17TH ADVANCED BEAM DYNAMICS WORKSHOP ON
FUTURE LIGHT SOURCES

Research with Coherent X Rays at the Mainz Microtron MAMI

H. Backe, University of Mainz

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ARGONNE NATIONAL LABORATORY, ARGONNE, IL U.S.A.

Research with Coherent X Rays at the Mainz Microtron MAMI

(H. Backe, 17th Advanced Beam Dynamics Workshop on Future Light Sources, Argonne, April 6-9, 1999)

1. The Mainz Microtron MAMI

2. Novel Interferometer with two Spatially Separated, Phase Correlated X-Ray Sources

2.1. Soft X-Ray Interferometer with Undulator Radiation

2.2. Development of a Hard X-Ray Interferometer with Transition Radiation

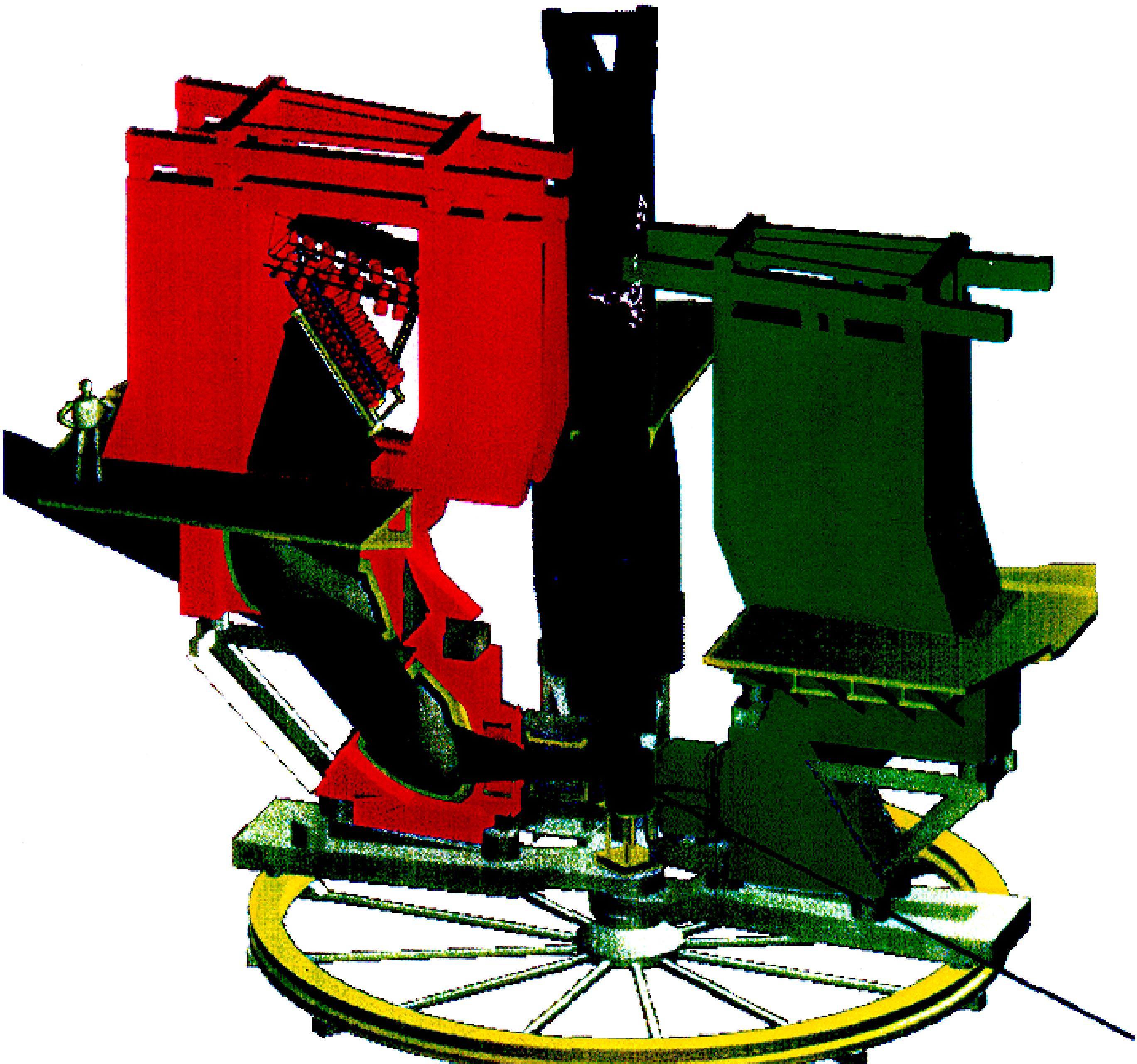
3. Transition Radiation as a Hard X-Ray Source

4. Investigation of Parametric X Radiation

5. Investigation of Smith-Purcell Radiation

6. Conclusion

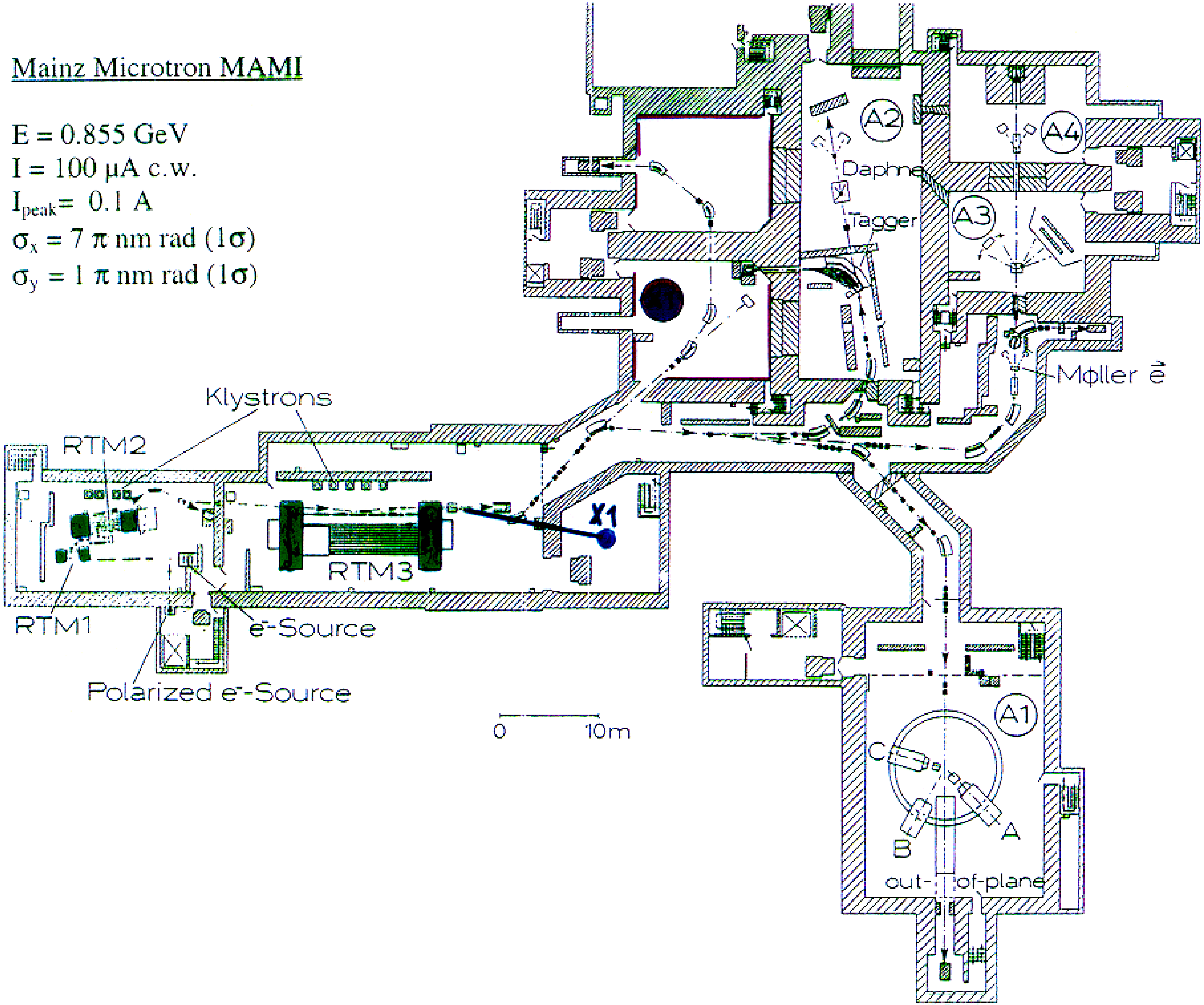
A1: Three Spectrometer Facility at MAMI



A: $\alpha > 23°$	B: $\alpha > 15°$	C: $\alpha > 55°$
$p < 735\,MeV/c$	$p < 870\,MeV/c$	$p < 655\,MeV/c$
$\Delta\Omega = 28\,msr$	$\Delta\Omega = 5.6\,msr$	$\Delta\Omega = 28\,msr$
$\Delta p/p = 20\%$	$\Delta p/p = 15\%$	$\Delta p/p = 25\%$

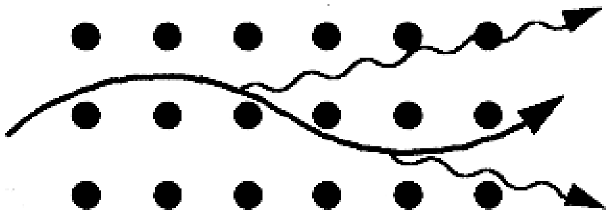
Mainz Microtron MAMI

E = 0.855 GeV
I = 100 μA c.w.
I_{peak} = 0.1 A
$\sigma_x = 7\ \pi$ nm rad (1σ)
$\sigma_y = 1\ \pi$ nm rad (1σ)

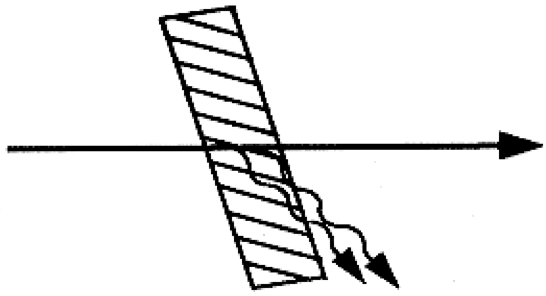


Processes

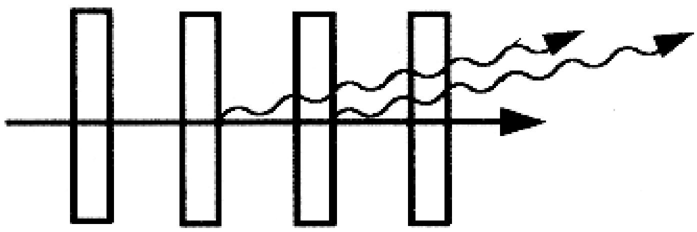
Channeling Radiation (CR)



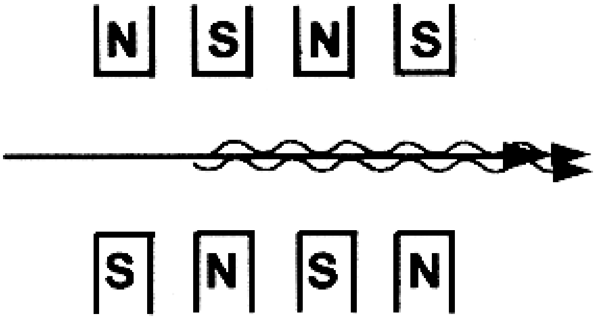
Parametric X-rays (PXR)



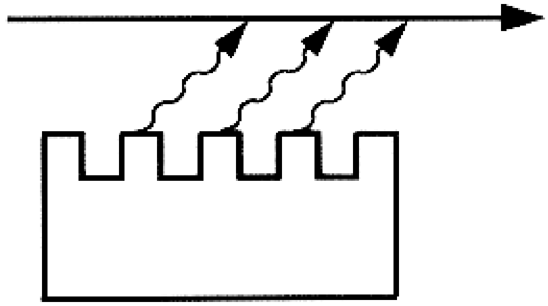
Transition Radiation (TR)



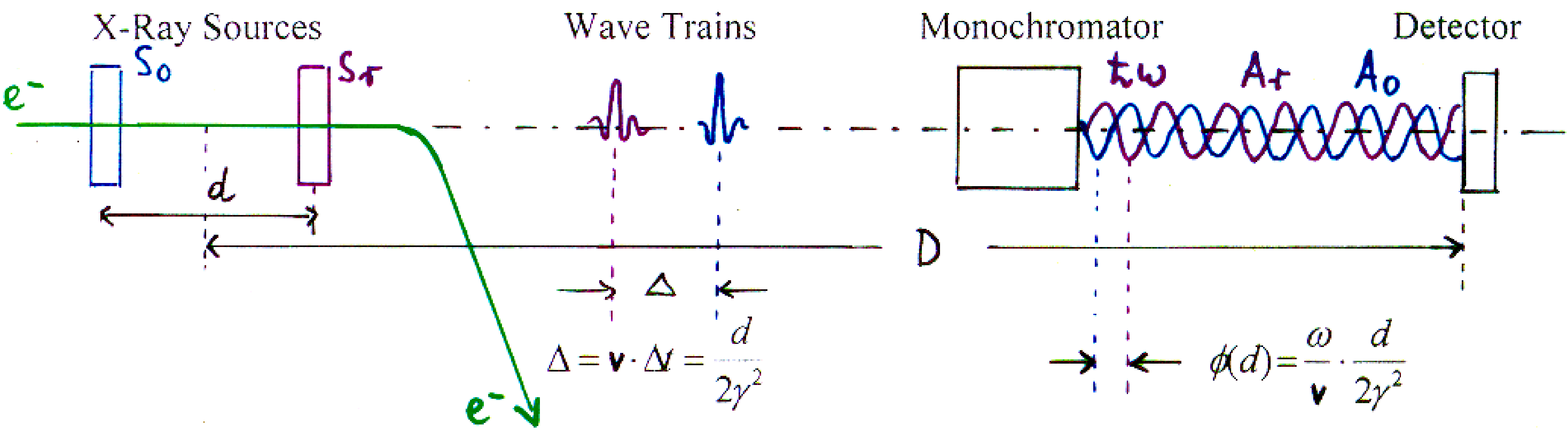
Undulator Radiation (UR)



Smith-Purcell Radiation (SP)



2. Novel Interferometer with two Spatially Separated, Phase Correlated X-Ray Sources



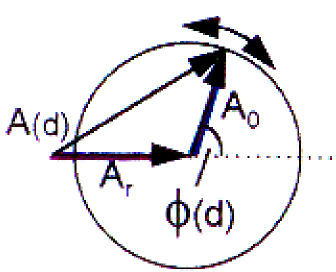
$$\Delta = v \cdot \Delta t = \frac{d}{2\gamma^2}$$

$$\phi(d) = \frac{\omega}{v} \cdot \frac{d}{2\gamma^2}$$

$$E = \gamma \cdot m_e c^2, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}$$

Interference Oscillations

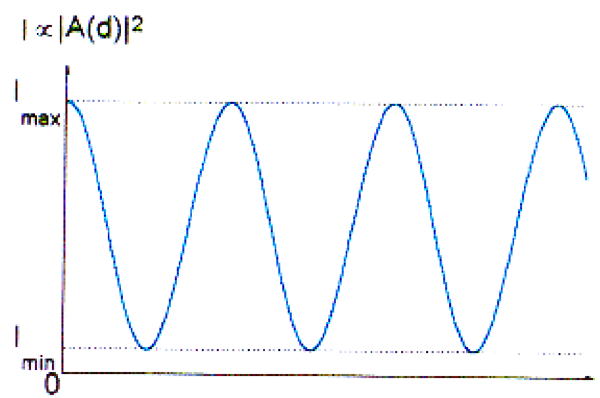
$$|A(d)|^2 = |A_r|^2 + |A_0|^2 + 2 \cdot |A_r| \cdot |A_0| \cos\phi(d)$$



Coherence:

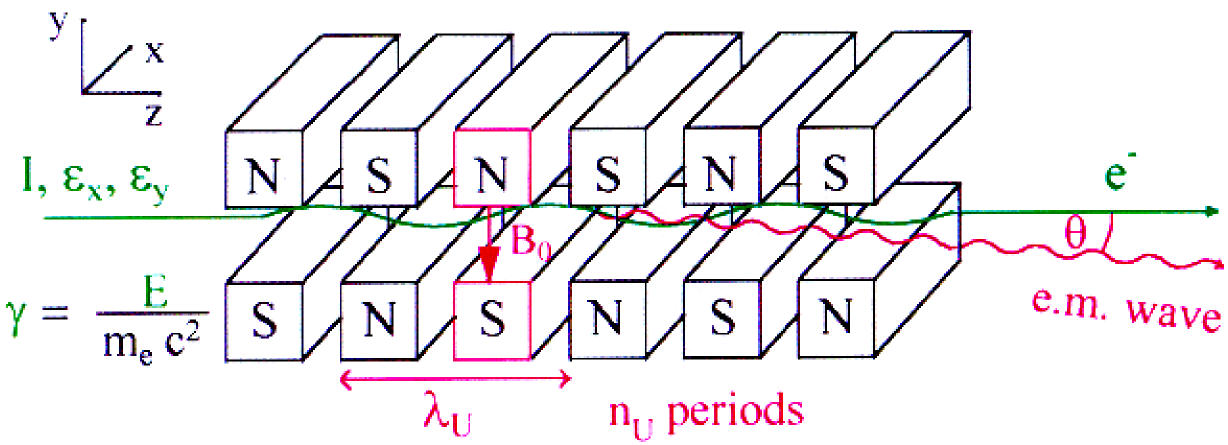
$$C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

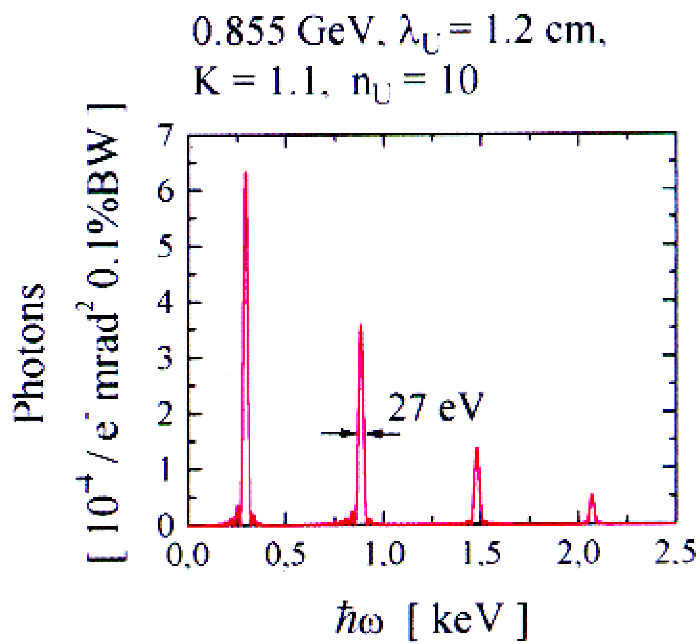
(C = 1 ideally)



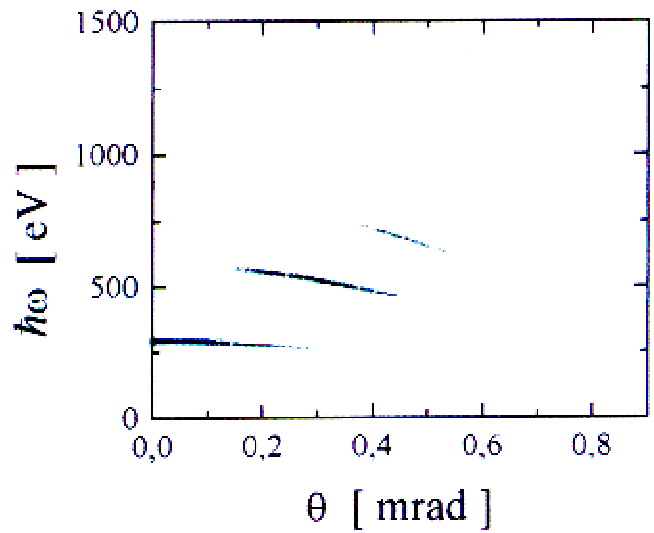
2.1. Soft X-Ray Interferometer with Undulator Radiation

Undulator Radiation



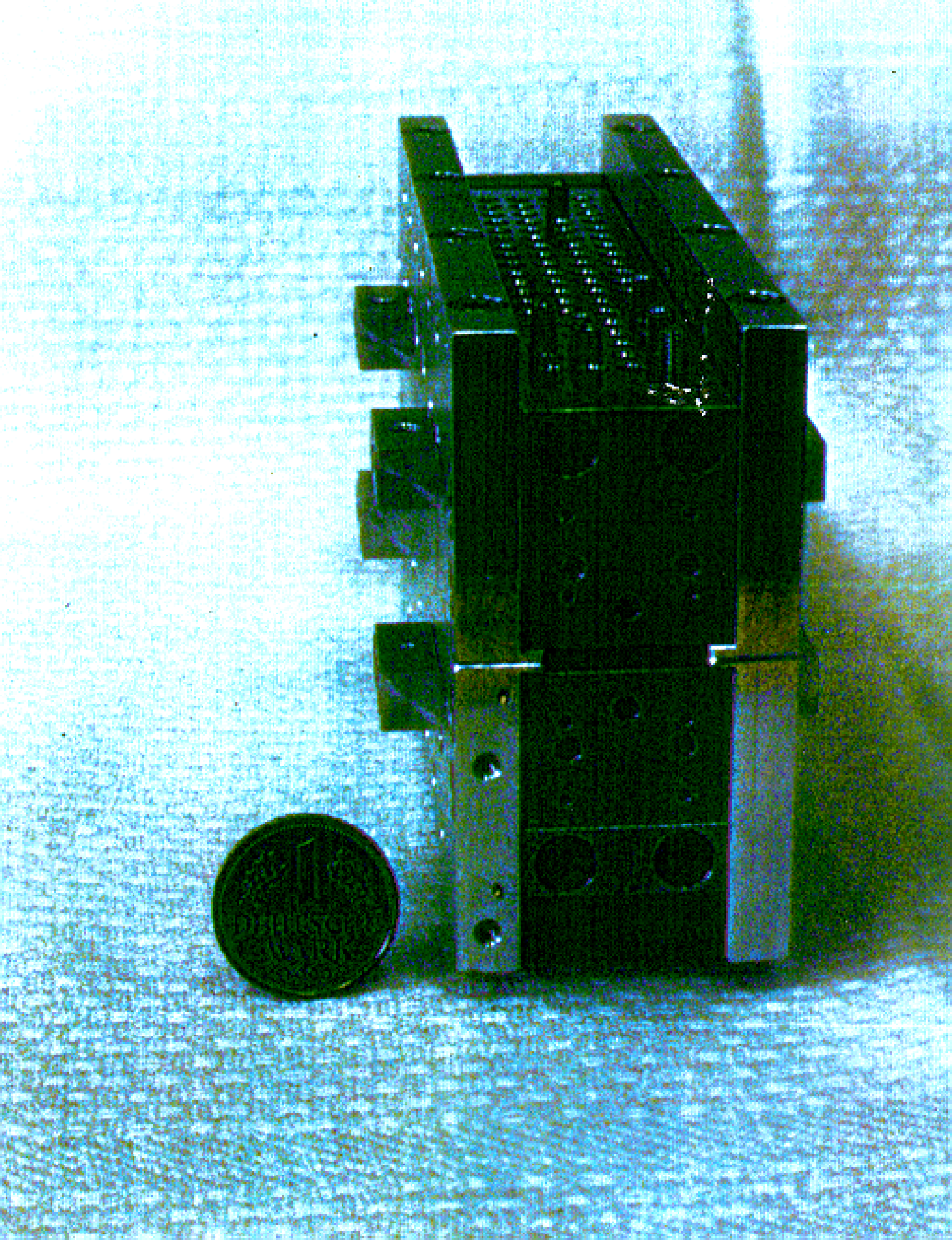


$$\lambda_L = \frac{\lambda_U}{2\gamma^2}\left(1 + K^2/2 + (\gamma\theta)^2\right)$$



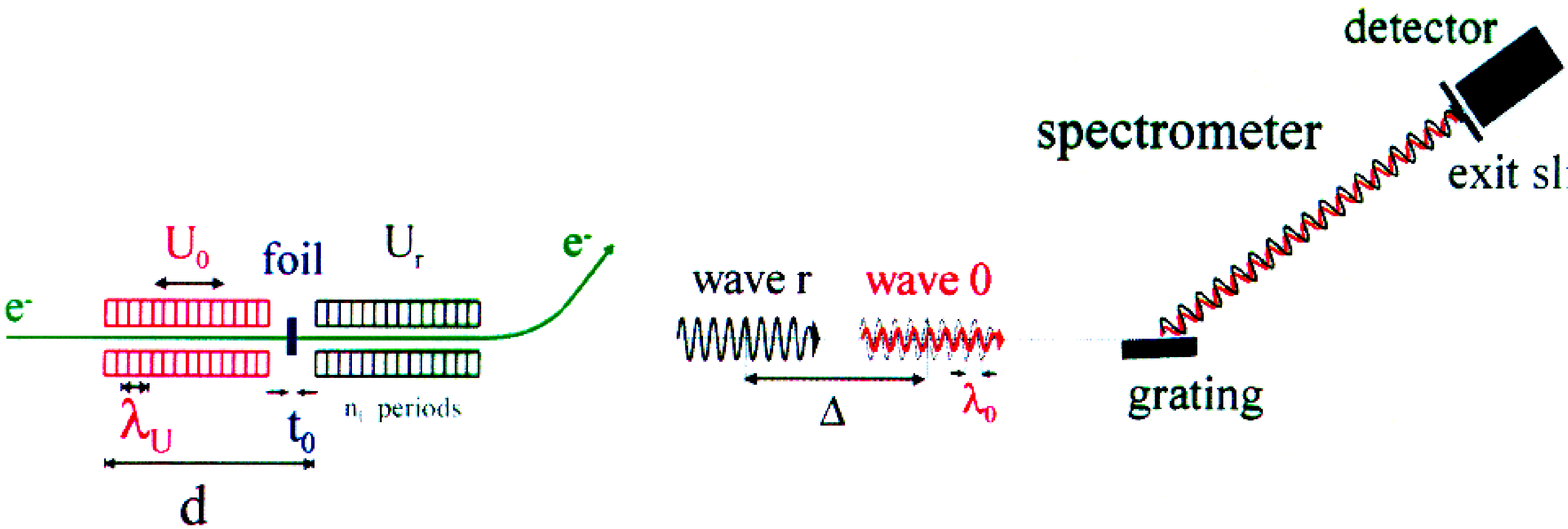
Undulator parameter $K = \frac{e B_0 \lambda_U}{2\pi m_e c}$

[] H. Motz et al., *Journal of Applied Physics* **24**, 826 (1953)



Basic Principle of the Interferometer (S. Dambach et al. PRL 80 (1998) 547

$$n(\omega) = 1 - \delta(\omega) - i\beta(\omega)$$

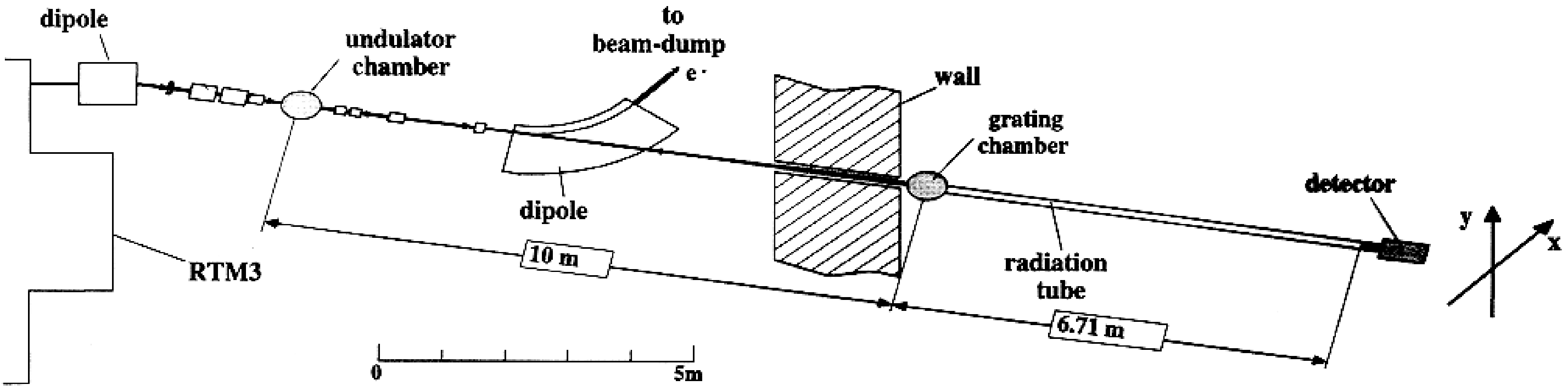


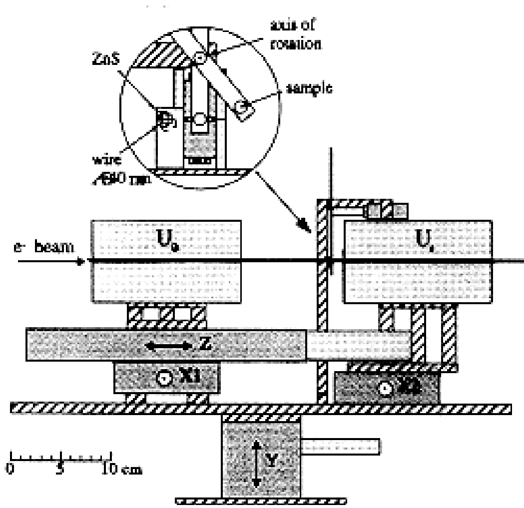
$$\Delta = \frac{d}{2\gamma^2}\left(1 + (\gamma\theta)^2\right) + \frac{K^2}{4\gamma^2} L_U \qquad \lambda_0 = \frac{\lambda_U}{2\gamma^2}\left(1 + \frac{K^2}{2} + (\gamma\theta)^2\right)$$

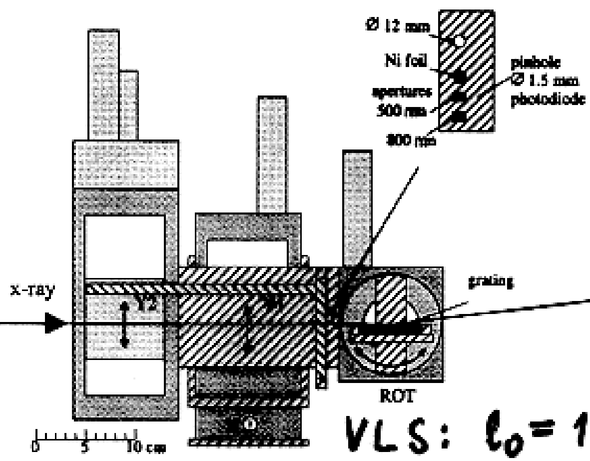
Intensity:

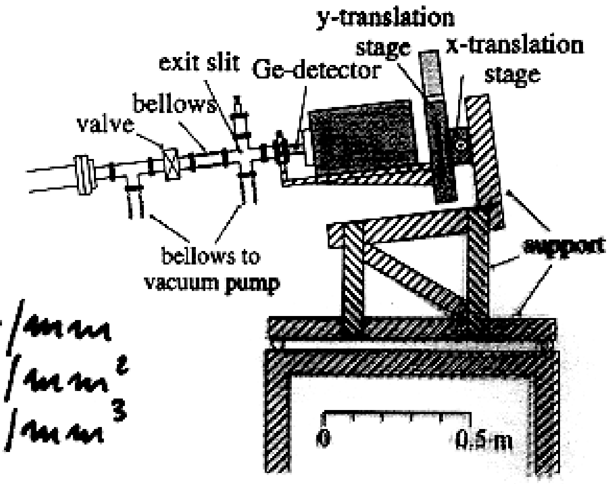
$$I = |A_r|^2 + |A_0|^2 \exp\left[-2\,\frac{\omega}{c}\,\beta(\omega)t_0\right] + 2\,|A_r||A_0| \exp\left[-\,\frac{\omega}{c}\,\beta(\omega)t_0\right] \cos\left[\frac{\omega}{c}(\Delta + \delta(\omega)t_0)\right]$$

