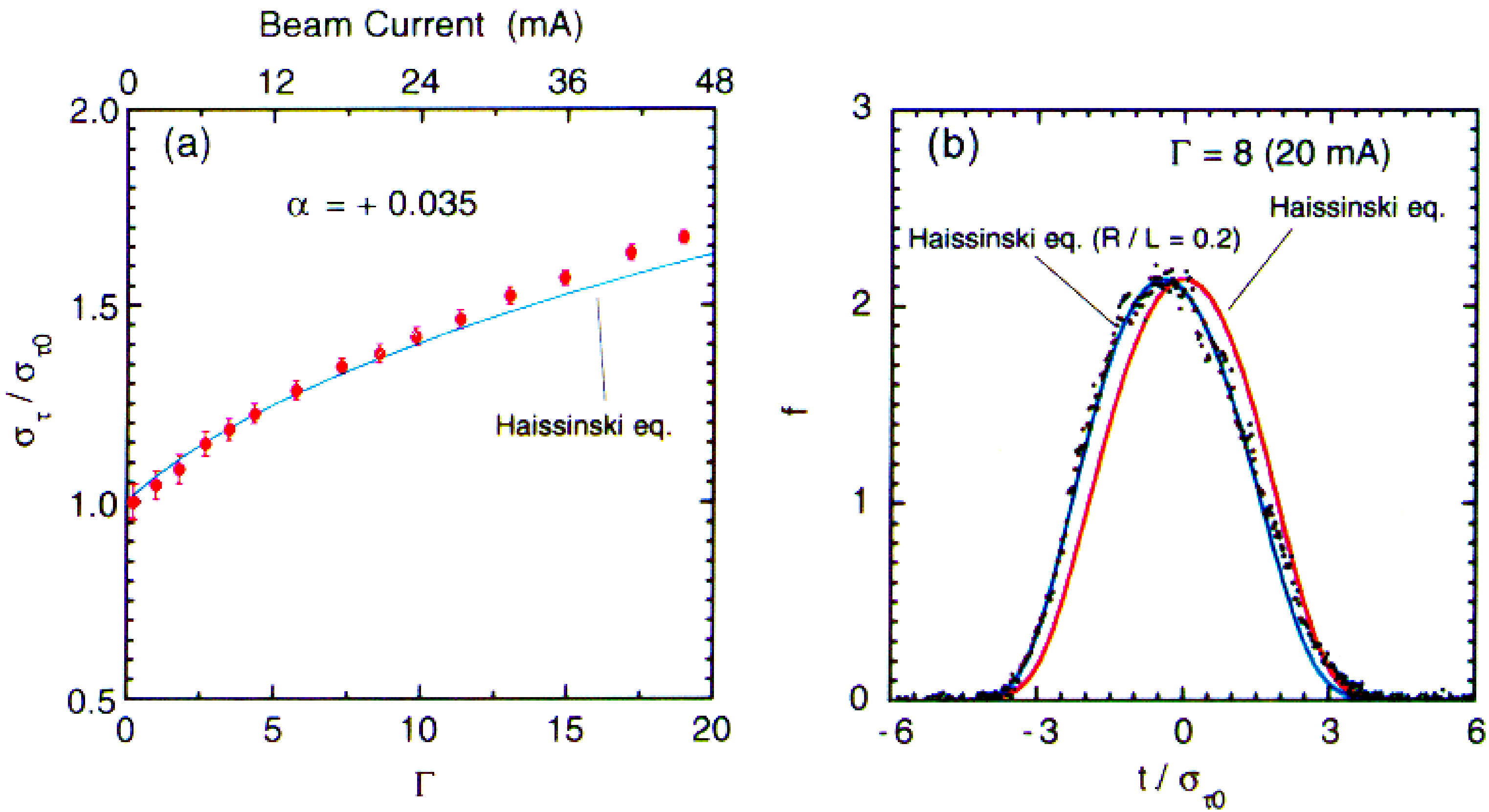


Bunch length;	Inductive impedance (***L***) dominant
Centroid;	Resistive impedance (***R***) dominant (Loss factor)
Bunch shape;	Asympetric due to resistive impedance

Experimental; Bunch length and shape (UVSOR)

Bunch lengthening

Bunch shape



UVSOR ring: $L = 130\ nH$ $\left(\text{Im}\,[Z/n] = 0.73\ \Omega\right)$ $\text{Re}\,[Z/n] \approx 0.15\ \Omega$ (bunch length dependence)

