NONLINEAR HARMONIC GENERATION IN SASE FELs

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THE SCALING OF THE GAIN LENGTH

To understand the scaling of thegain length, consider the 1D case

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) E_x(z,t) = \frac{4\pi}{c^2}\frac{\partial}{\partial t} J_x(z,t)$$

where

$$E_x = \frac{1}{2}\sum_h \hat{E}_h(z)\exp\left[ihk_0(z-ct)\right] + \text{c.c.}$$

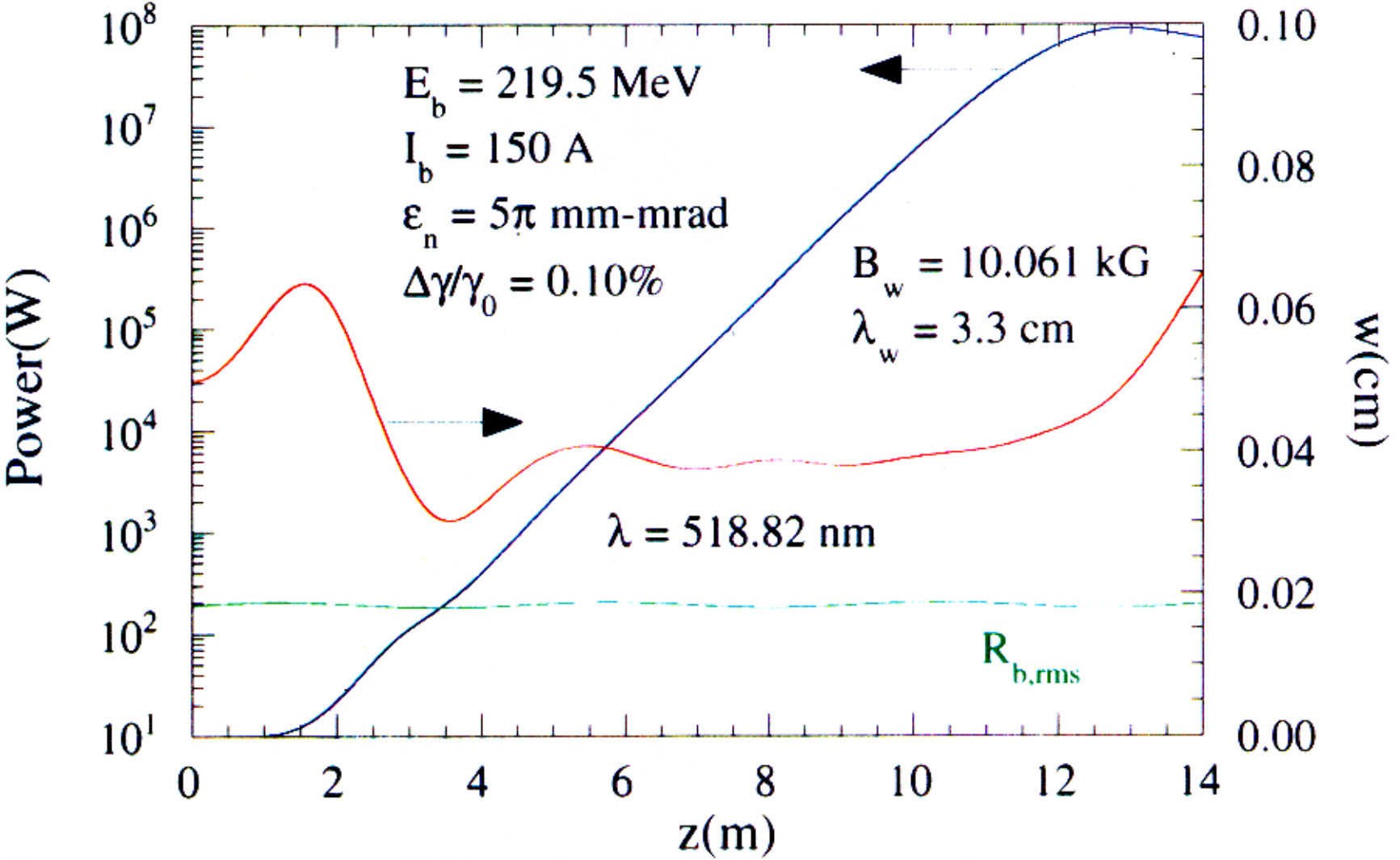
and the source current is given in terms of a nonlinear conductivity