Experimental Setup









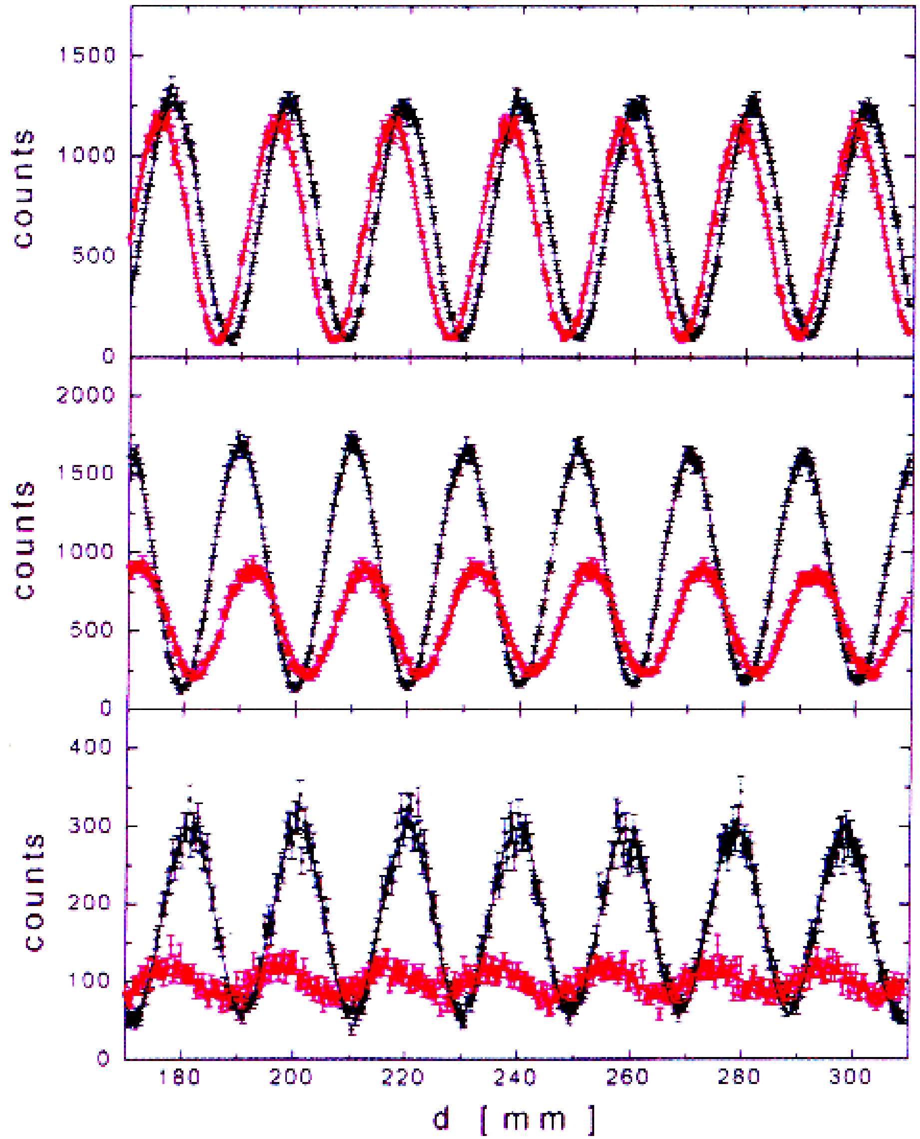
VLS: $l_0 = 1400/mm$

$l_1 = 0.48/mm^2$

$l_2 = 1.07/mm^3$

Oscillations

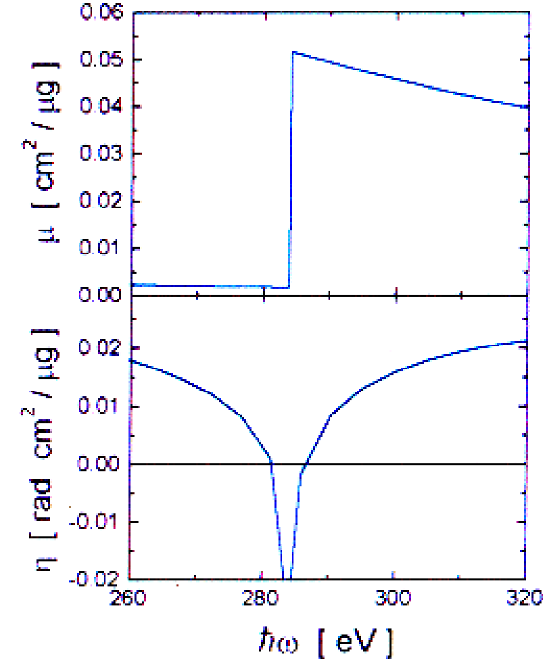
black: without foil
red: 65 μg/cm² C-foil



λ_{osc} = 20.68 mm
$\hbar\omega$ = 279.6 eV

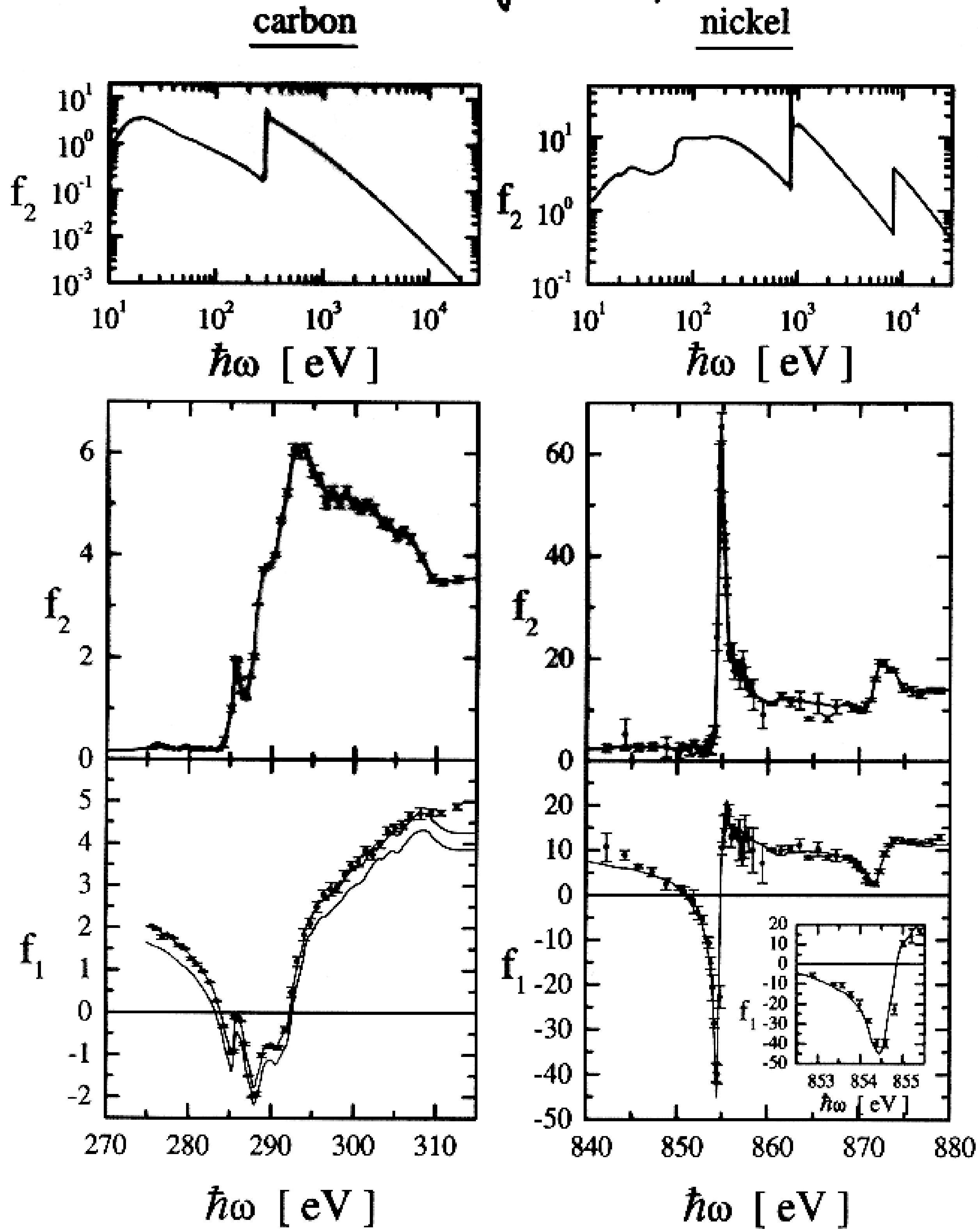
λ_{osc} = 20.09 mm
$\hbar\omega$ = 287.8 eV

λ_{osc} = 19.50 mm
$\hbar\omega$ = 299.4 eV



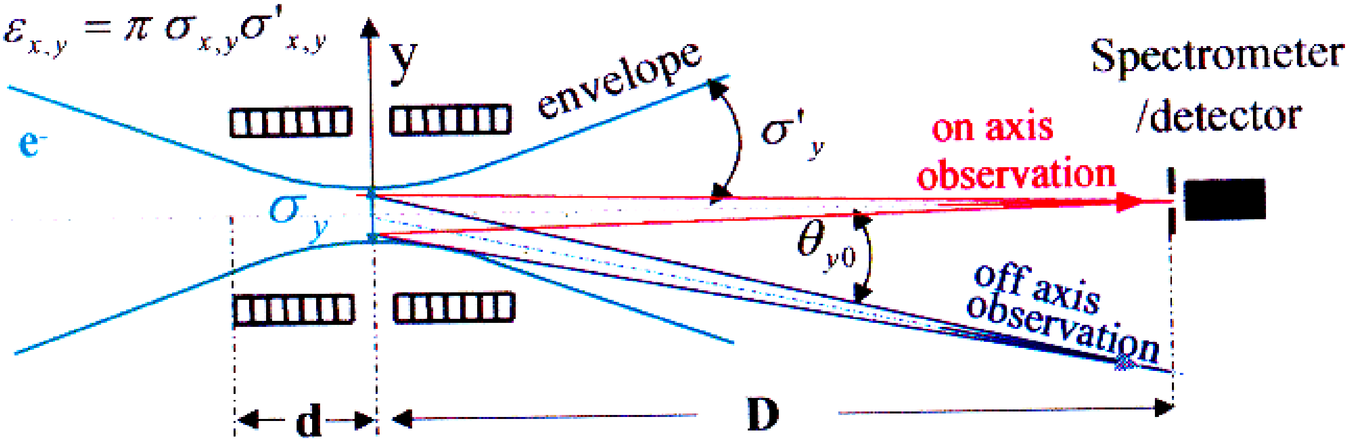
[] Henke et al. *Atomic Data and Nuclear Tables*, Vol.54, No.2 (1993)

Atomic Scattering Factors f_1, f_2 and Kramers - Kronig - Dispersion Relation



Coherence Conditions

(Undulator Interferometer)



Phase difference between wave trains

$$\phi = \frac{\omega}{c} \cdot \left(\frac{K^2(y)}{4 \cdot \gamma^2} L_U + \frac{d}{2\gamma^2} \left(1 + (\gamma\theta)^2\right) \right)$$

fluctuates because of

- finite beam emittance ($\delta\theta$, $\delta K(y)$)
- workings of the beam spot ($\delta K(y)$)
- spreading ($\delta\gamma$, $\delta\omega$, ...)

Example: **Coherence conditions** from beam emittance

$$\frac{d}{D}\frac{\varepsilon_{x,y}}{\pi} \le \frac{\lambda}{4\pi}$$

($C \ge 0.71$)

on-axis

and

$$\theta_{x0,y0} \cdot d \cdot \sqrt{\frac{\varepsilon_{x,y}}{\pi D}} \le \frac{\lambda}{2\pi}$$

($C \ge 0.43$)

off-axis observation

At $\lambda = 41.3$Å (300 eV), $\varepsilon_x/\pi = 7$ nm rad, $D = 16.7$ m:

coh. length $d \le 0.78$ m, coh. angle $\theta_{x0} \le 4.1 \cdot 10^{-5}$ rad

Conceivable Applications of the Undulator Interferometer

- Beam diagnostic tools

- Study of small angle scattering of relativistic electrons in thin foils

- Spectroscopy at $O_{IV/V}$ absorption edges of transuranium elements with 10 ng samples

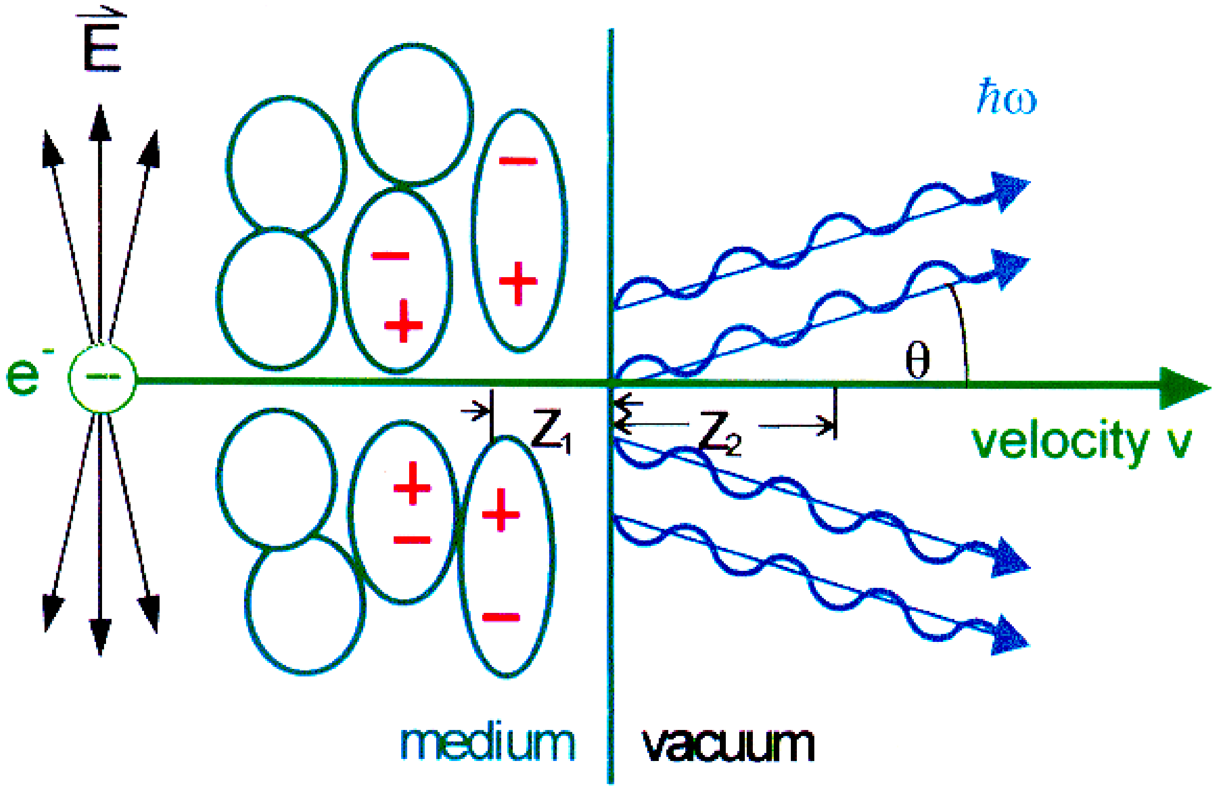
- Development into a Fourier-Transform Spectrometer