Longitudinal Instability and Energy Spread

Solution of Vlasov eq. <= Liouville's theorem => Preserve longitudinal phase space **size**

Phase stability criterion: $$\frac{d}{dt}\left(V_{RF} + V_{wake}\right) < 0$$

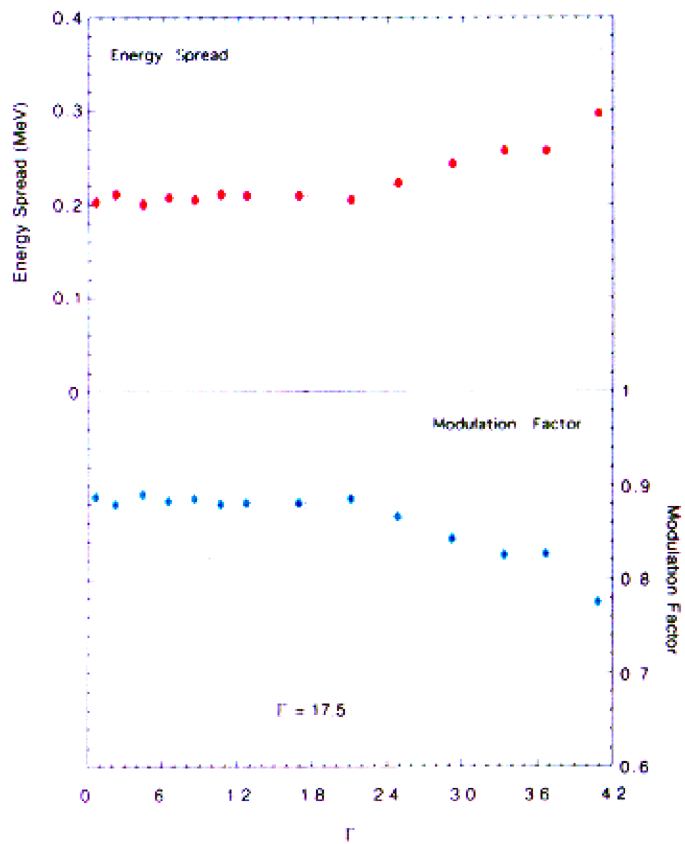
Assuming a parabolic bunch shape at <u>the high current</u> !

The bunch shape: $$I(t) = \frac{3\,\tilde{I}\,h}{4\sqrt{5}\,\sigma_p f_{RF}}\left(1 - \frac{1}{5\,\sigma_p^{\,2}}\,t^2\right)$$

$$\frac{d}{dt}\left(V_{RF} + V_{wake}\right) = -2\pi f_{RF}\,\hat{V}_{RF} + \frac{3\,\tilde{I}\,h\,L}{10\sqrt{5}\,\sigma_p^{\,3} f_{RF}} < 0 \qquad \Gamma = \frac{\tilde{I}\,h\,L}{2\pi f_{RF}^{\,2}\,\hat{V}_{RF}\,\sigma_0^{\,3}}$$

$$\Gamma < \frac{10\sqrt{5}}{3}\left(\frac{\sigma_p}{\sigma_0}\right)^3 \quad \text{where} \quad \sigma_p = \frac{3\sqrt{2\pi}}{4\sqrt{5}}\,\sigma\text{, then} \qquad \Gamma < \frac{9\sqrt{2\pi^3}}{16}\left(\frac{\sigma}{\sigma_0}\right)^3$$

