17TH ADVANCED BEAM DYNAMICS WORKSHOP ON
FUTURE LIGHT SOURCES

Care and Feeding of Coherent Synchrotron Radiation

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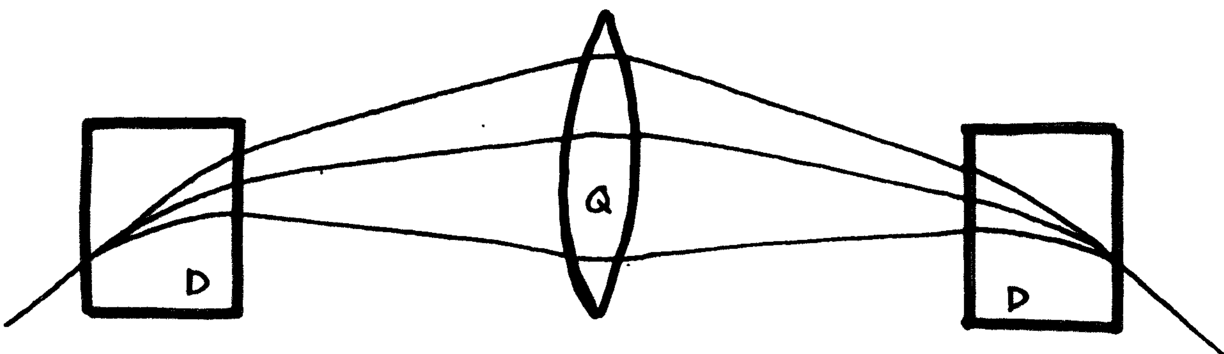
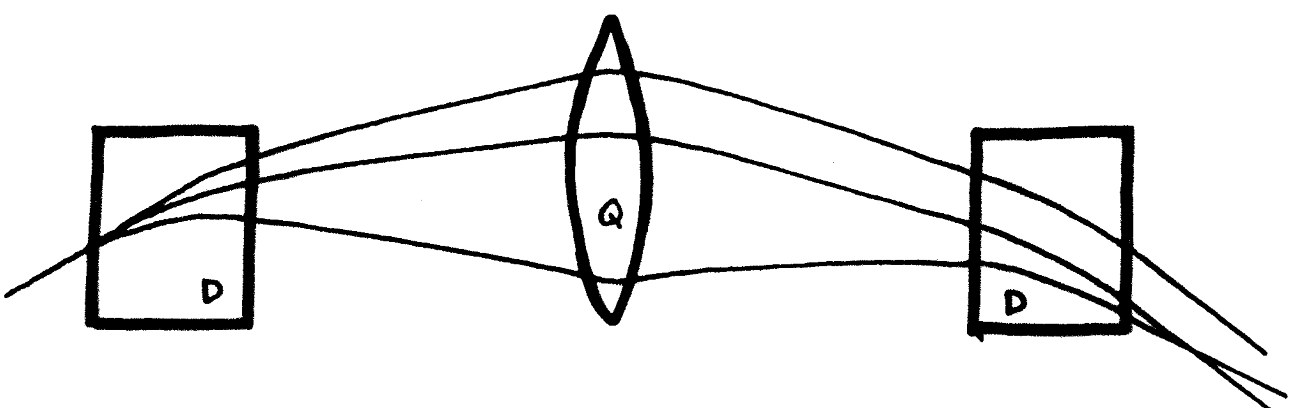
INDUCED ENERGY SPREAD AND EMITTANCE GROWTH

Example: Achromatic bend through angle θ.

Let ΔE = particle's energy change during bend.