- Measurement of the magnetic circular birefringence of thin magnetic films

2.2. Development of a Hard X-Ray Interferometer with Transition Radiation

Transition Radiation

from single interface



single interface intensity:

$$I_0 = \frac{d^2 N_0}{(d\hbar\omega/\hbar\omega)\, d\Omega} = \frac{\alpha\theta^2\omega^2}{16\,\pi^2 v^2}\,(Z_1 - Z_2)^2$$

formation length:

$$Z_i = \frac{4c}{\omega}\,(\gamma^{-2} + \theta^2 + \omega_p^2/\omega^2)^{-1}$$

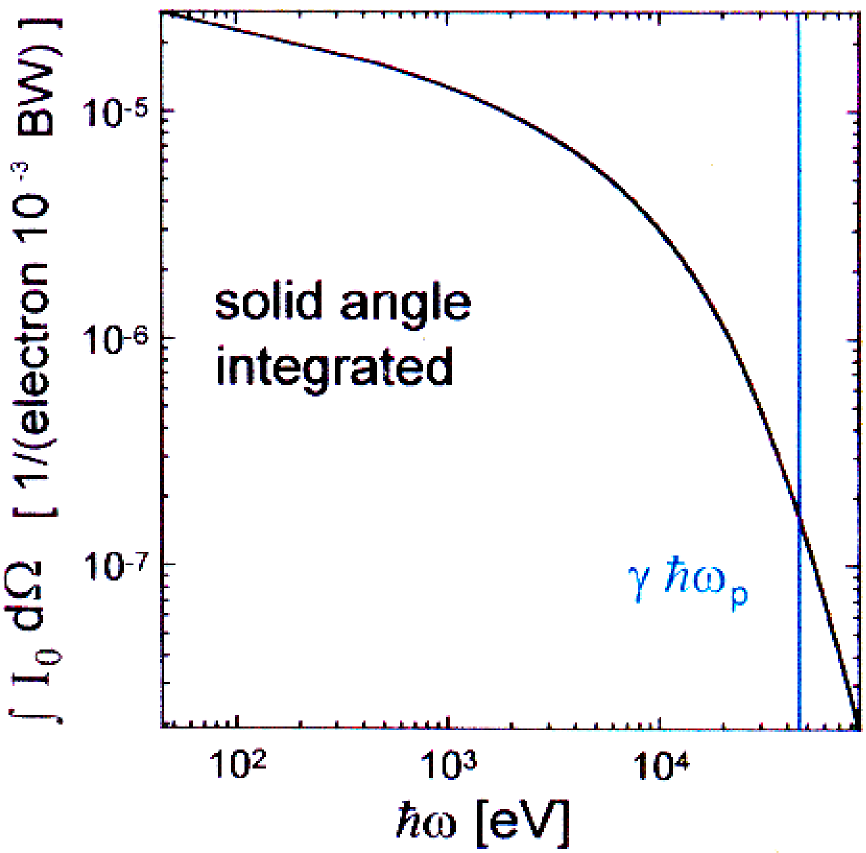
plasma frequency $\omega_p^2 = 4\pi\, r_e\, c^2\, n_a Z$

atomic density n_a

classical electron radius $r_e = 2.818$ fm

Features of Transition Radiation

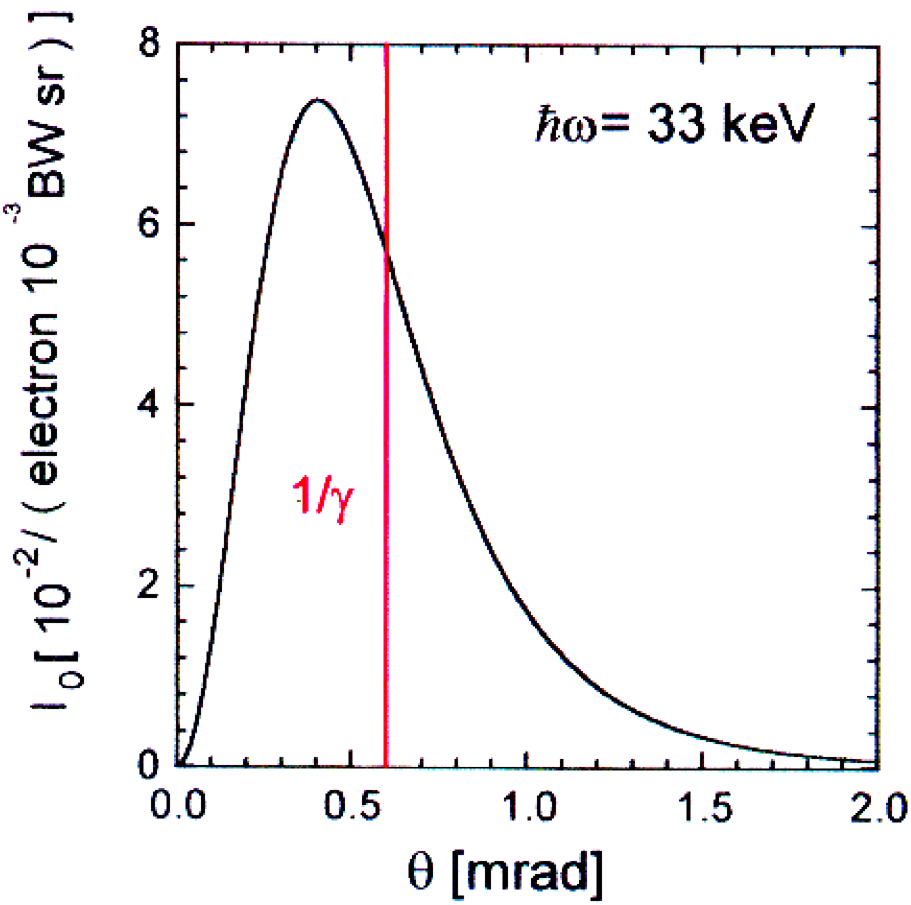
(single interface)



$\hbar\omega_p$ = 26.2 eV (Be)

γ = 1673

$\gamma \hbar\omega_p$ = 44 keV

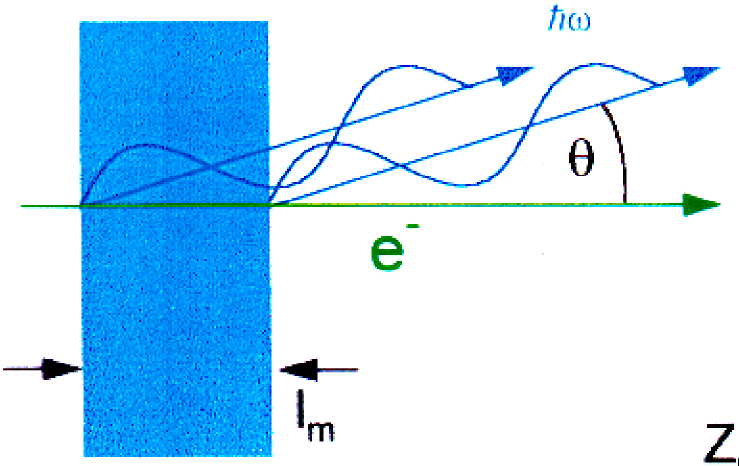


$\theta_{max} \approx 1/\gamma$

$\approx$ 0.6 mrad

Transition Radiation

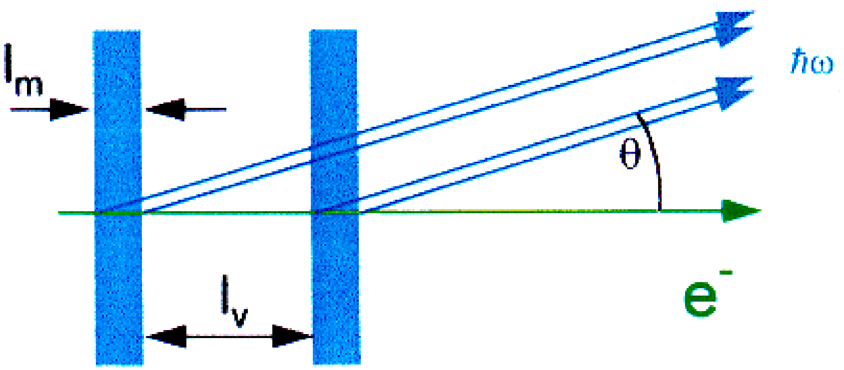
from single foil:



$$\frac{d^2 N_0}{(d\hbar\omega/\hbar\omega)\, d\Omega} = I_0 \cdot 4\sin^2\left(\frac{l_m}{Z_m}\right)$$

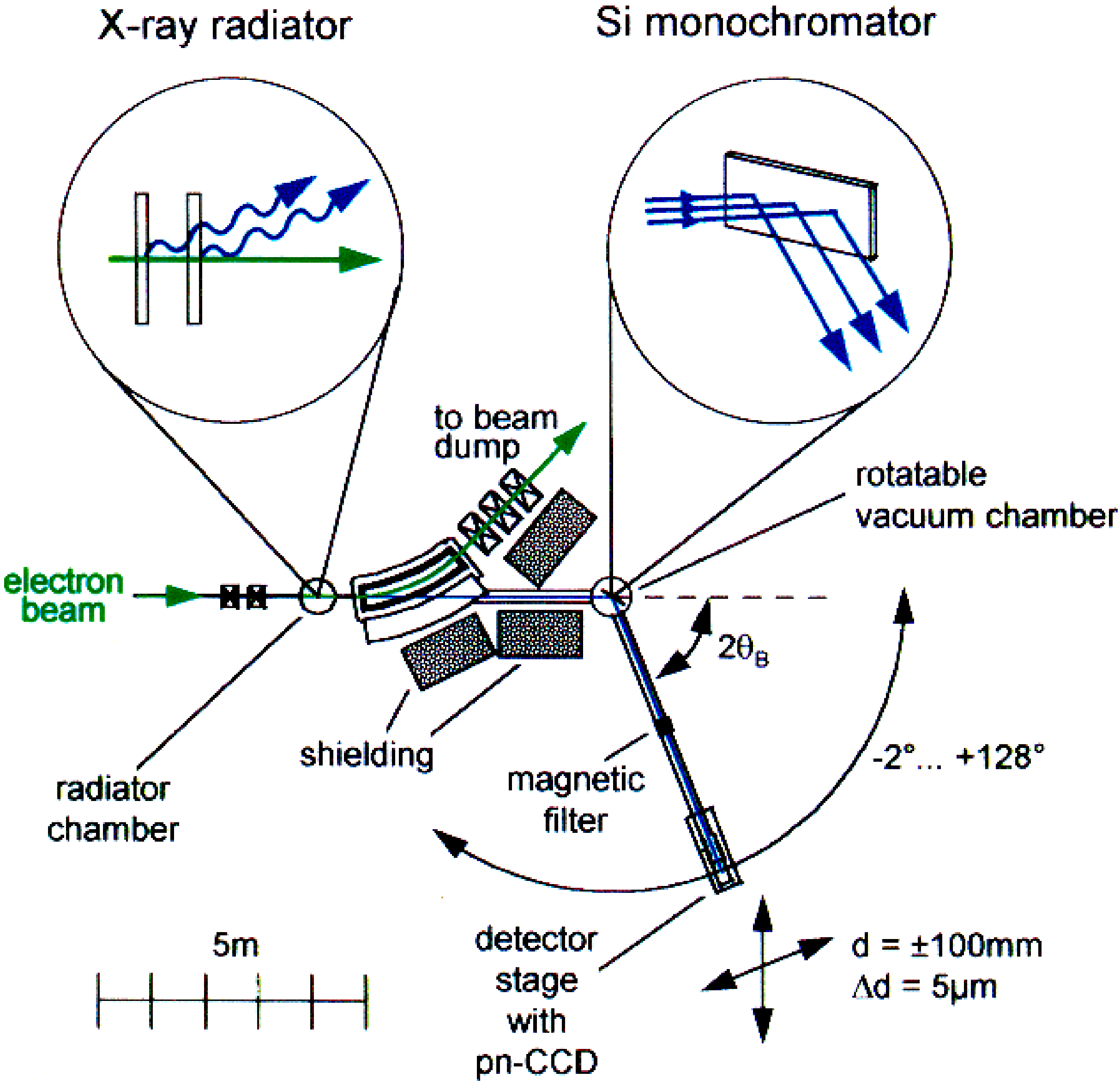
$$Z_m = \frac{4c}{\omega}\left(\gamma^{-2} + \theta^2 + \omega_p^2/\omega^2\right)^{-1}$$

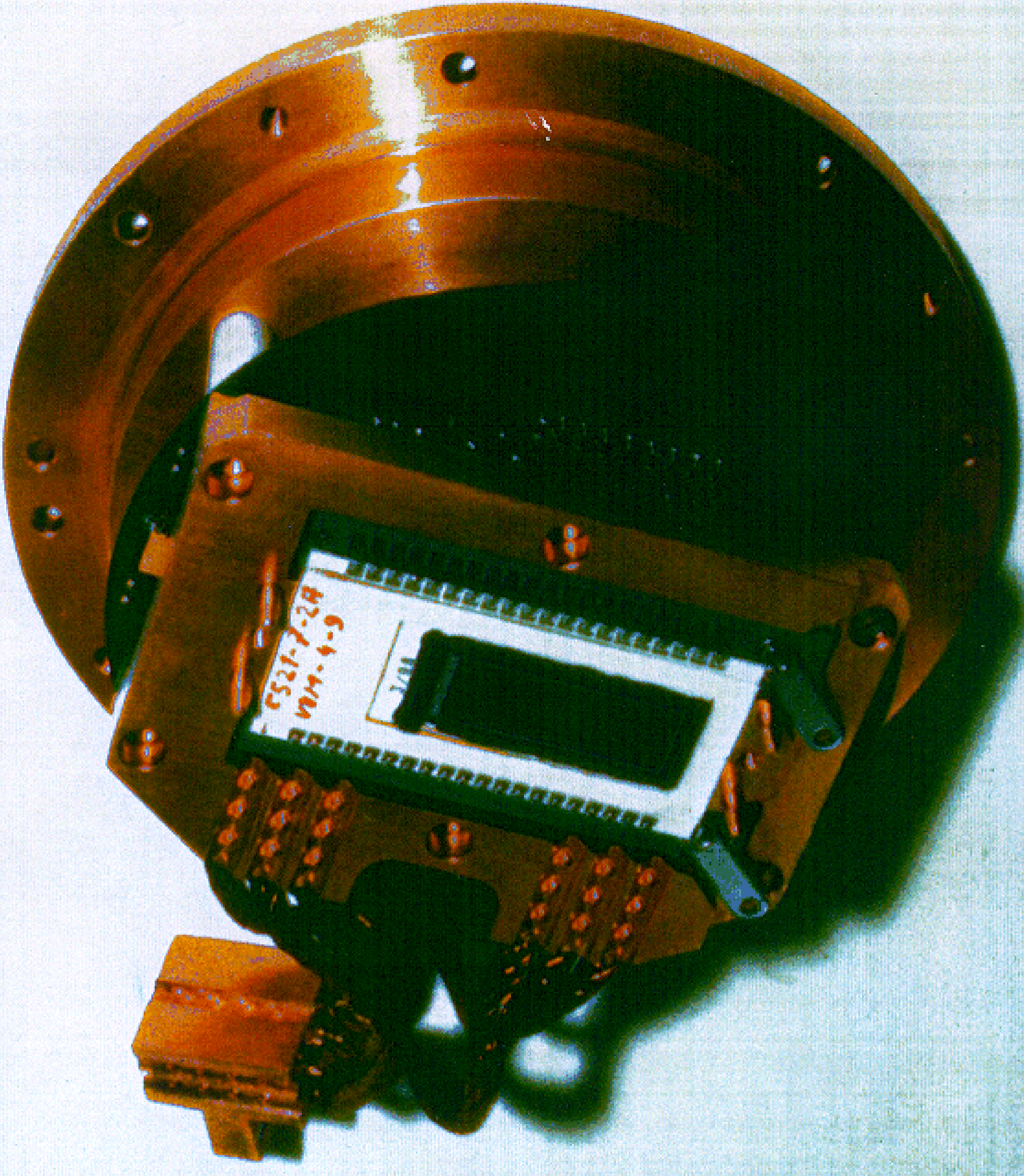
from two foils:



$$\frac{d^2 N_0}{(d\hbar\omega/\hbar\omega)\, d\Omega} = I_0 \cdot 4\sin^2\left(\frac{l_m}{Z_m}\right) \cdot 4\cos^2\left(\frac{l_m}{Z_m} + \frac{l_v}{Z_v}\right)$$