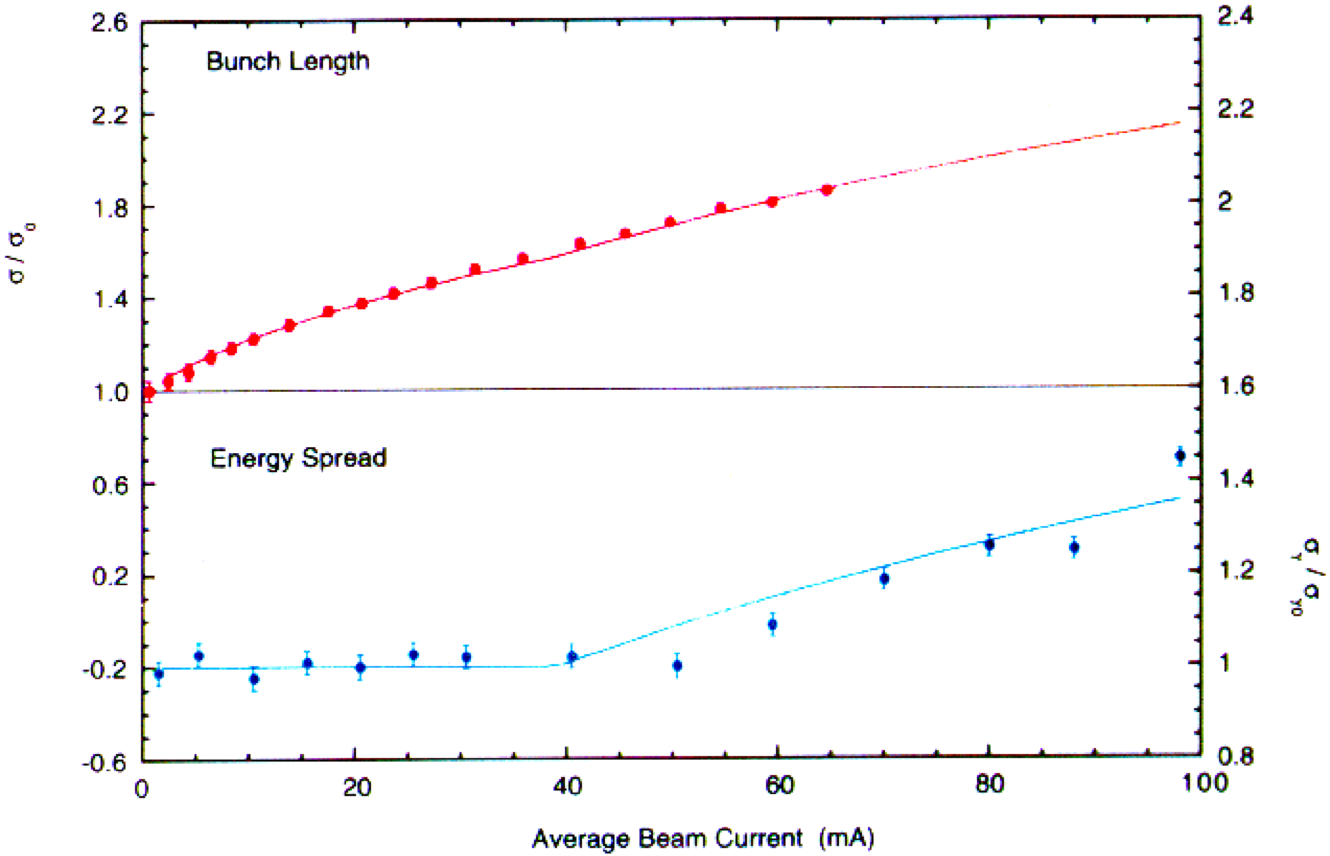
$$\Gamma < 4.43\left(\frac{\sigma}{\sigma_0}\right)^3 \qquad \underline{\Gamma \lesssim 17.5}$$



Energy spread may be increased avobe the upper bound ! !

Ideal case; Inductive impedance dominant (**R=0**) and no mode-coupling

Since $\sigma_0 \propto \frac{\delta E}{E} (= \sigma_\gamma)$ if $\Gamma > 17.5$ maybe $\sigma_0{}' = \left(\frac{\Gamma}{17.5}\right)^{1/3} \sigma_0,$ $\sigma_\gamma{}' = \left(\frac{\Gamma}{17.5}\right)^{1/3} \sigma_\gamma$ and $\Gamma' = 17.5$



What We Should Do on the UVSOR Ring

Need more peak current !! for FEL

How about "short bunch" !?

$\sigma_0 \longrightarrow \frac{1}{2}\sigma_0$

$\alpha \longrightarrow \frac{1}{4}\alpha$

$\hat{V}_{RF} \longrightarrow 4\hat{V}_{RF}$

To realize a short bunch ...