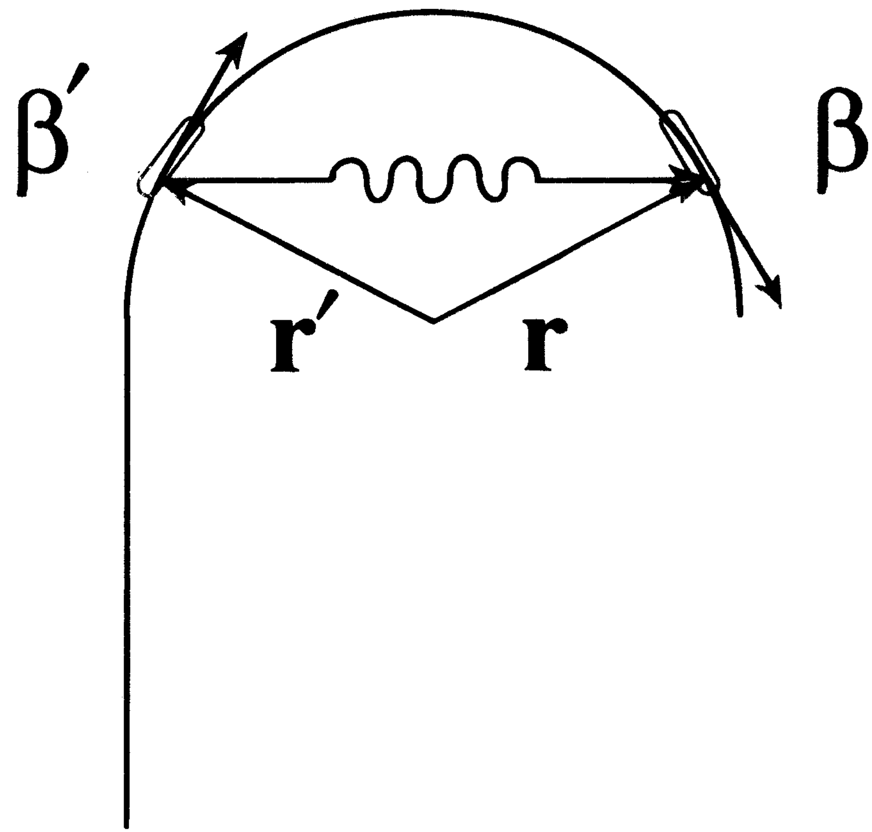
$\Delta E = 0$ (ideal case) $\Delta\theta = 0$



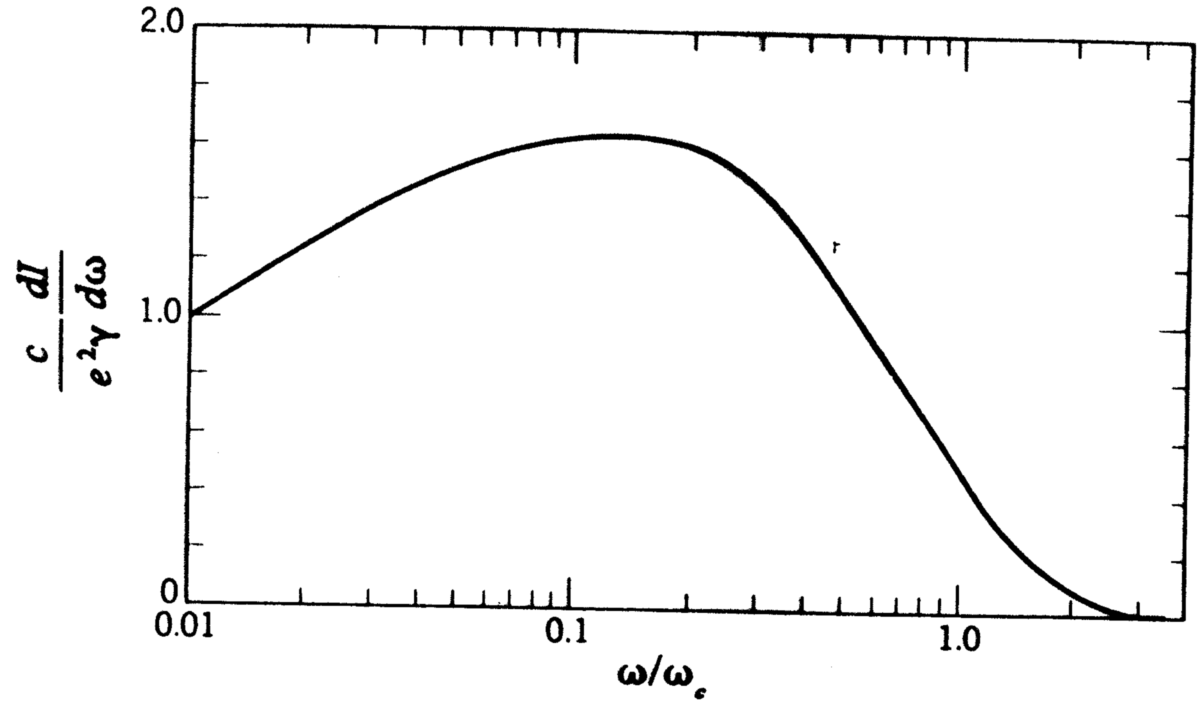
$\Delta E \neq 0$ $\Delta\theta = \theta(\Delta E/E)$