Experimental Set-up

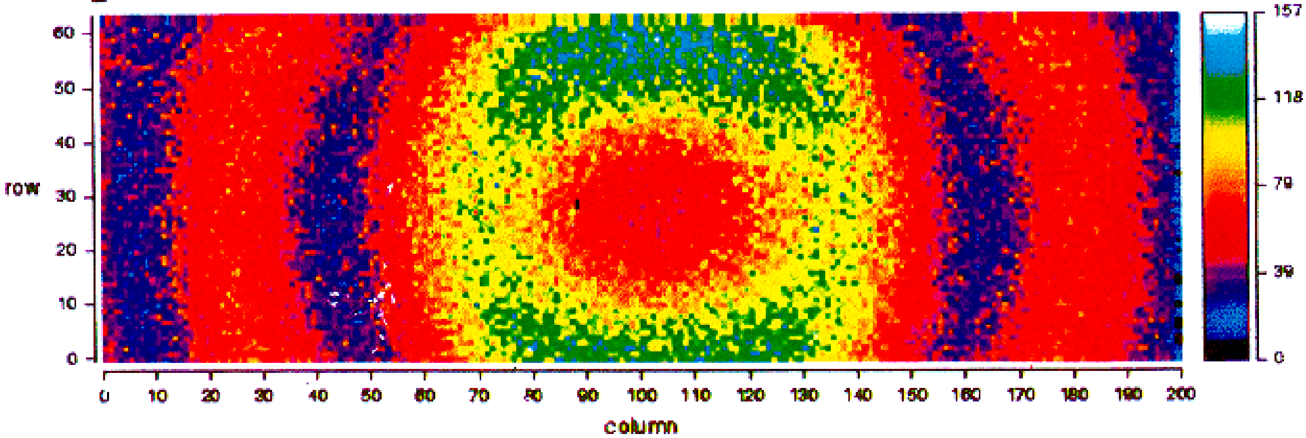




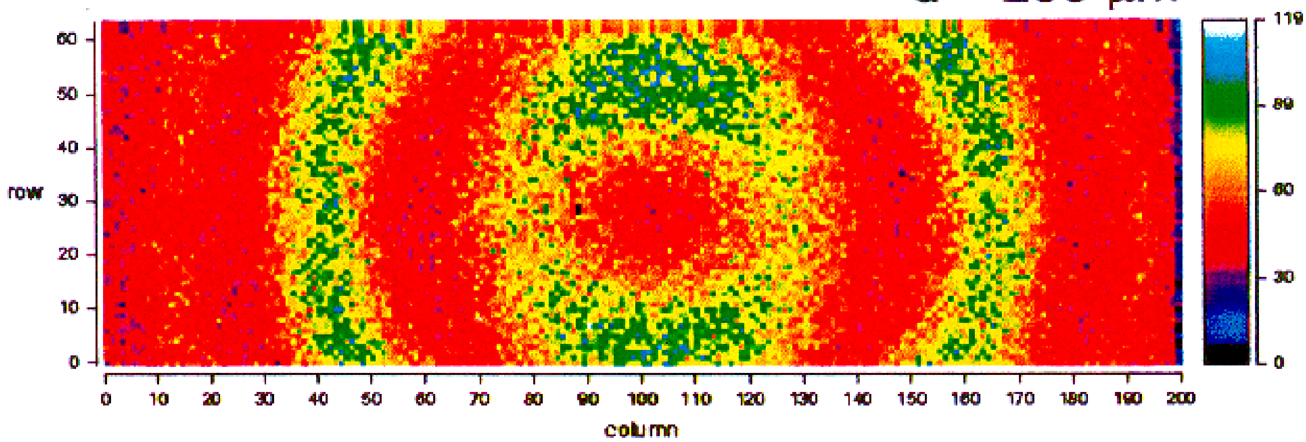
Interferences with two 15 μm beryllium foils

$\theta_B = 22.8°$, $\hbar\omega = 5100$ eV

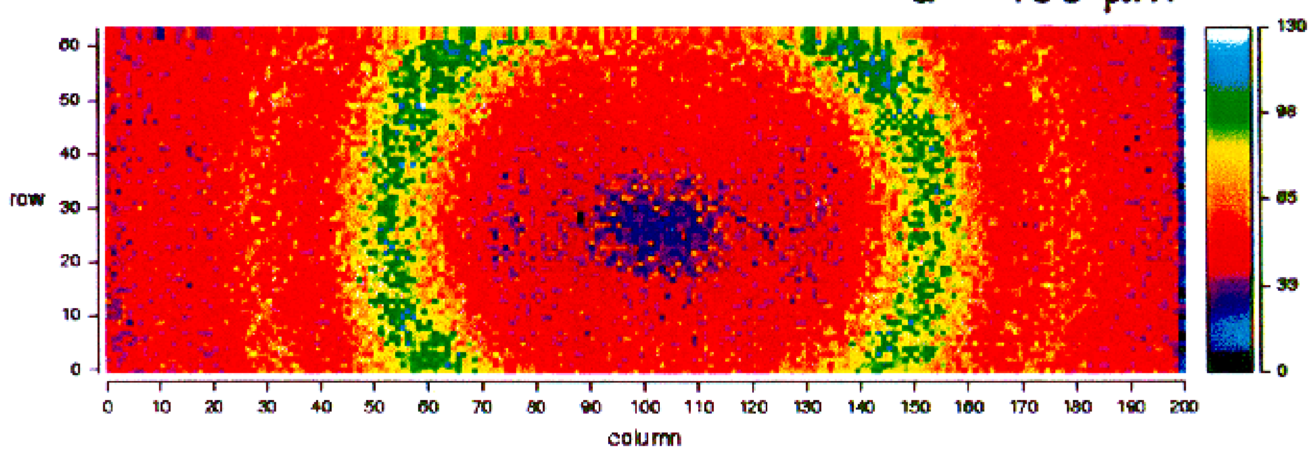
d = 50 μm



d = 250 μm



d = 450 μm

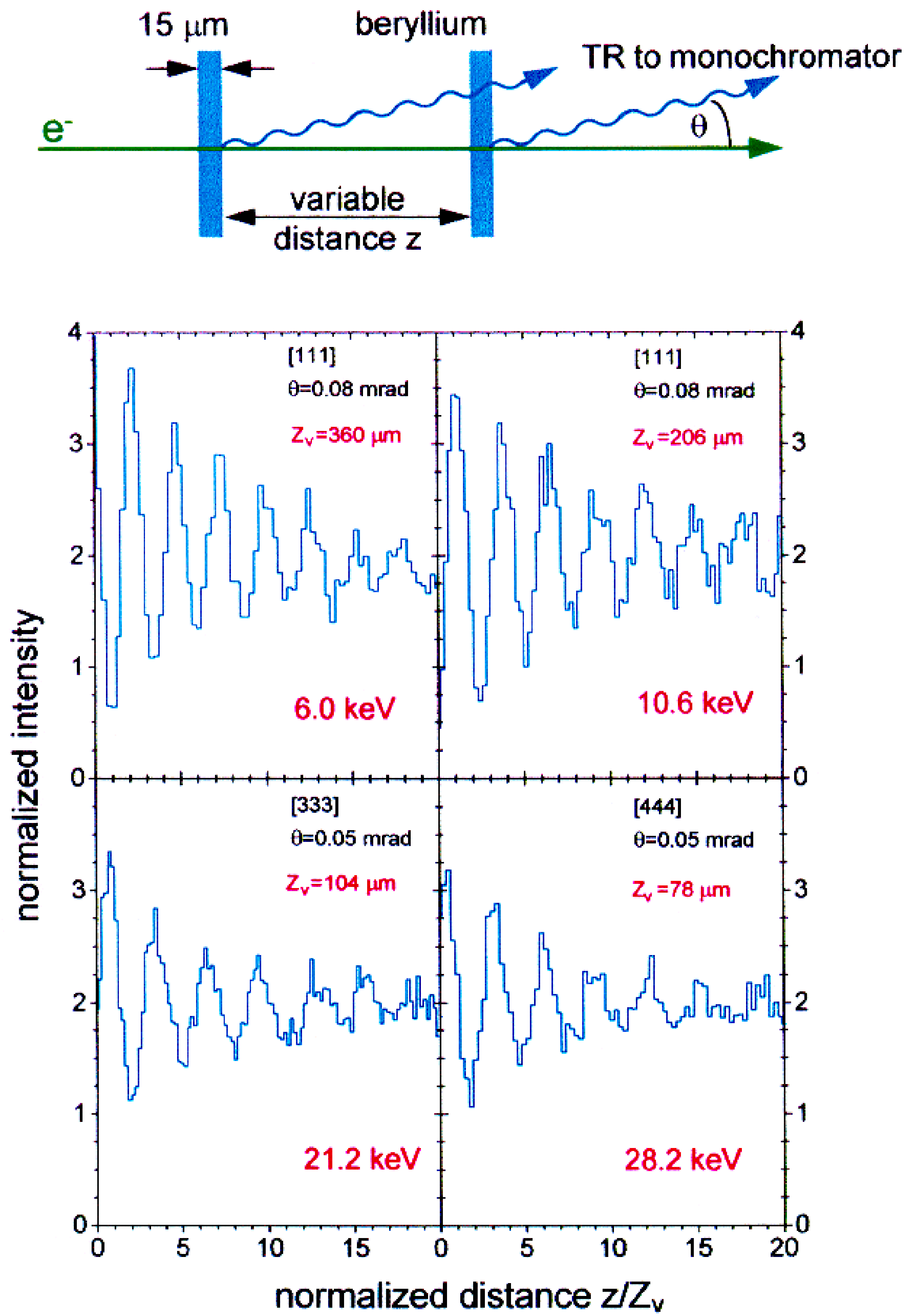


E_{e^-} = 855 MeV

I_{e^-} = 4 nA

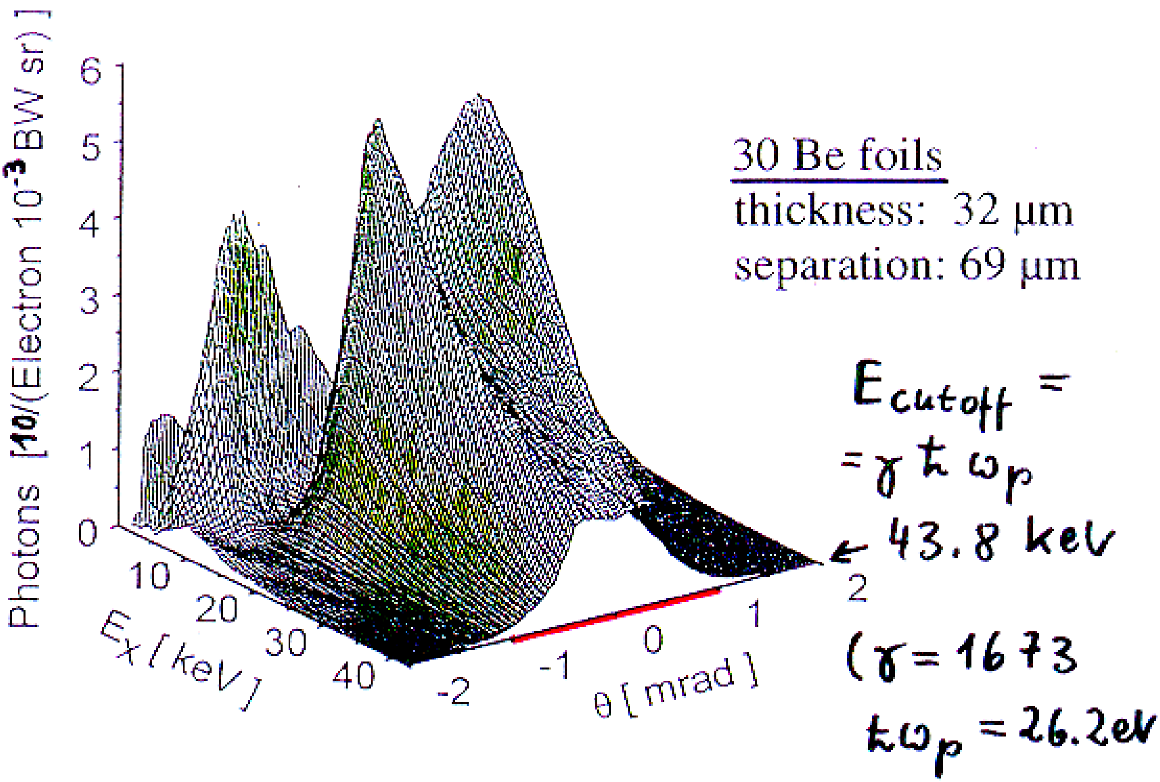
16.6 frames/s

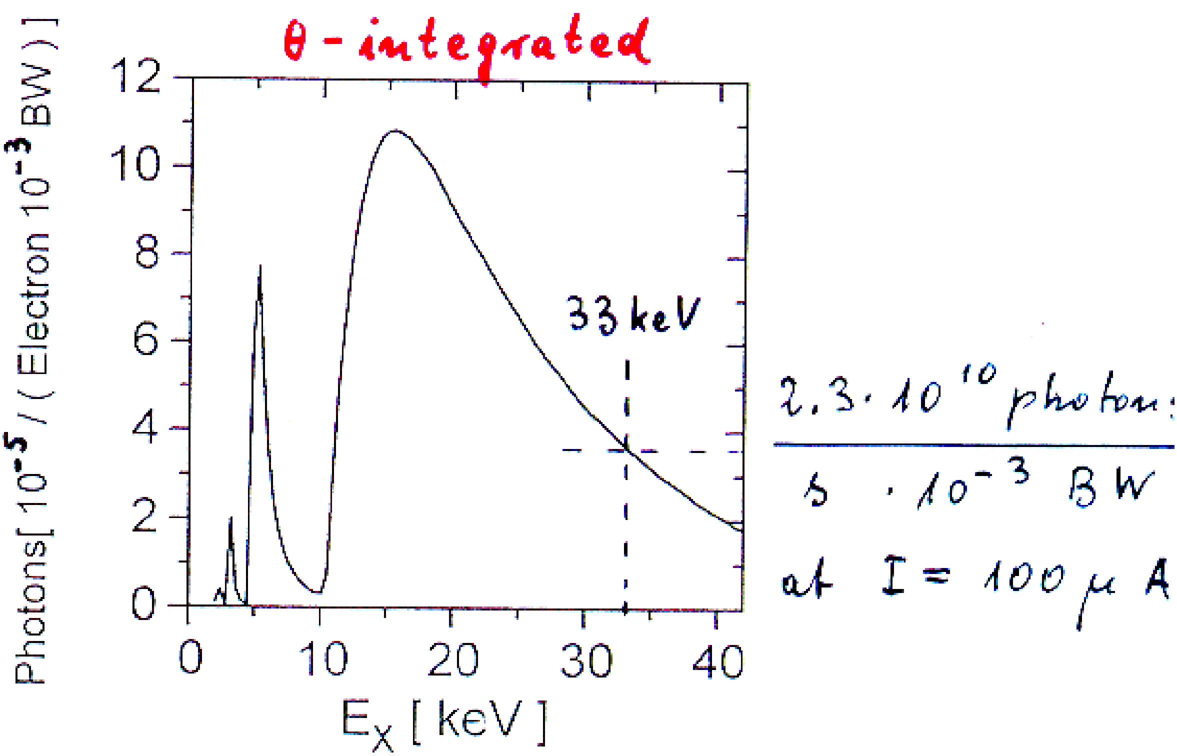
2000 frames each


15 μm
beryllium
TR to monochromator
e-
θ
variable
distance z
[111]
θ=0.08 mrad
Zv=360 μm
6.0 keV
[111]
θ=0.08 mrad
Zv=206 μm
10.6 keV
[333]
θ=0.05 mrad
Zv=104 μm
21.2 keV
[444]
θ=0.05 mrad
Zv=78 μm
28.2 keV
normalized intensity
normalized distance z/Zv