Narural bunch length $\sigma_0 = \frac{1}{f_{RF}}\sqrt{\frac{E/e\,h}{2\pi}\frac{\alpha}{\hat{V}_{RF}}}$

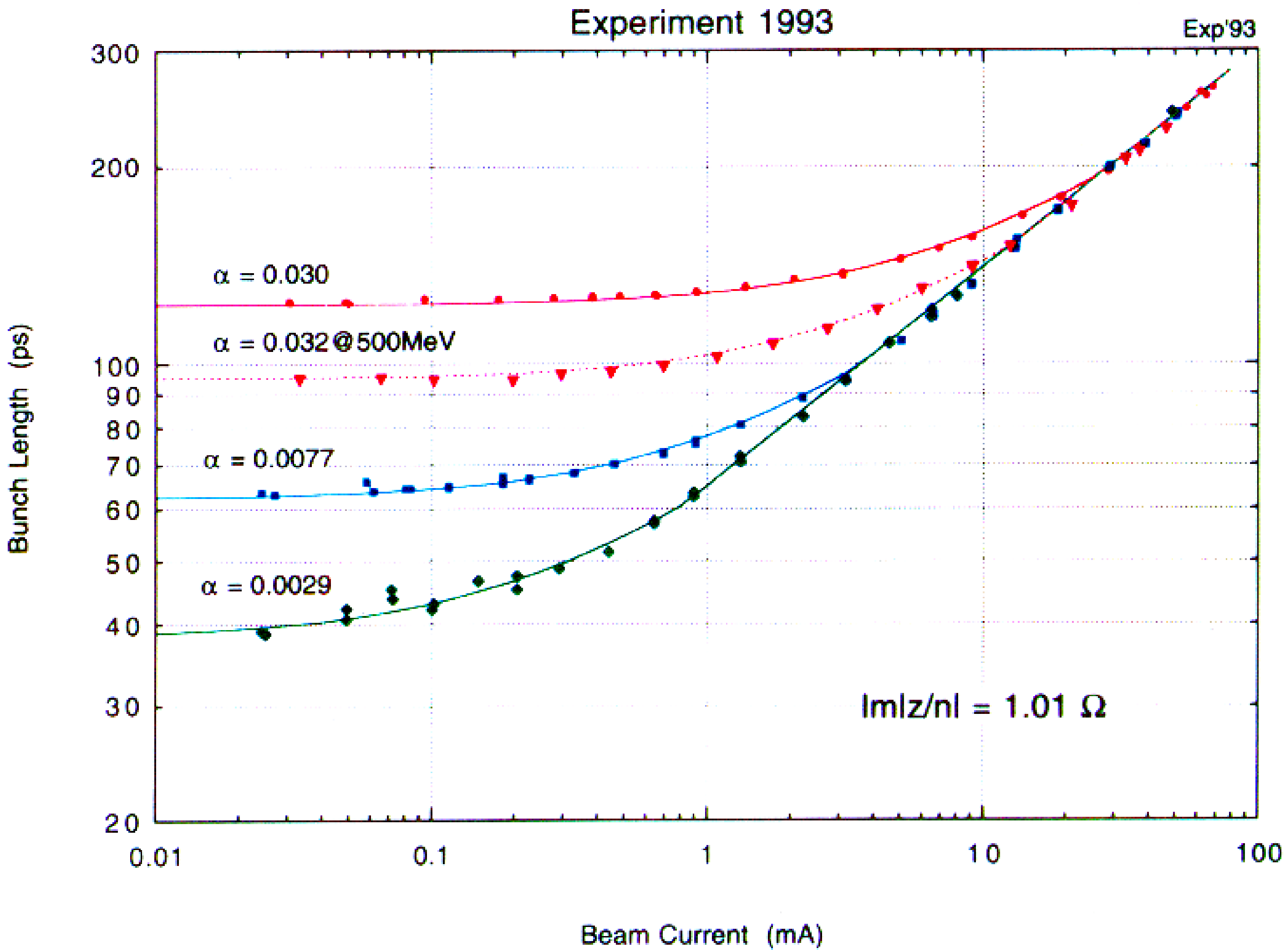
1) Low momentum compaction factor $\sigma_0 \propto \sqrt{\alpha}$