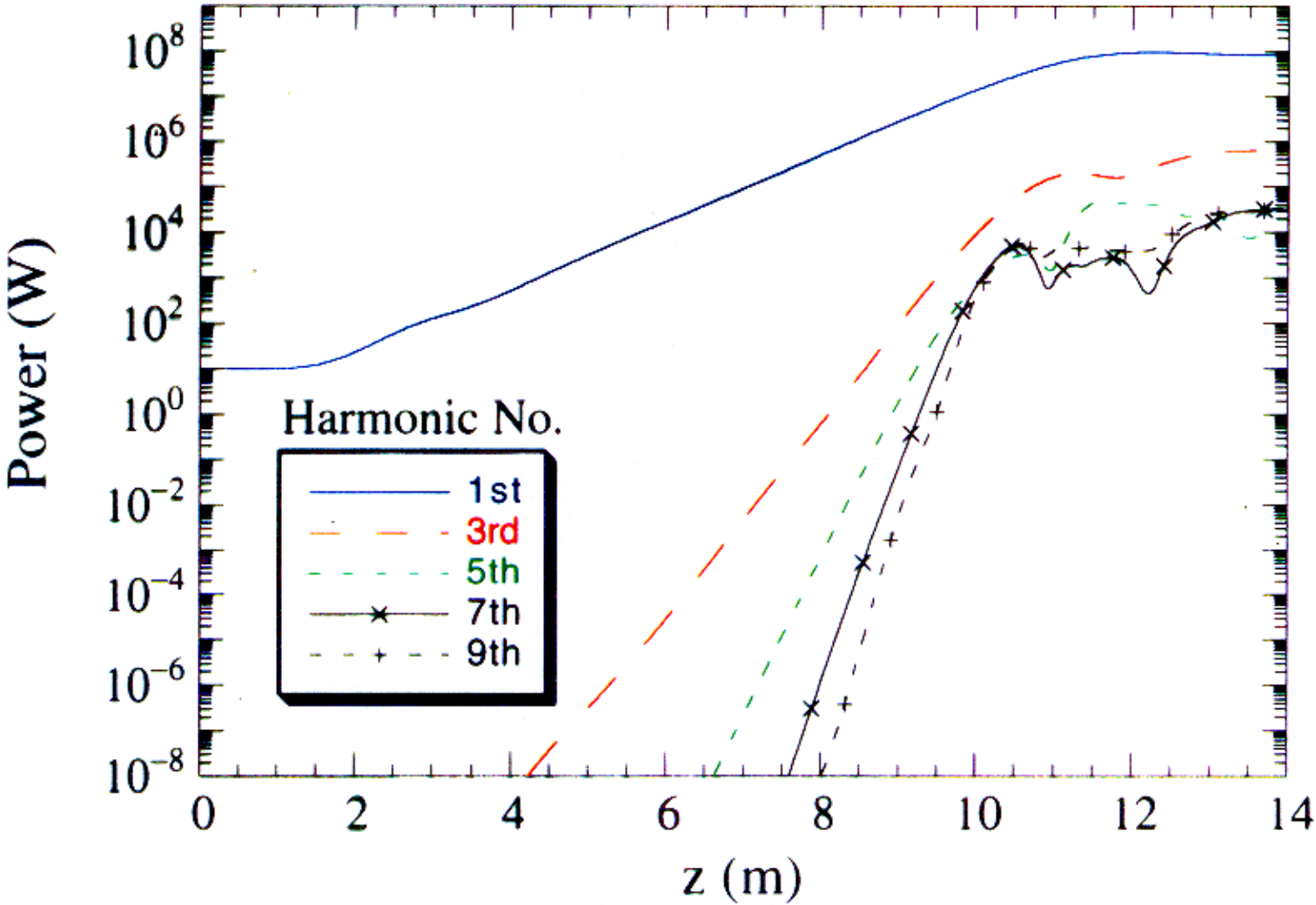
$$J_x = \frac{1}{2}\sum_h \left[\sigma_h^{(L)}\hat{E}_h + \sigma_h^{(NL)}\hat{E}_1^h\right]\exp\left[ihk_0(z-ct)\right] + \text{c.c.}$$