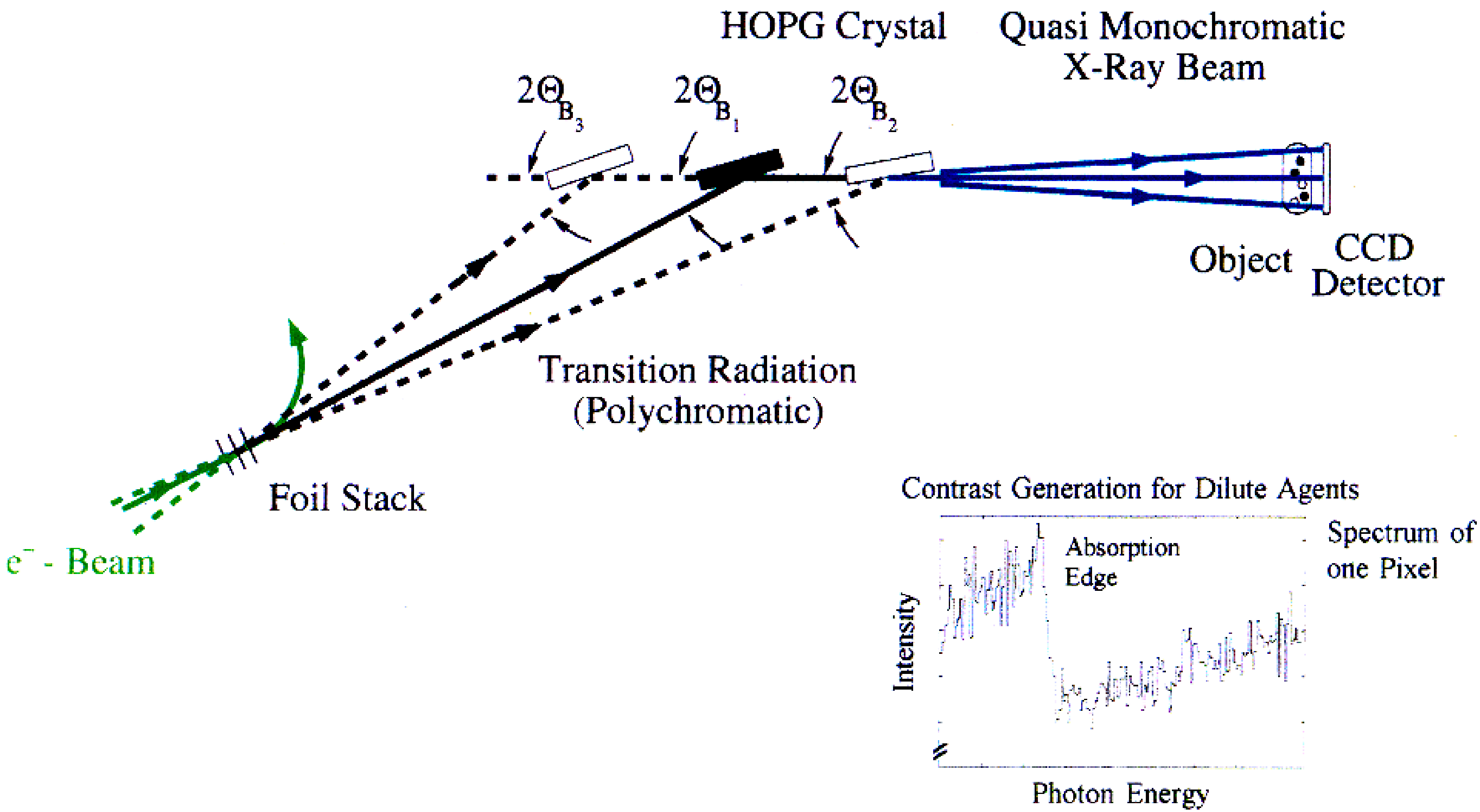
3. Transition Radiation
as a
Hard X-Ray Source

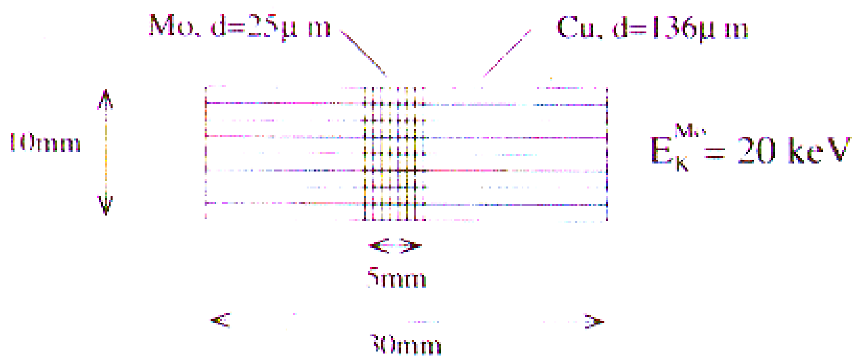
Calculations



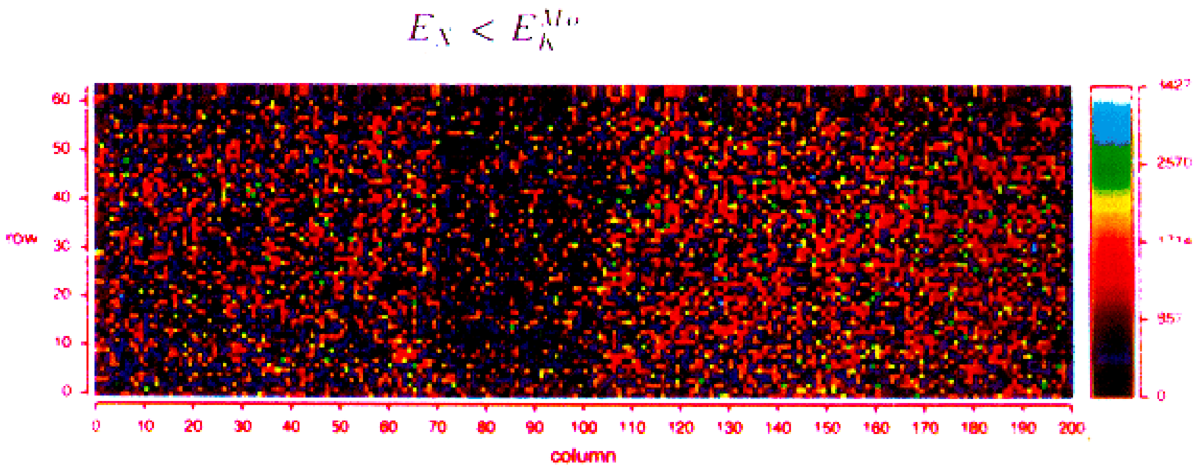


Principle of the Fast Tunable Monochromator for Digital Subtraction Imaging

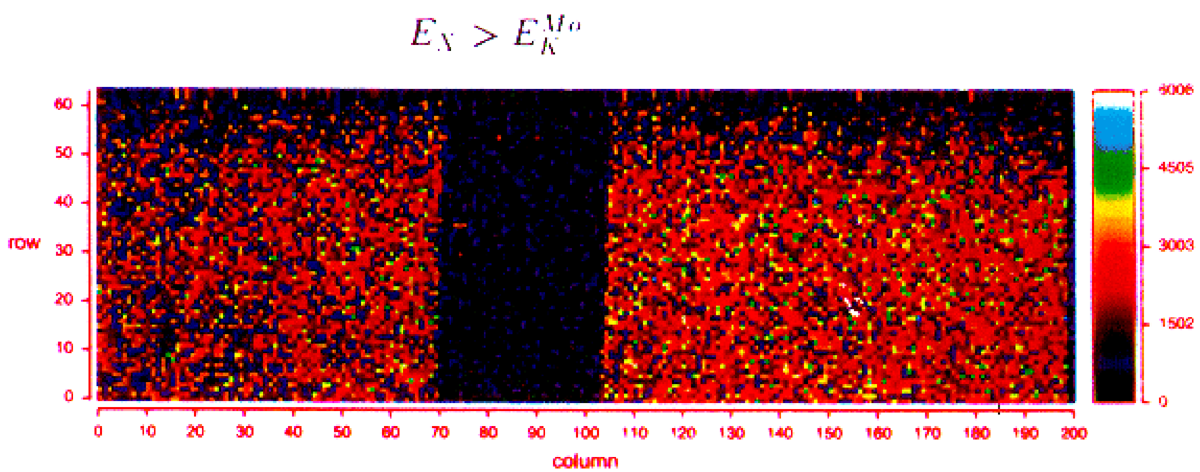




Mo, d=25μ m
Cu, d=136μ m
10mm
5mm
30mm
$E_K^{Mo} = 20$ keV

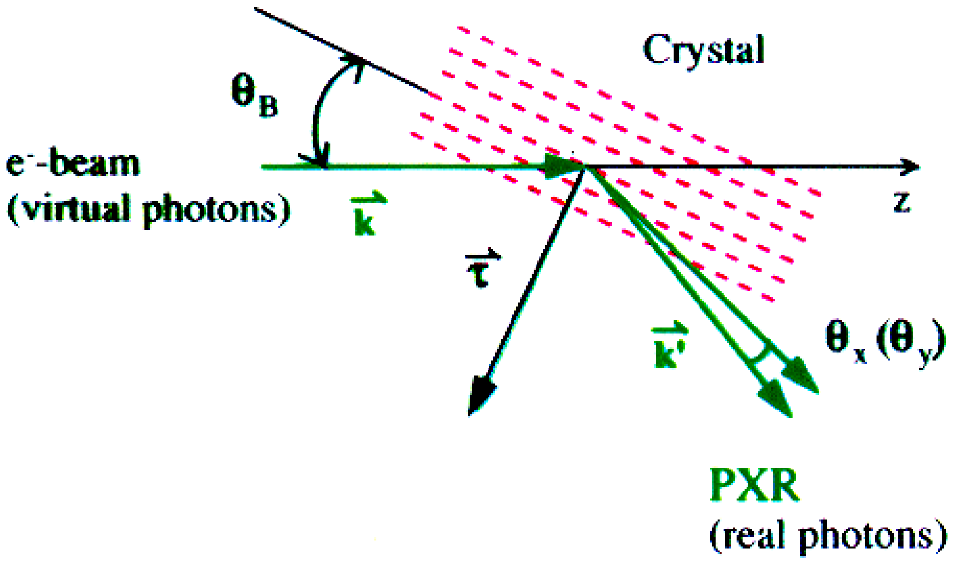


$E_X < E_K^{Mo}$
row
column
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
2570



$E_X > E_K^{Mo}$
row
column
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
6006
4505
3003
1502

4. Parametric X-Ray Radiation



Bragg Condition

$$\vec{k}'^2 = (\vec{k} + \vec{\tau})^2$$

$$n\,\lambda = 2\,d\,\sin\theta_B$$

Feranchuk-Ivashin Model

$$\frac{d^2N}{d\theta_x d\theta_y} = \frac{\alpha}{\pi}$$ — coupling strength

$$\cdot\,|\chi_\tau|^2$$ — crystal structure factor

$$\cdot\,L_{eff}$$ — effective absorption length (self absorption)

$$\cdot\,\frac{\hbar\omega_B/\hbar c}{4\sin^2(\theta_B)}\,\frac{\theta_x^2\cos^2(2\theta_B)+\theta_y^2}{(\theta_x^2+\theta_y^2+\theta_{ph}^2)^2}$$ — angular distribution

$$L_{eff} = \int e^{-f(z)/L_a} dz \qquad \theta_{ph} = (1/\gamma^2 + |\chi_0|)^{1/2} \quad , |\chi_0| \simeq \left(\frac{\omega_p}{\omega}\right)^2$$

$f(z)$ - distance to crystal surface ; ω_p - plasma frequency

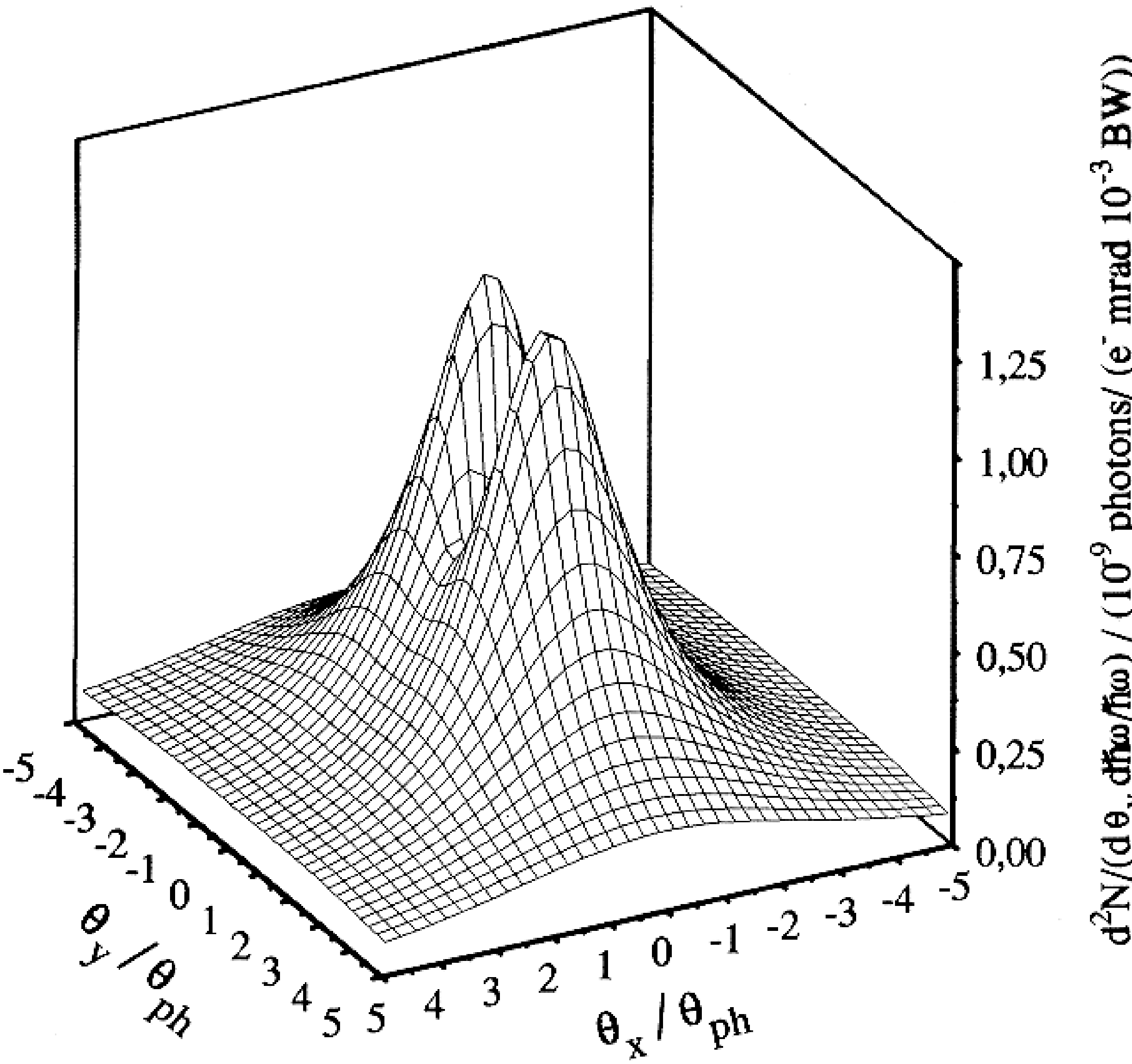
L_a - absorption length

Theoretical Angular Distribution

(I.D Feranchuk & A.V. Ivashin: J.Physique, 46 (1985)1981)