2) High peak RF voltage $\sigma_0 \propto \sqrt{1/\hat{V}_{RF}}$

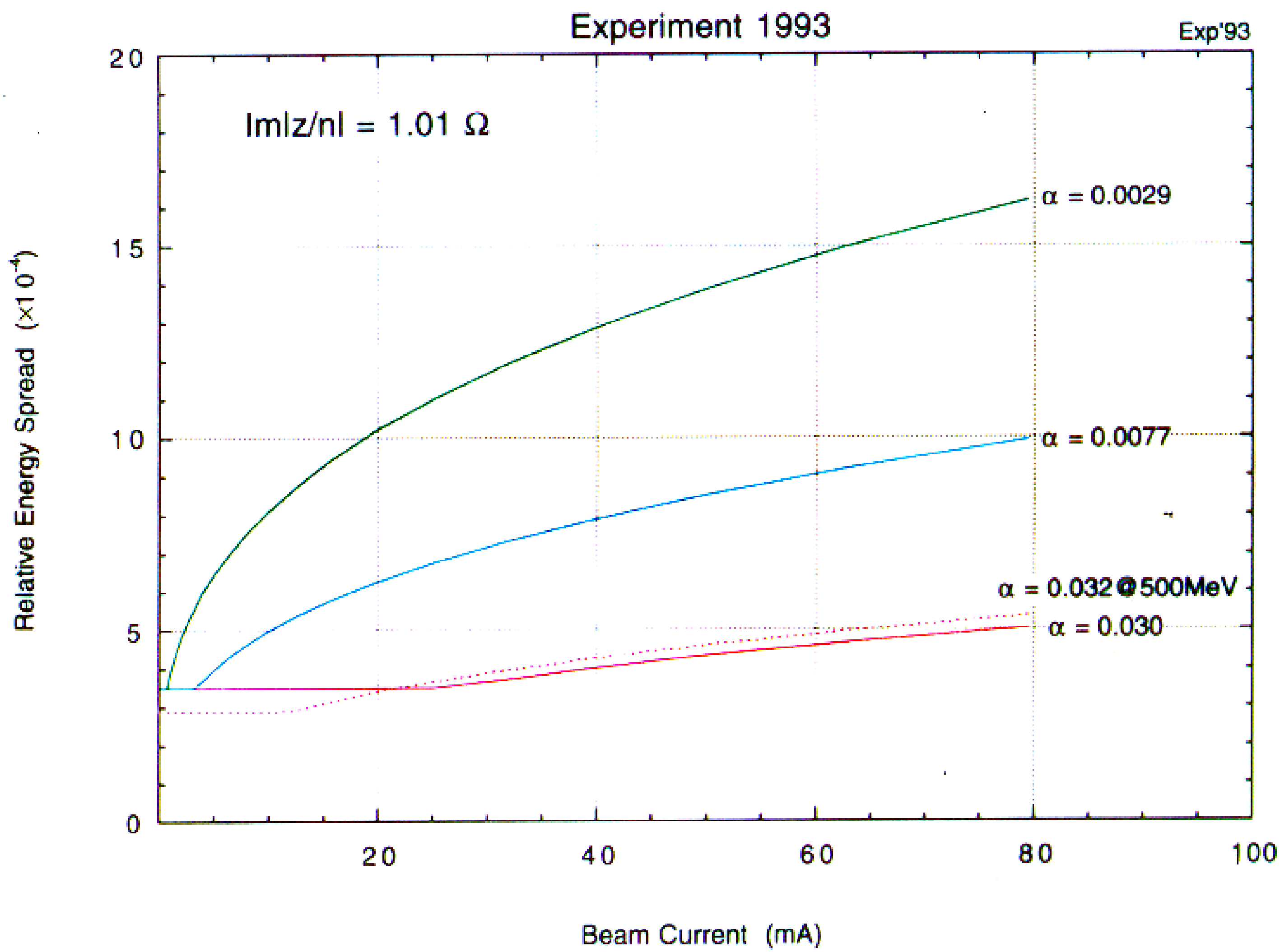
However $\Gamma = \frac{\tilde{I}\,h\,L}{2\pi f_{RF}^{\,2}\,\hat{V}_{RF}\,\sigma_0^{\,3}}$ $\sigma_0 \to \frac{1}{2}\sigma_0$ $\alpha \to \frac{\alpha}{4}$ $\Gamma \to 8\Gamma$ $\hat{V} \to 4\hat{V}$ $\Gamma \to 2\Gamma$

Short bunch increases Γ => Low threshold current for phase unstable (increase of energy spread)