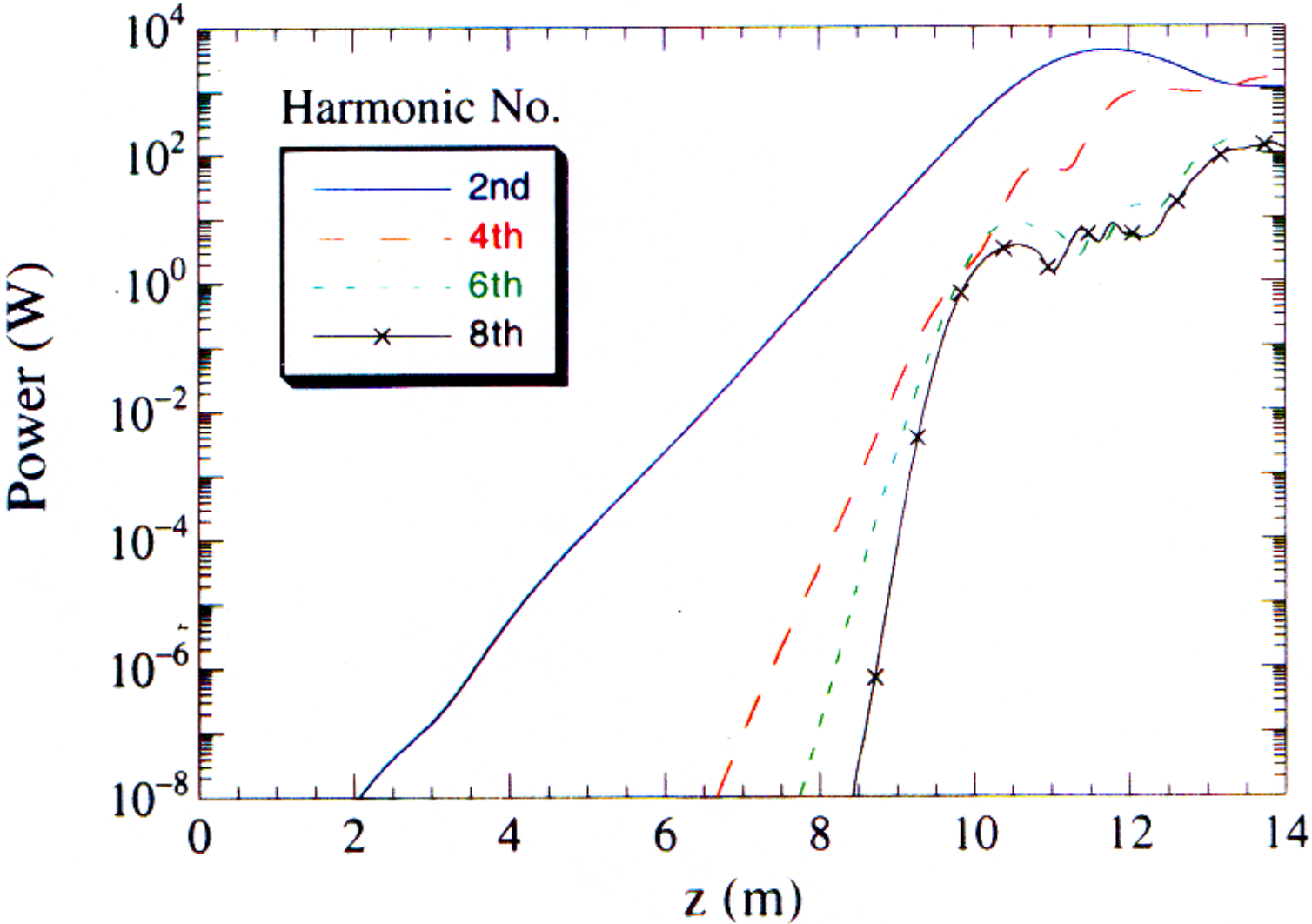
The $\sigma_h^{(L)}$ term describes the linear interaction. The nonlinear terms arise from bunching at the various harmonics. Bunching at the fundamental dominates, and

$$\frac{d}{dz}\hat{E}_h \cong -\frac{2\pi}{c}\left(\sigma_h^{(L)}\hat{E}_h + \sigma_h^{(NL)}\hat{E}_1^h\right)$$

If the fundamental grows as $\hat{E}_1^h \approx \exp\left[h\Gamma z\right]$ then the harmonic will grow as $\hat{E}_1 \approx \exp\left[\Gamma z\right]$ and the gain length scales as $L_G \approx (2h\Gamma)^{-1}$.


E_b = 219.5 MeV
I_b = 150 A
ε_n = 5π mm-mrad
$\Delta\gamma/\gamma_0$ = 0.10%
B_w = 10.061 kG
λ_w = 3.3 cm
λ = 518.82 nm
$R_{b,rms}$
Power(W)
w(cm)
z(m)
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0.10
0.08
0.06
0.04
0.02
0.00
0
2
4
6
8
10
12
14


Power (W)
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
Harmonic No.
1st
3rd
5th
7th
9th
0
2
4
6
8
10
12
14
z (m)