Naively expect emittance growth $\propto \Delta E_{rms}$.

BUNCH SELF-INTERACTION



Synchrotron Radiation Spectrum for Circular Motion



$$\omega_c = 3\gamma^3 \frac{c}{R} \longleftrightarrow \lambda_c = \frac{2\pi}{3\gamma^3} R.$$

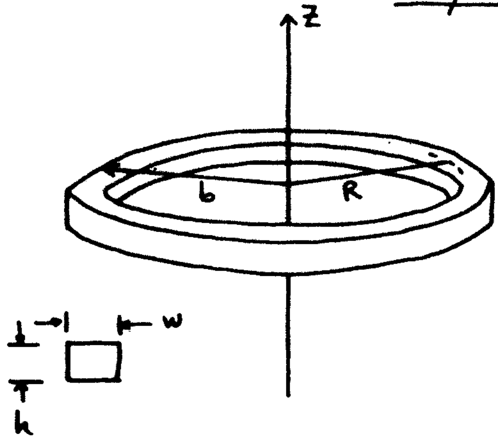
IR Demo: $E = 42$ MeV $\Rightarrow \gamma = 83$, $R = 1$ m, $\sigma \sim 1$ ps.

$$\Rightarrow \lambda_c = 3.6\,\mu\text{m} \simeq 0.01\,\text{ps}.$$

For $\lambda \gtrsim \sigma \gg \lambda_c$ (<u>CSR regime</u>),

$$\frac{dI}{d\omega} \simeq 3.25\,\frac{e^2}{c}\left(\frac{\omega R}{c}\right)^{1/3} \Rightarrow \underline{\gamma\text{-independent.}}$$

Synchronism Condition



Conducting walls increase the phase velocity of EM waves:

$$v_p^{EM} = \frac{\omega}{k}$$
$$\simeq \frac{c}{bk}\left[\left(\frac{\pi p b}{h}\right)^2 + n^2 + \left(\frac{3\pi}{4}\right)^{2/3} n^{2/3}\right]^{1/2}$$

n = azimuthal (θ) index

p = axial (z) index

Condition for pronounced coupling: $v_p^{beam} = v_p^{EM}$

$$v_p^{beam} = \frac{\omega_0}{k_0} = \frac{\omega_0}{k} n \; ; \quad \omega_0 = \frac{\beta c}{R}$$

$$\Rightarrow \quad \omega_0 n = \omega(n)$$

Lowest-order coupled mode (with $\beta = 1$):

$$n_c \simeq \pi \frac{R}{h}\left(\frac{R}{w}\right)^{1/2} .$$

For a square toroid, $w = h = 2\rho$, and

$$\boxed{n_c \simeq \pi \left(\frac{R}{2\rho}\right)^{3/2}}$$

Same considerations apply in, e.g., a 180°-bend.

Suppression of Coherent Synchrotron Radiation Effects

RMS harmonic for Gaussian longitudinal bunch profile:

$$f(t) = \frac{\exp\left(-t^2/2(\sigma_\ell/c)^2\right)}{(\sigma_\ell/c)\sqrt{2\pi}} \Rightarrow F(\omega) = \exp\left(-\omega^2/2(c/\sigma_\ell)^2\right).$$

$$\omega = n\omega_0 = nc/R \qquad \Rightarrow \qquad F(n) = \exp\left(-\frac{n^2\sigma_\ell^2}{2R^2}\right)$$

$$n_{rms} = \frac{R}{\sigma_\ell}$$

Conditions for CSR suppression:

$$n_{rms} \ll n_c$$

$$R \gg \frac{8\rho^3}{\pi^2\sigma_\ell^2} \Rightarrow \text{CSR is unimportant.}$$

Storage rings satisfy this criterion.