Bunch Lengthening at Different Momentum Compaction

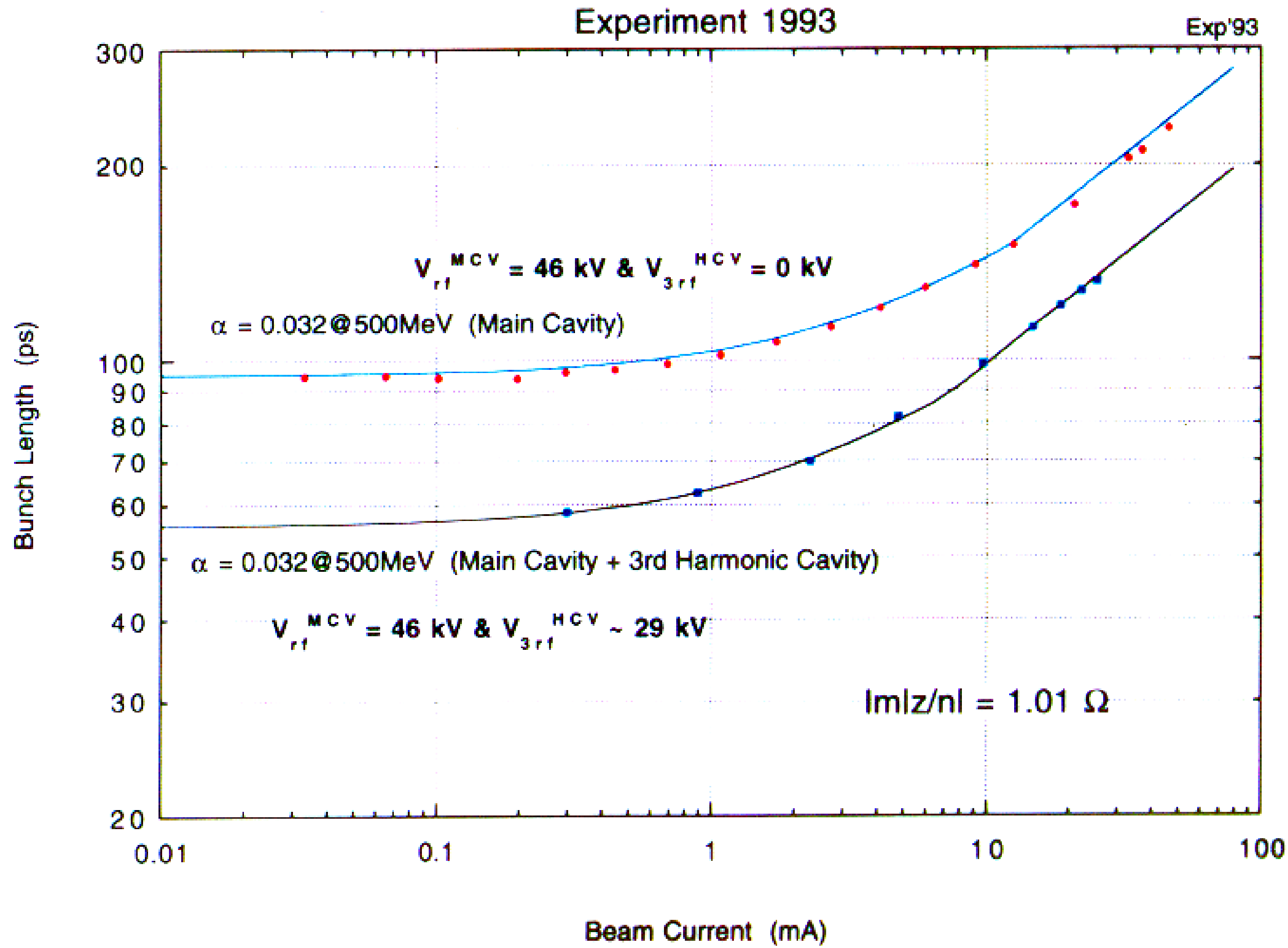


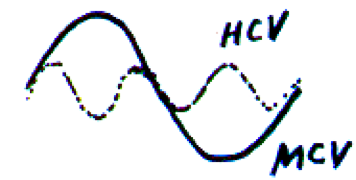
Energy Spread at Different Momentum Compaction