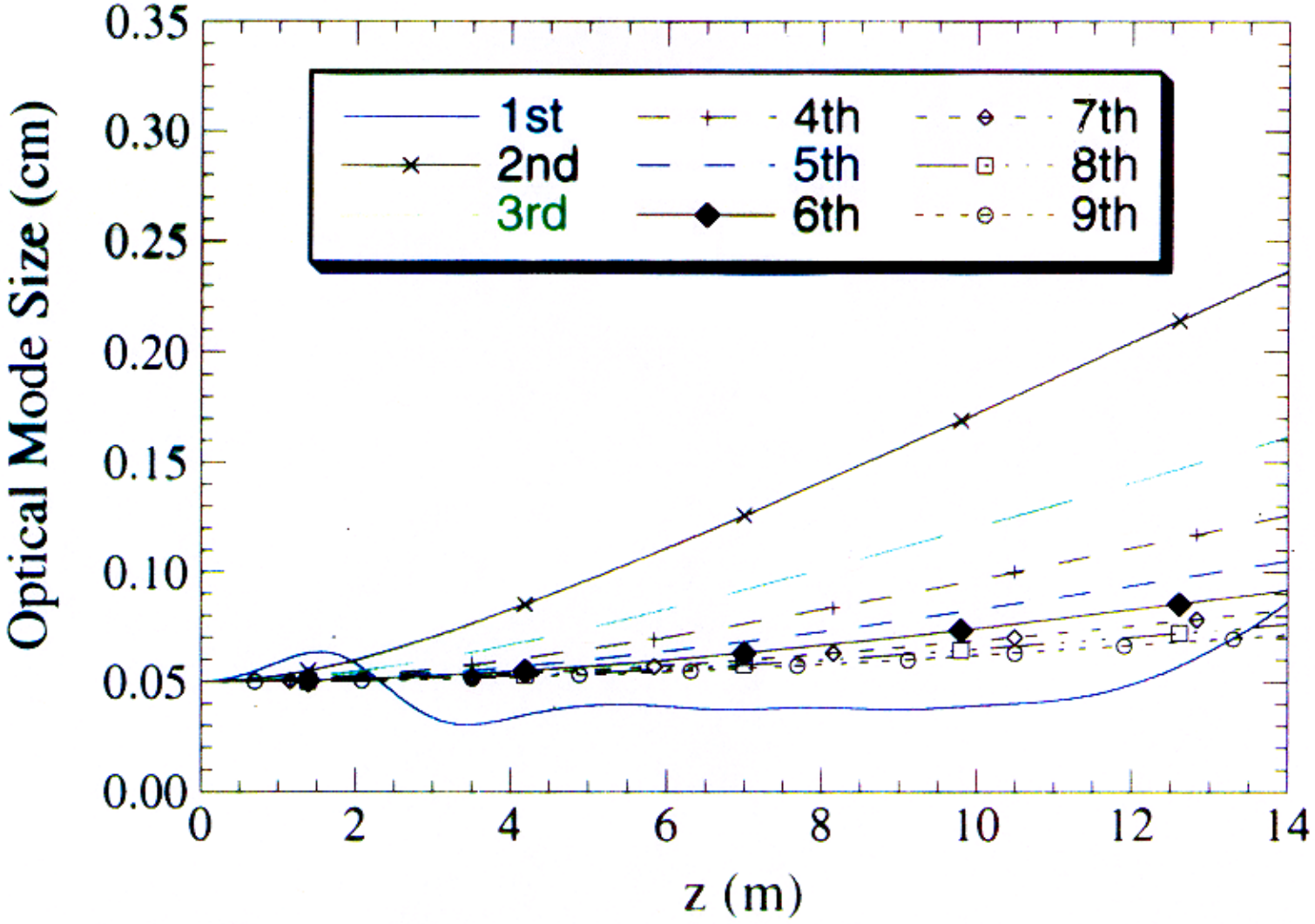

Harmonic No.
2nd
4th
6th
8th
Power (W)
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
2
4
6
8
10
12
14
z (m)

HARMONIC POWERS & GAIN LENGTHS

The scaling of the powers and gain lengths for the fundamental and harmonics are shown in the Table. To within a very good approximation, the gain lengths scale inversely with the harmonic number. The odd harmonic powers are substantial and drop off relatively slowly with increasing harmonic number, while the even harmonic powers are relatively small. This is expected for planar wiggler geometries.

The scaling of the gain with the harmonic number is characteristic of the nonlinear mechanism, and is a well-known phenomenon in TWTs. Of course, the linear gain lengths decrease with increasing harmonic number.

Harmonic No.	Gain Length (m)	Power
1	0.592	95.5 MW
2	0.335	4.28 kW
3	0.201	797 kW
4	0.165	1.57 kW
5	0.124	47.5 kW
6	0.098	175 W
7	0.089	32.5 kW
8	0.065	144 W
9	0.072	31.8 kW


1st
2nd
3rd
4th
5th
6th
7th
8th
9th
Optical Mode Size (cm)
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10
12
14
z (m)