Apply to 180°-bend for FEL 1st pass:

$$R = 1\,\text{m}$$

$$\frac{8\rho^3}{\pi^2\sigma_\ell^2} = \frac{8\,(0.025)^3}{\pi^2\underbrace{(0.0006)^2}_{=2\,\text{ps}}} = 35\,\text{m}$$

$$R \ll \frac{8\rho^3}{\pi^2\sigma_\ell^2}$$

$\Rightarrow$ CSR <u>is</u> a concern in the FEL.

Ingredients of CSR Theory

Single-particle hamiltonian:

$$H = c\sqrt{\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + (mc)^2} + e\phi ,$$

$$\begin{bmatrix} \mathbf{A}(\mathbf{r},t) \\ \\ \phi(\mathbf{r},t) \end{bmatrix} = e \int d\mathbf{r}' dt' \, \frac{\delta\left(t - t' - \frac{1}{c}|\mathbf{r} - \mathbf{r}'|\right)}{|\mathbf{r} - \mathbf{r}'|} \begin{bmatrix} \boldsymbol{\beta}' \\ \\ 1 \end{bmatrix} n(\mathbf{r}', t') .$$

Change in total single-particle energy:

$$\frac{dH}{dt} = e\,\frac{\partial \mathcal{V}}{\partial t} \; ; \quad \mathcal{V} \equiv \phi - \boldsymbol{\beta}\cdot\mathbf{A} .$$

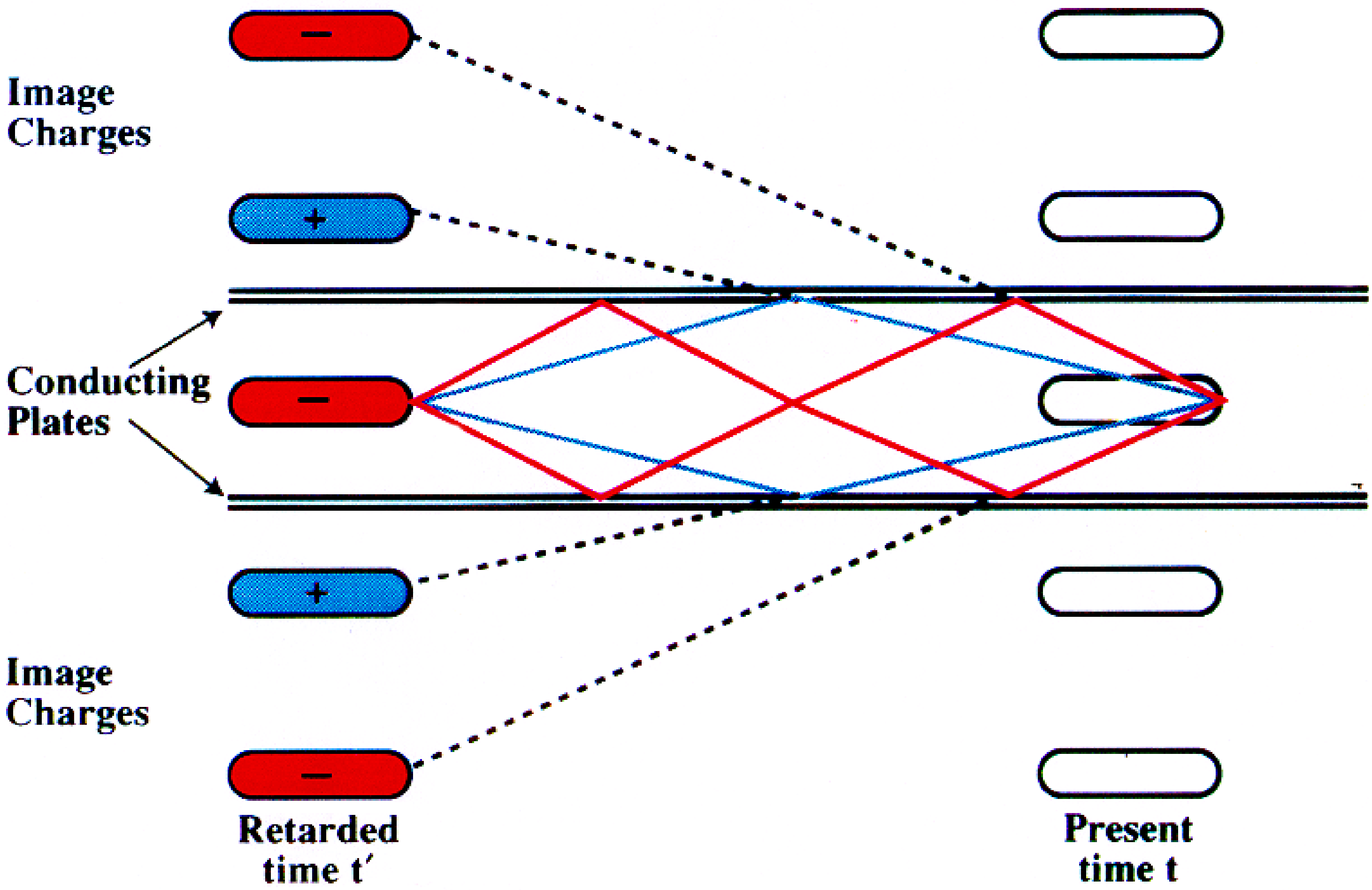
Change in kinetic single-particle energy:

$$mc^2 \frac{d\gamma}{dt} = -e\,\frac{d\phi}{dt} + e\,\frac{\partial \mathcal{V}}{\partial t} .$$

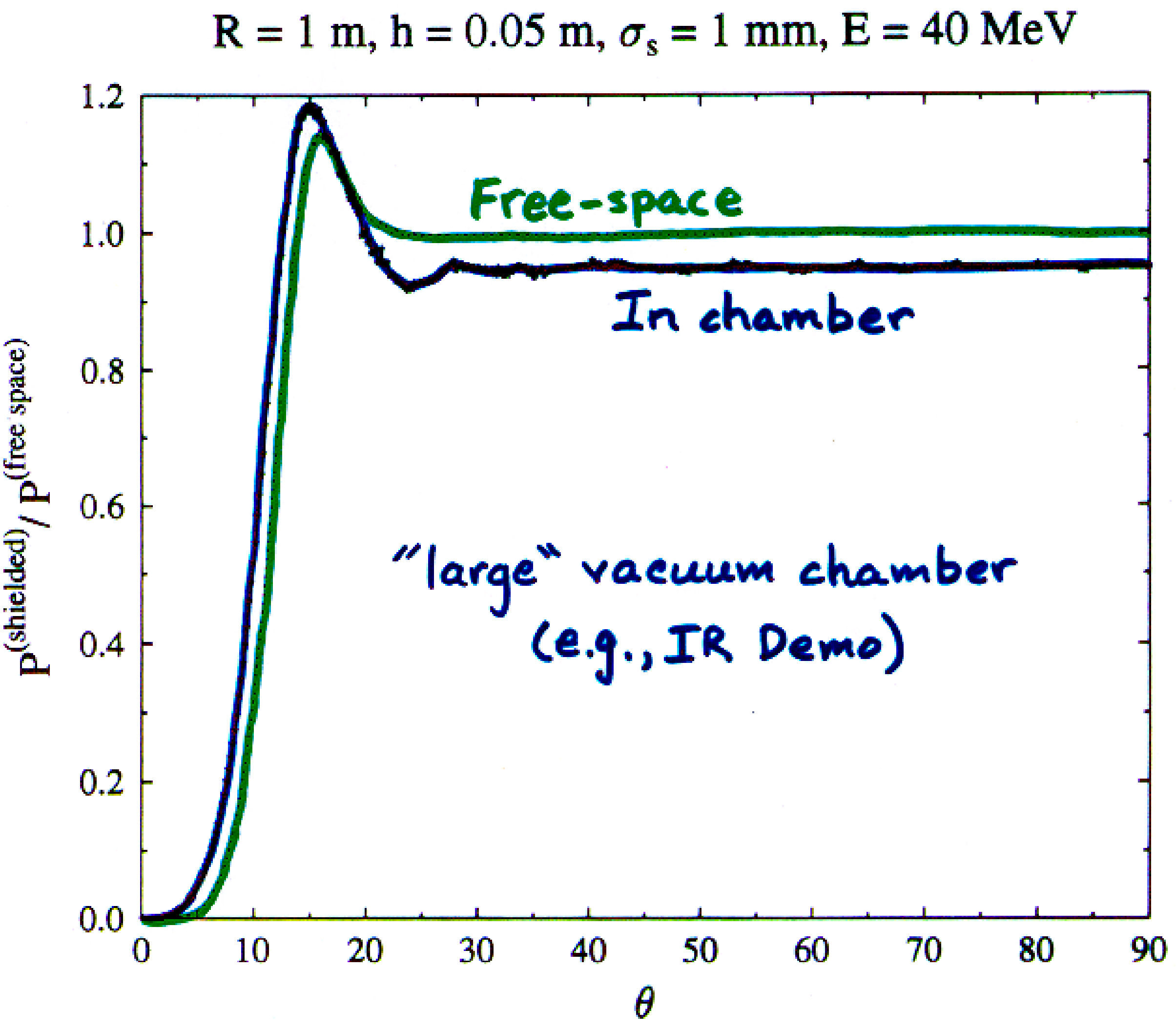
Rate of work done by bunch against itself:

$$P(t) = -\int d\mathbf{r} \;\; mc^2 \frac{d\gamma(\mathbf{r},t)}{dt} \; n(\mathbf{r},t) .$$

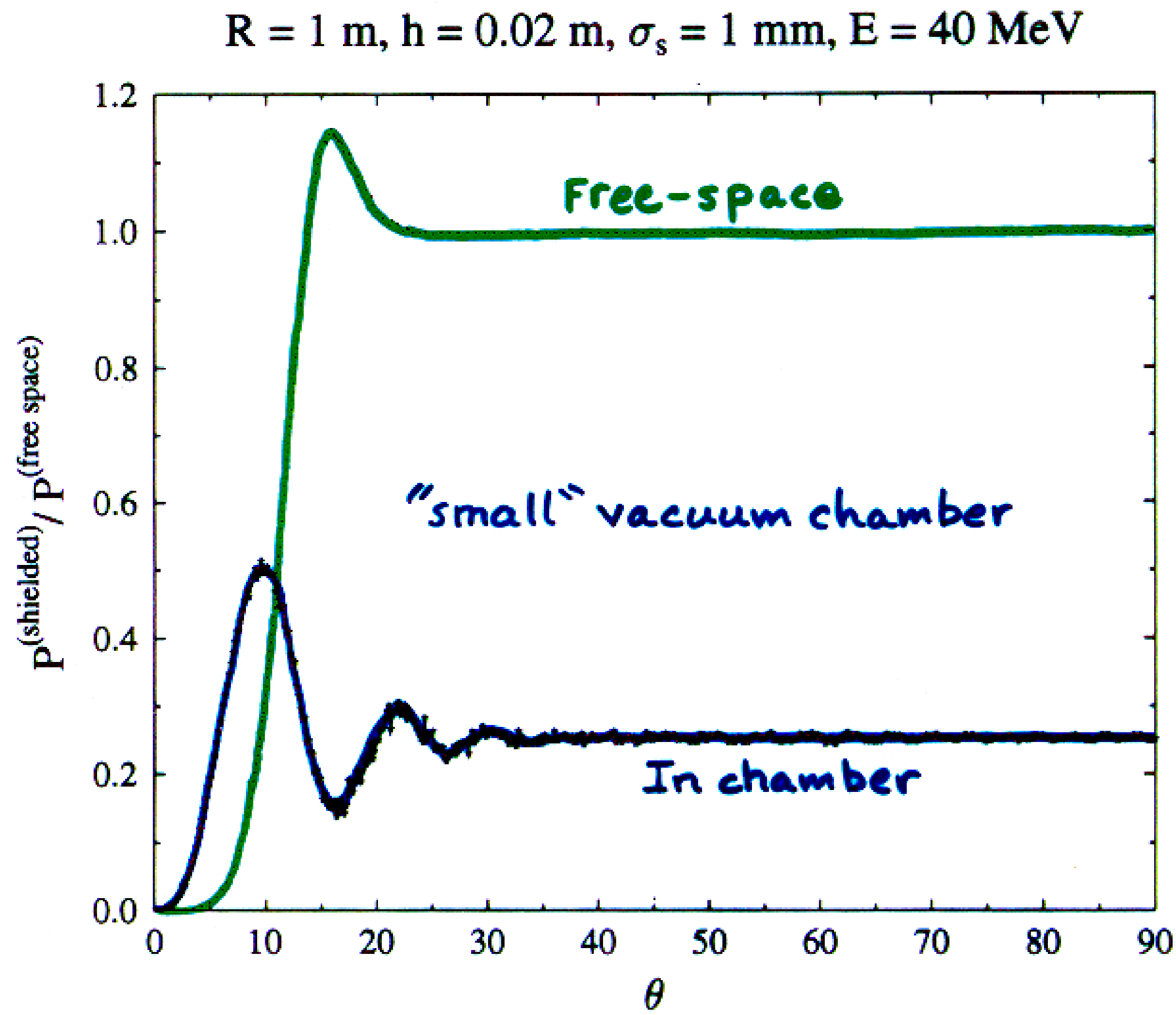
BUNCH SELF-INTERACTION IN VACUUM CHAMBER



TRANSIENT SELF-INTERACTION



TRANSIENT SELF-INTERACTION



Example Application

"Canonical" assumptions:

1. rigid-line-charge gaussian bunch,
2. CSR is principally dispersive,
3. long magnet ($\Rightarrow$ steady state),
4. big vacuum chamber ($\rightarrow$ free space).

Result: $\epsilon = \sqrt{\epsilon_0^2 + (\Delta\epsilon)^2}$;

$$\boxed{|\Delta\epsilon\,(\text{mm-mrad})| = 0.039\; Q(\text{pC})\,\sigma_x(\text{mm}) \left[\frac{R(\text{m})}{\sigma^4(\text{mm})}\right]^{1/3} |g(\theta)|;}$$

$|g(\theta)| = |\theta \sin\theta + \cos\theta - 1|$,

θ = bend angle,

Q = bunch charge,

σ_x = rms transverse bunch size,

R = bend radius,

σ = rms bunch length.

For IR Demo 180° magnet with "average" bunch profile:

$\underline{\epsilon_0 \sim 8\text{ mm-mrad}}$, $\sigma \sim 2.5$ mm, $Q = 135$ pC,

$\sigma_x \sim 7.5$ mm, $R \doteq 1$ m, $\theta = \pi$

$\Rightarrow |\Delta\epsilon| = 23$ mm-mrad $\Rightarrow$ $\underline{\epsilon = 24\text{ mm-mrad}}$.

Is Space Charge Important?

Answer: Yes if $\dfrac{\text{Debye length } \lambda_D}{\text{rms bunch radius } R} \lesssim 1$.

$$\frac{\lambda_D}{R} = \frac{v_{Th}}{R\omega_p} \simeq \left[\frac{E_b(\text{MeV})\,\ell(\text{mm})}{Q(\text{pC})}\right]^{1/2} \frac{\epsilon_n(\text{mm-mrad})}{2R(\text{mm})};$$

v_{Th} = thermal speed,
ω_p = plasma frequency,
E_b = electron-beam energy,
ℓ = rms bunch length,
Q = bunch charge,
ϵ_n = normalized rms emittance.

With representative beam parameters:

$\ell \sim 0.5$ mm,
$Q = 135$ pC,
$\epsilon_n \sim 10$ mm-mrad,
$R \sim 1$ mm,

we find

$\boxed{\text{space charge is important if } E_b \lesssim 10 \text{ MeV.}}$

IRFEL LATTICE PERFORMANCE