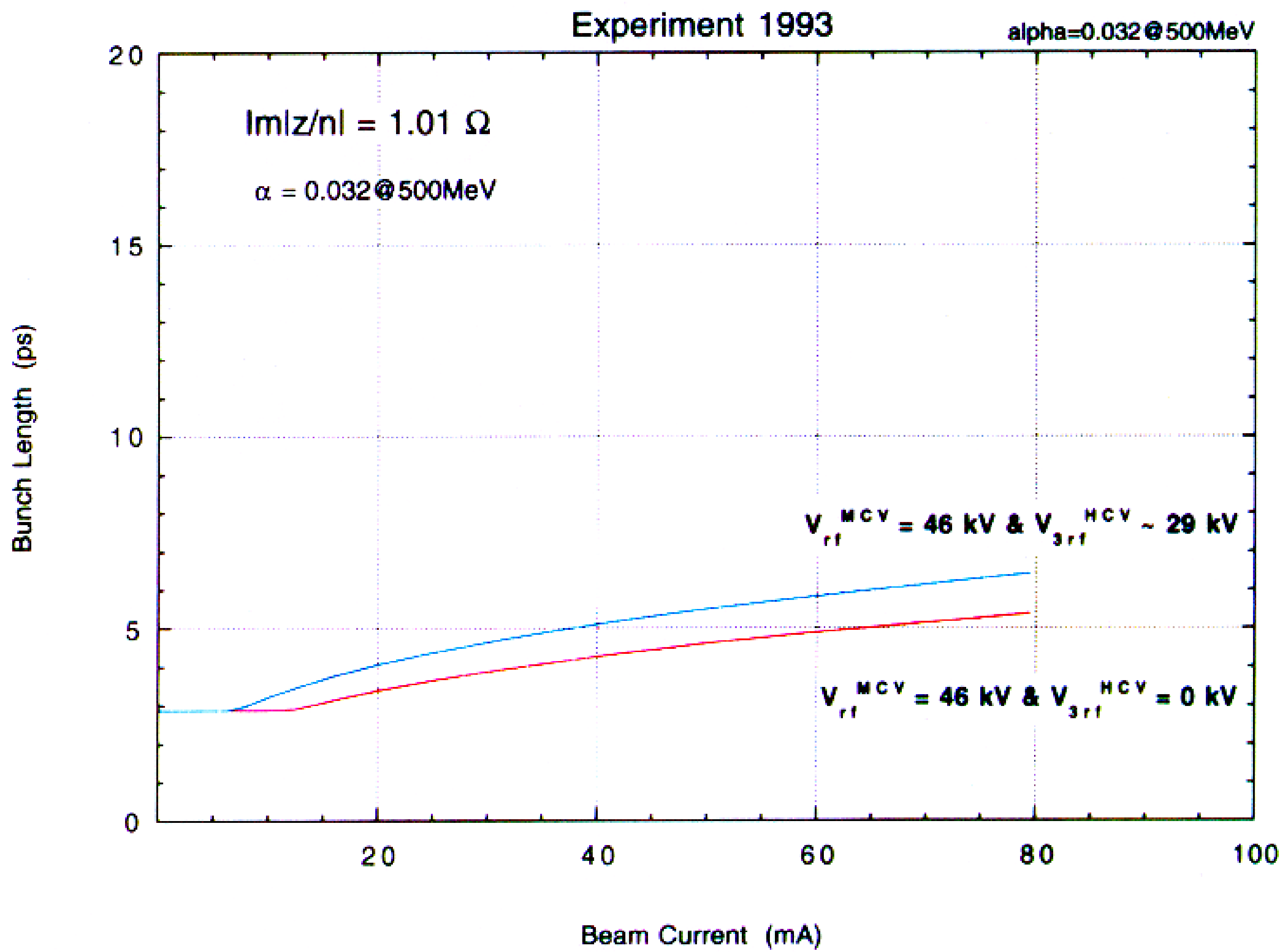
Bunch Length with Harmonic Cavity (steep effective RF slope)