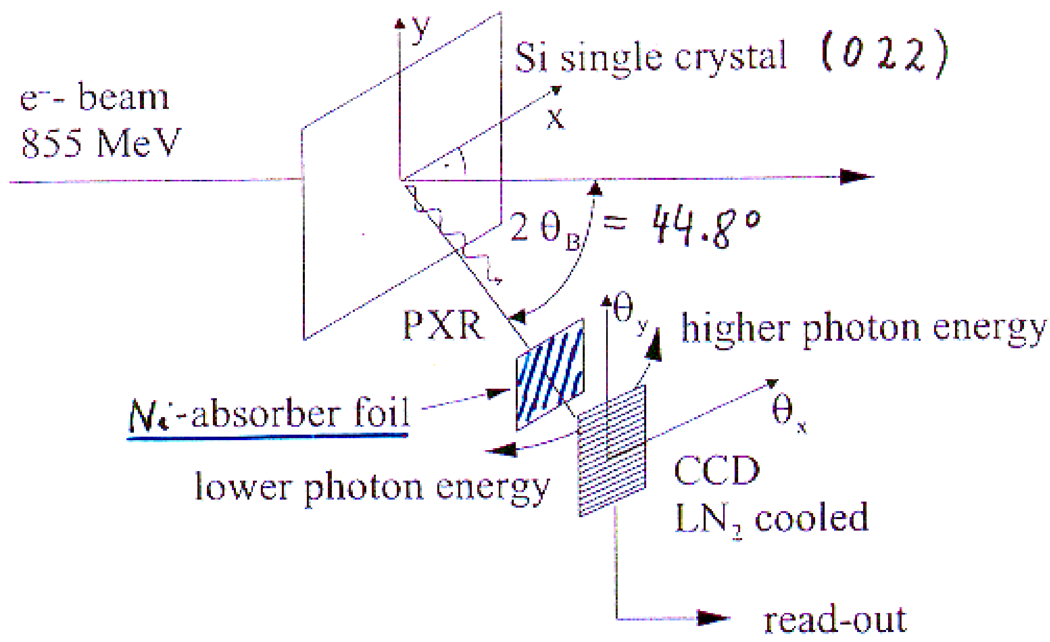
Si (111), $\theta_B = 22.5°$

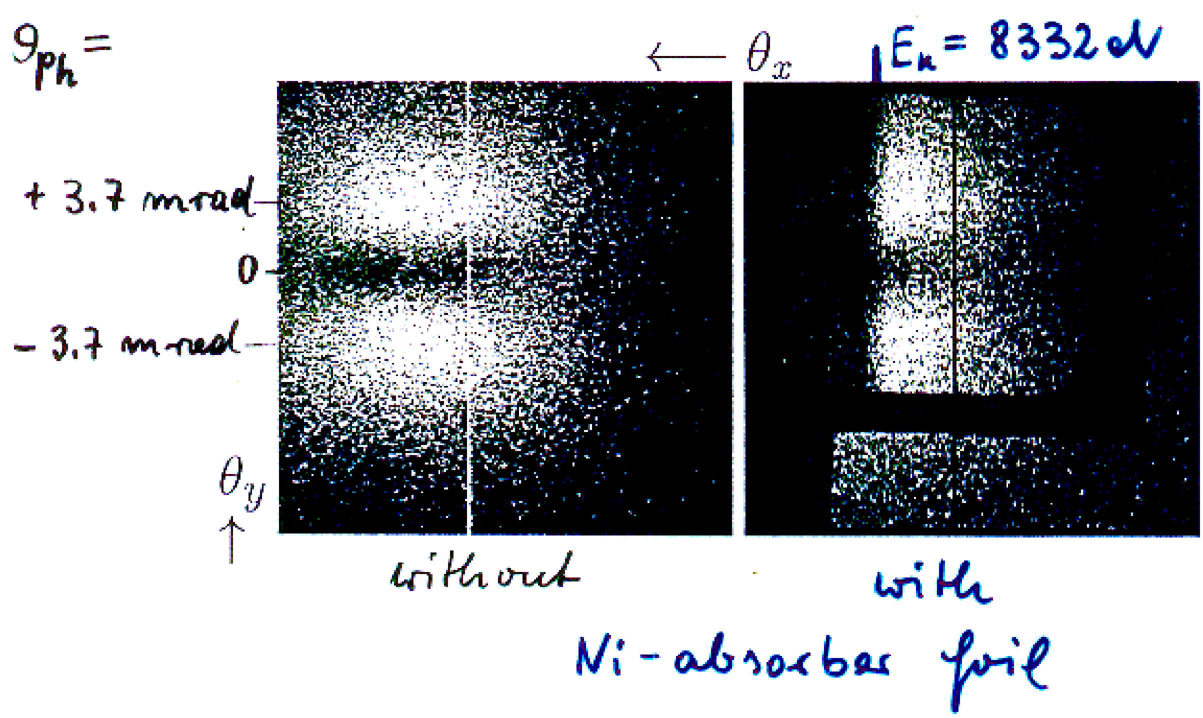
$$\theta_{ph} = (1/\gamma^2 + |\chi_0|)^{1/2} \quad , |\chi_0| \simeq \left(\frac{\omega_p}{\omega}\right)^2, \hbar\omega_p = 31\text{eV}$$
$$\gamma = 1673$$

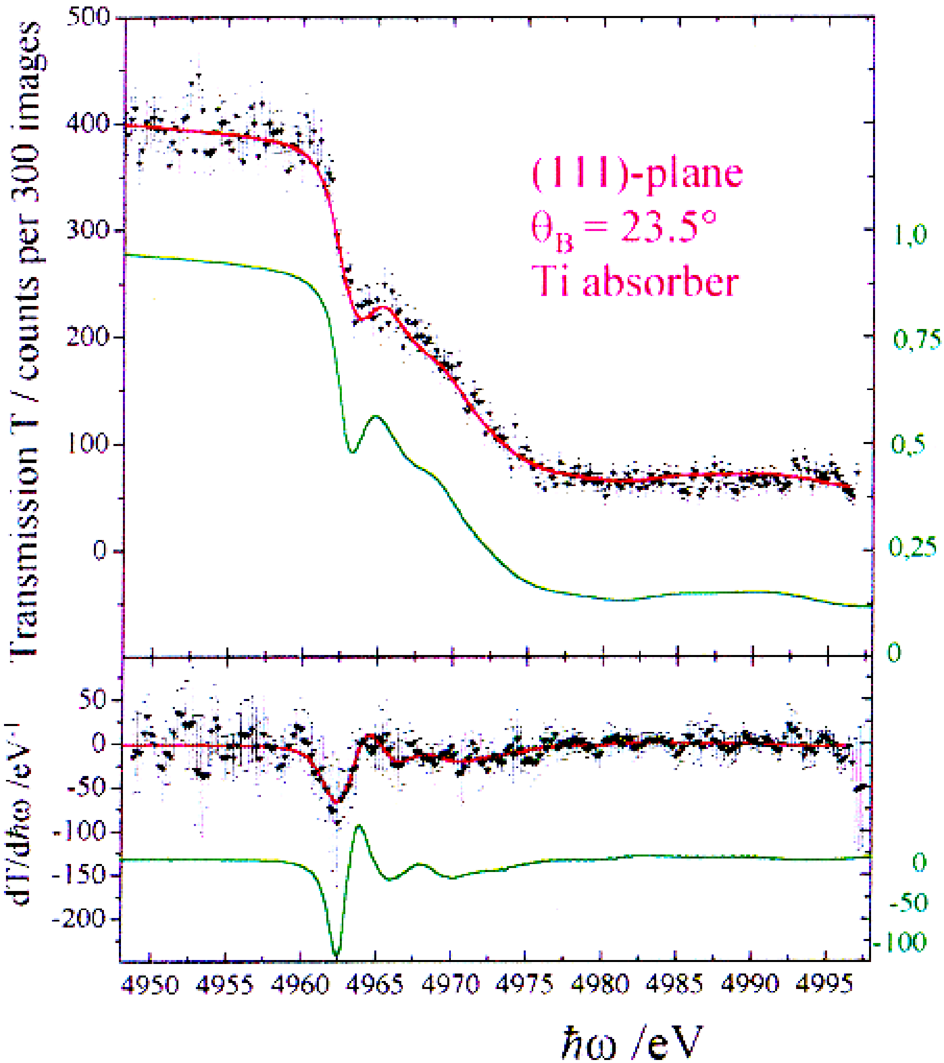


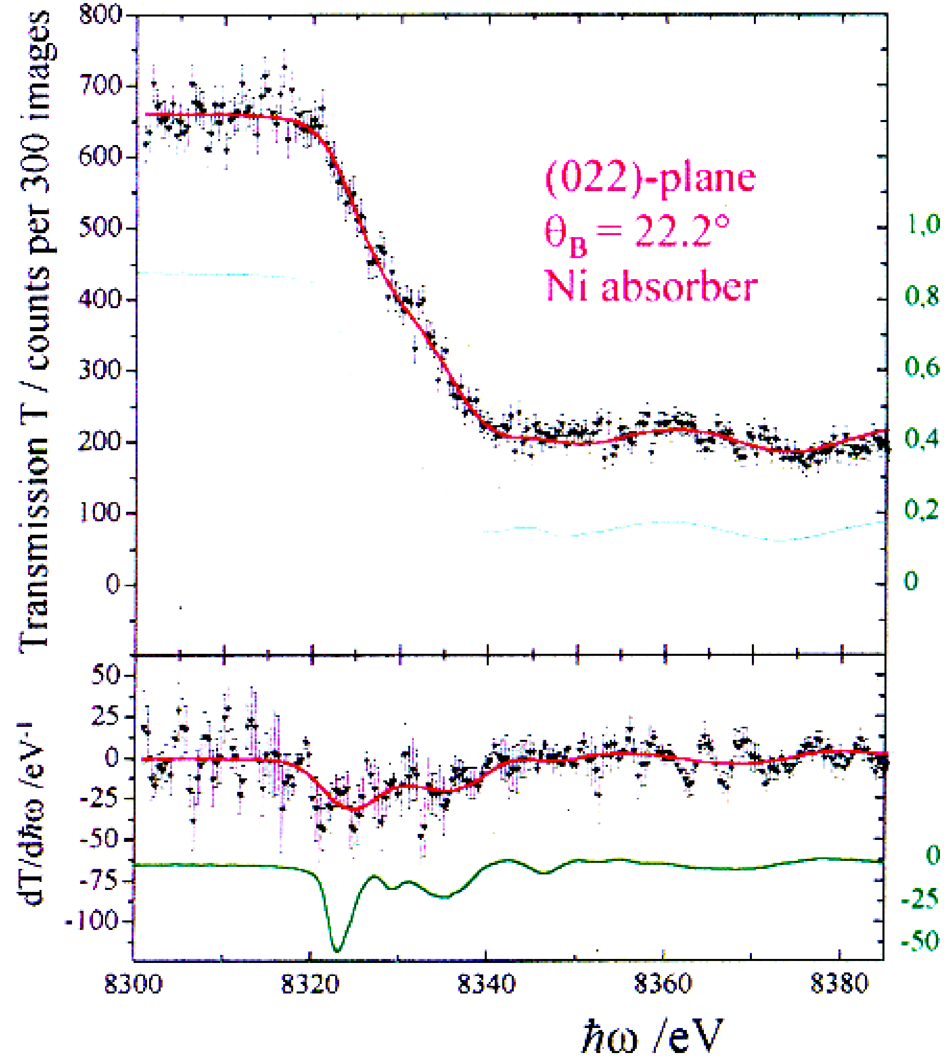
How Narrow is the Line Width of Parametric X-ray Radiation

K.-H. Brenzinger et al., Phys. Rev. Lett. 79 (1997) 2462









Fitfunction:

$$f_{fit}(E) = N \int \left[T_{\mu a}(E') \otimes g(E', \theta_x)\right] I_{PXR}(\theta_x, \theta_y)\, d\theta_y$$

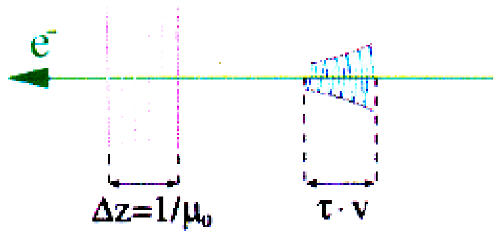
$$\Delta E_{PXR} = \Delta E_{intr.} \otimes \Delta E_{geo}$$

$\Delta E_{PXR,(111)} \leq 1.2$ eV (95%CL)

$\Delta E_{PXR,(022)} \leq 3.5$ eV (95%CL)

Contributions to the Line Width

1. Natural Linewidth:



$$\Gamma = \frac{\hbar}{\tau} \cong 4.6 \cdot 10^{-5} eV \cdot (\mu_0 / \rho)[cm^2/g]$$

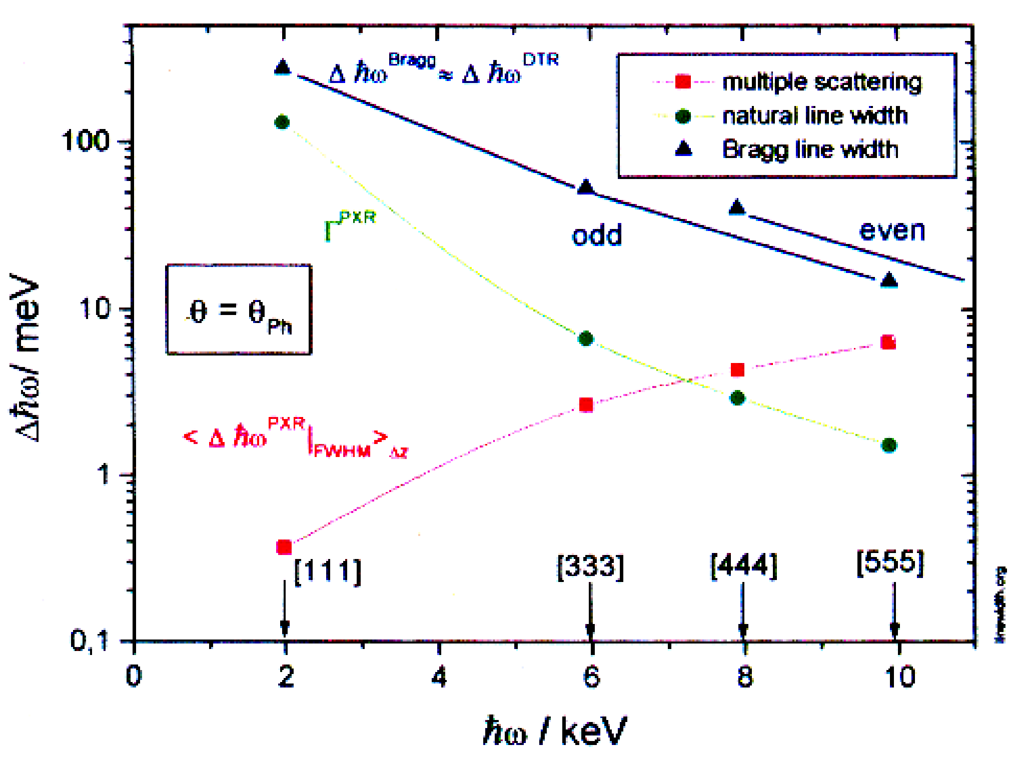
2. Mean contribution from multiple scattering of the electrons in the crystal, characterized for silicon (ρ =2.33 g/cm³, F = 0.9, pc = 855 MeV) by variance [G.R. Lynch, O.I. Dahl, NIM B58 (1991) 6]

$$\sigma_\alpha^2 \cong \frac{8.90 \cdot 10^{-7}}{(\mu_0/\rho)[cm^2/g]} \left[\ln\left(\frac{4.88 \cdot 10^4}{(\mu_0/\rho)[cm^2/g]} \right) - 1 \right]$$

$$\boxed{\left\langle \Delta\hbar\omega^{PXR}\big|_{FWHM} \right\rangle_{\Delta z} \cong 0.5\,\sigma_\alpha \cdot \hbar\omega_p} \qquad \theta = \theta_{Ph} \cong \frac{\omega_P}{\omega}$$