$$\hat{V}_{eff} = \hat{V}_{main} + 3 \cdot \hat{V}_{HCV}$$

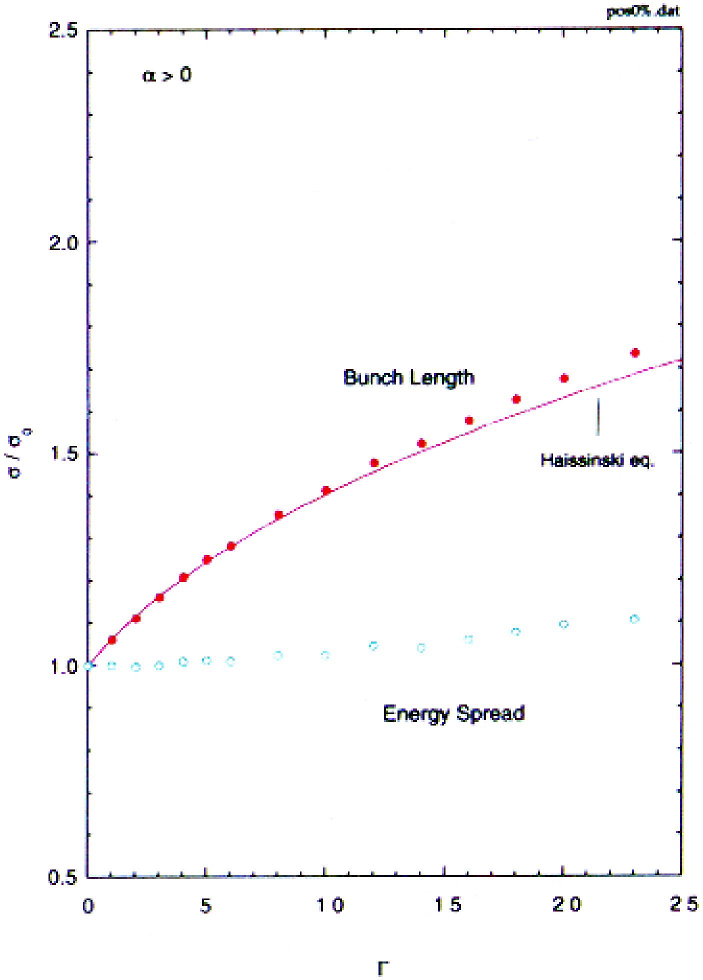


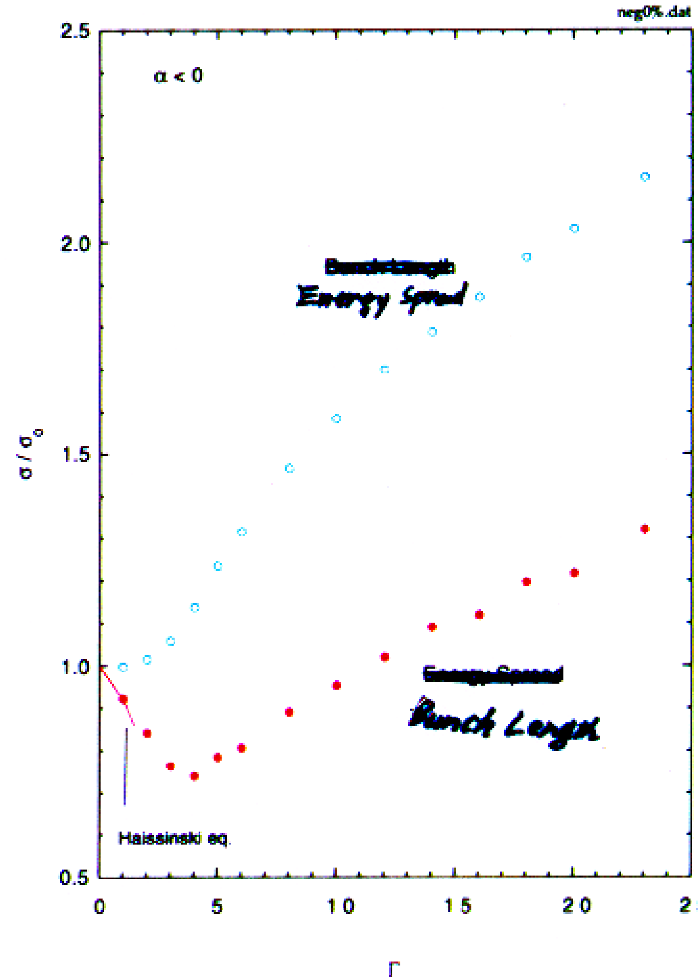


Energy Spread with Harmonic Cavity (steep effective RF slope)



Simulation of Potential-Well Distortion





$\alpha < 0$ No solution Haissinski eq
at $\Gamma > 1.56$

Normalization to Γ

$$\Gamma = \frac{\tilde{I}\,L}{|\alpha|\,(E_0/e)\,\sigma_{\varepsilon 0}{}^2\sigma_{\tau 0}}$$

Good fit with $L = \cancel{70}\ 130\ nH \quad \Rightarrow \quad Im\left[Z/n\right] = \frac{L}{\cancel{\omega_{rev}}\ f_{rev}} \approx \cancel{2.5}\ 0.8\ \Omega$