3. Contributions from finite detector and beam spot size can be neglected.

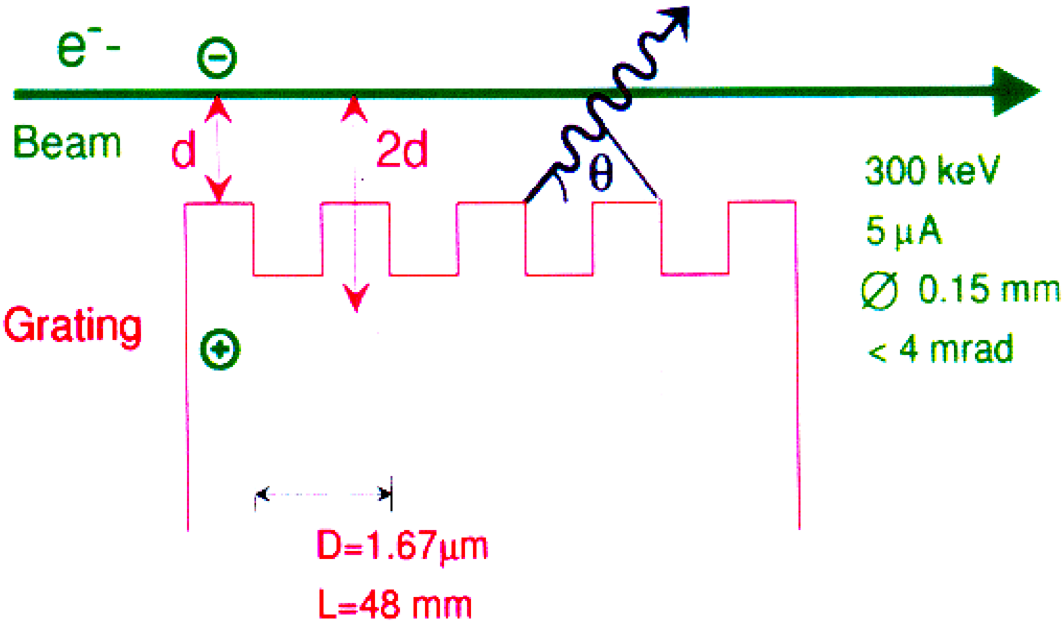
Line Width of PXR and DTR



5. Investigation of Smith-Purcell Radiation

The Experiment of Smith and Purcell

S.J. Smith, E.M. Purcell; *Phys. Rev. 92 (1953) 1069*



Superposition of Huygens elementary waves

⇒ Coherence or Smith–Purcell condition

$$n \cdot \lambda = D \cdot (1/\beta - \cos\theta)$$

n	:	diffraction order
λ	:	wavelength of the emitted radiation
D	:	spacing of grooves
β	:	reduced electron velocity, $\beta = \frac{v}{c}$
θ	:	angle of observation

Intensity predicted by dipole model :

K.Ishiguro, T.Takao; *Optica Acta 8 (1961) 25*

Spectral Intensity of Smith-Purcell Radiation compared with Undulator Radiation

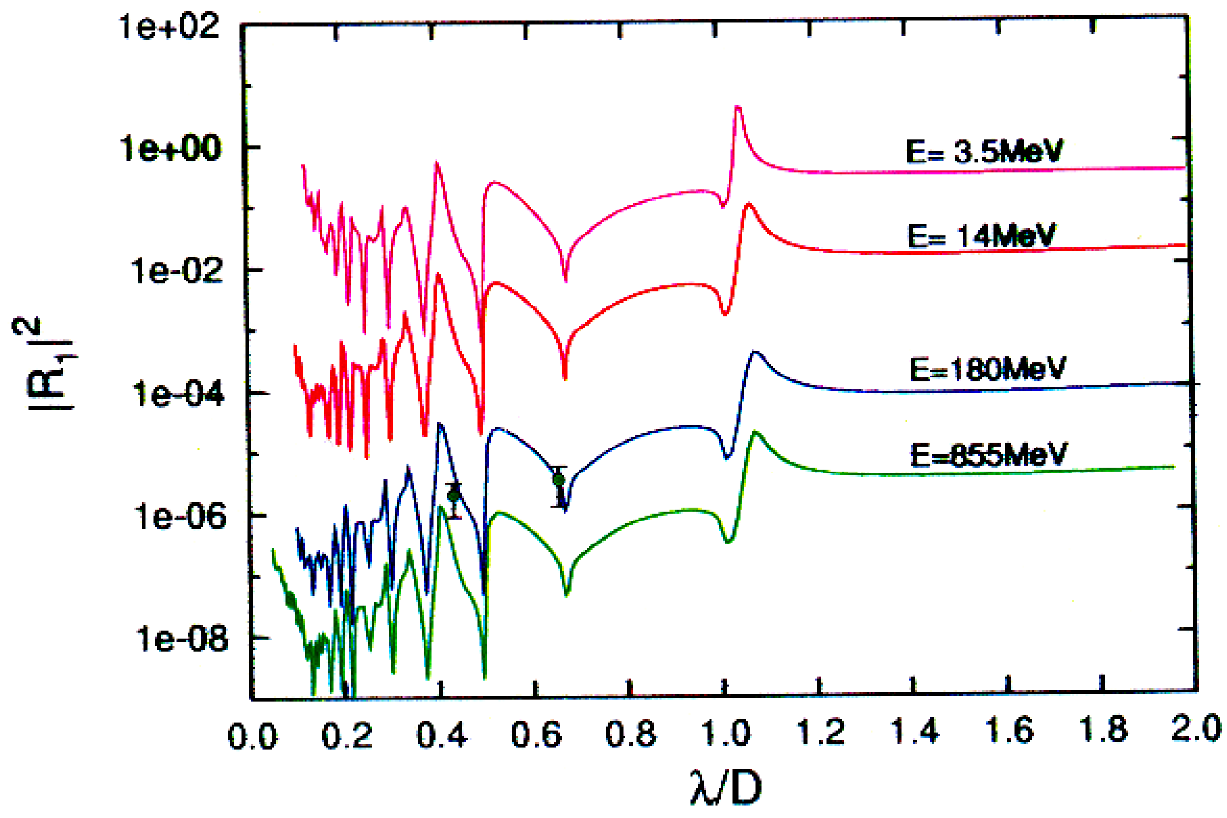
Undulator Radiation

$$\frac{d^2\dot{N}_{Und}}{d\Omega \cdot d\hbar\omega / \hbar\omega} = \alpha \cdot \frac{I}{e} \cdot \frac{D}{\lambda} \cdot N_w^2 \cdot \boxed{\frac{K^2/2}{1+K^2/2+(\theta\gamma)^2}} \boxed{n^2 \cdot F_j(n)} \cdot \left(\frac{\sin(\pi \cdot N_w \cdot \delta\omega/\omega)}{\pi \cdot N_w \cdot \delta\omega/\omega}\right)^2$$

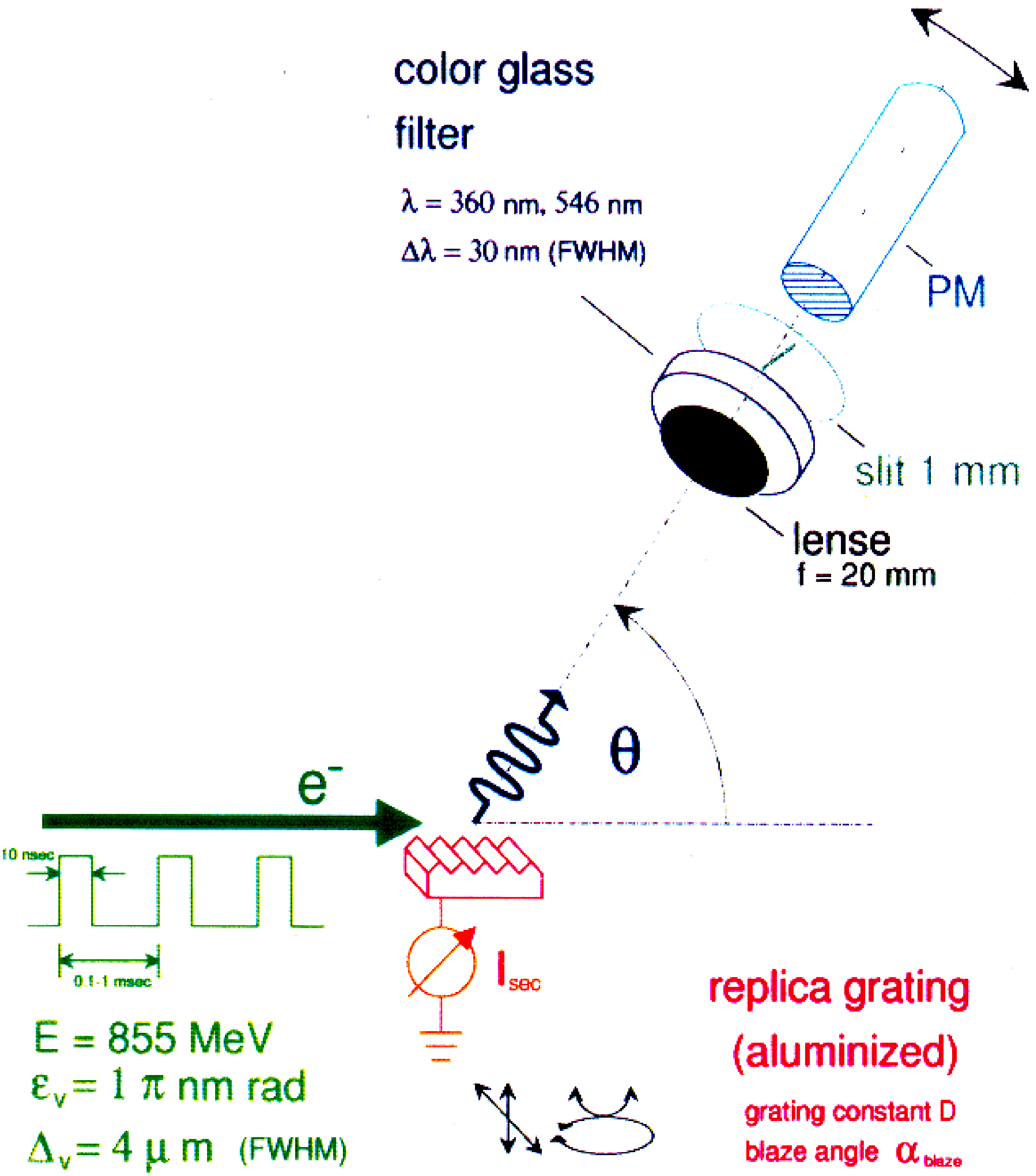
$$\boxed{F_j(n) \cong 1}$$

Smith-Purcell Radiation ($\phi = 90°$)

$$\frac{d^2\dot{N}_{SP}}{d\Omega \cdot d\hbar\omega / \hbar\omega} = \alpha \cdot \frac{I}{e} \cdot \frac{D}{\lambda} \cdot N_w^2 \cdot \boxed{\frac{\sin^2\theta}{1/\beta - \cos\theta}} \boxed{\exp\left(-\frac{y_0}{\beta\gamma \cdot \lambda/(4\pi)}\right) |R_n(\theta,\phi)|^2} \cdot \left(\frac{\sin(\pi \cdot N_w \cdot \delta\omega/\omega)}{\pi \cdot N_w \cdot \delta\omega/\omega}\right)^2$$



Experimental Setup



6. Conclusion

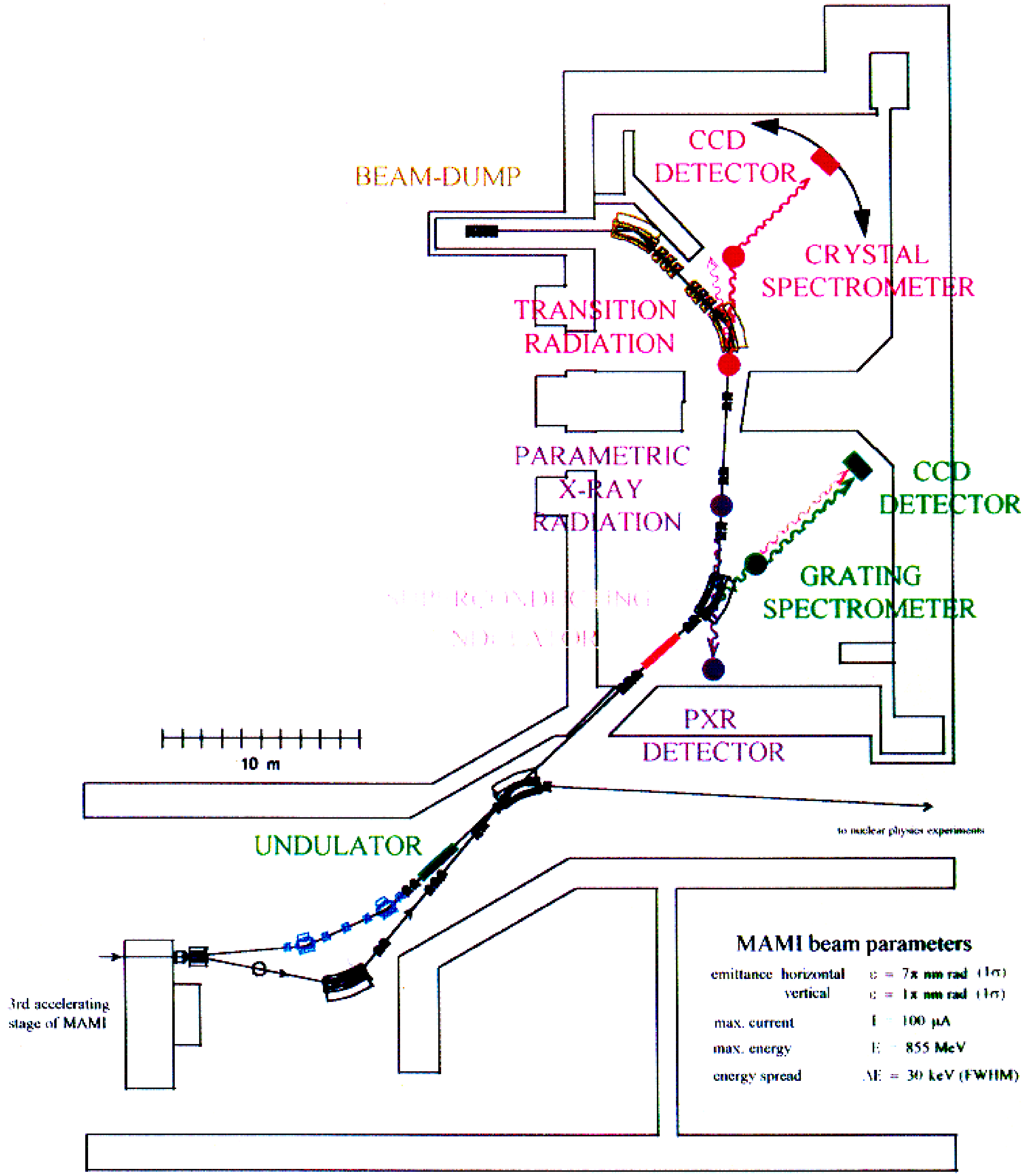
- ***Novel interferometer*** have been developed ***with two spatially separated, phase correlated X-ray sources*** based on undulator and transition radiation for in the soft and hard X-ray region, respectively

- ***Transition Radiation*** from a foil stack ***is a viable hard X-ray source*** for applications

- ***Parametric X Radiation (PXR) is a very compact monochromatic X-ray source.*** Some interesting features, as e.g. the narrow line width, remain to be explored

- ***Smith-Purcell Radiation is to weak a source*** even in the visible spectral region

X-ray Radiation Research at MAMI



H.Backe, N.Clawiter, S.Dambach, Th.Doerk, H.Euteneuer, F.Hagenbuck, K.-H.Kaiser, O.Kettig, G.Kube, W.Lauth, J.Lind, H.Mannweiler, H.Schöpe, D.Schroff, A.Steinhof, Th.Walcher

X1-Kollaboration, Institut für Kernphysik,
Johannes Gutenberg-Universtät Mainz

Th.Kerschner, H.Koch, H.Matthäy, A.Wilms, M.Zemter

Institut für Experimentalphysik, Lehrstuhl I, Ruhr-Universität Bochum

L.Strüder

MPI-Halbleiterlabor München

P.Holl, J.Kemmer, R.Stötter, C.v.Zanthier

KETEK GmbH Oberschleißheim

H. Backe[1]
V.G. Baryshevsky[3]
N. Clawiter[1]
S. Dambach[1]
Th. Doerk[1]
N. Eftekhari[1]
H. Euteneuer[1]
F. Görgen[1]
A. Grubich[3]
F. Hagenbuck[1]
K.H. Kaiser[1]
Th. Kerschner[2]
O. Kettig[1]
H. Koch[2]
G. Kube[1]
W. Lauth[1]
J. Lind[1]
Olga Lugovskaya[3]
H. Matthäy[2]
H. Mannweiler[1]
H. Schöpe[1]
M. Schüttrumpf[2]
A. Steinhof[1]
Th. Tonn[1]
Th. Walcher[1]
A. Wilms[2]
M. Zemter[2]

[1]Institut für Kernphysik,
Universität Mainz

[2]Institut für Experimentalphysik I,
Universität.Bochum

[3]Institute for Nuclear Problems,
Minsk, Belarus