ANALYSIS OF THE FREQUENCY DEPENDENCE OF THE LONGITUDINAL COUPLING IMPEDANCE OF A SMALL HOLE IN A COAXIAL LINER *

Alexei V. Fedotov, Robert L. Gluckstern
Physics Dept., University of Maryland, College Park, MD 20742

Abstract

We recently developed a general analysis for an azimuthally asymmetric rectangular slot in the inner conductor of a coaxial liner, which allowed us to investigate the coupling impedance numerically. In the present paper we obtain analytic expressions for a small hole of arbitrary shape. Specifically, we go beyond the quasi-static (Bethe) approximation to explore and understand the structure of the impedance in the frequency region near the cutoffs of the inner beam pipe and outer coaxial structure. Finally, we extend our analytic analysis to a hole in a wall of finite thickness.

1 INTRODUCTION

In an earlier work, Gluckstern and Neri [1] analyzed the impedance of a small azimuthally symmetric pill-box in a beam pipe at frequencies of the same order as the cutoffs of the TM_{0n} modes in the pipe. They found that the admittance could be written as the sum of a term depending primarily on the pill-box width and thickness, and a term depending primarily on the pipe radius. In fact, the broad resonance used frequently by others to describe the behavior near cutoffs, was shown to be due to a change of sign of the imaginary part of the admittance.

We recently constructed a variational form for the impedance of a rectangular hole in the wall of a coaxial liner [2]. Our analysis allowed us to study numerically the frequency dependence of the coupling impedance of a transverse rectangular slot, small square hole [2] and a longitudinal rectangular slot, including the resonances due to the slot length [3]. However, it is possible to obtain an approximate analytic expressions, analogous to that obtained in [1], for a small hole of arbitrary shape which would allow us to understand the structure of the impedance in the frequency region near the cutoff of the beam pipe. In the case of a narrow pill-box the dominant contribution comes from the magnetic portion of the problem; therefore, in [1], only the magnetic part was considered. In the present paper we extend the analysis to the azimuthally asymmetric problem of a hole in the wall of a coaxial liner. We also consider both the electric and magnetic portions of the problem. Finally we extend our analysis to a hole in a wall of finite thickness.

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2 THE LONGITUDINAL COUPLING IMPEDANCE

The source fields in the frequency domain generated by the driving current

$$J_z(x,y,z;k) = I_0\delta(x)\delta(y)e^{-jkz} \tag{1}$$

are the following:

$$E_r^{(s)}(r,z;k) = Z_0 H_\theta^{(s)}(r,z;k) = \frac{Z_0 I_0}{2\pi r}e^{-jkz}, \tag{2}$$

$$E_z^{(s)}(r,z;k) = 0, \tag{3}$$

where $Z_0 = 120\pi$ [Ω], $k = \omega/c$. The definition of the frequency dependent longitudinal coupling impedance of any obstacle can be taken to be

$$Z_\parallel = \frac{-1}{I_0}\int_{-\infty}^{\infty} dz e^{jkz} E_z(0,\theta,z;k), \tag{4}$$

where $E_z(r,\theta,z;k)$ is the axial electric field in the frequency domain, with frequency dependence $\exp(j\omega t)$, where $\omega = kc$. This expression can be rewritten as

$$\frac{Z_\parallel(k)}{Z_0} = -\frac{1}{2\pi a Z_0 I_0}\int dS E_z(a,\theta,z;k)e^{jkz}, \tag{5}$$

where the surface integral is only over the hole, since E_z vanishes on the liner wall. Since the driving current on axis is proportional to $\exp(-jkz)$, the problem is simplified by obtaining results for an even driving current $\cos kz$ and an odd driving current $-j\sin kz$ separately [2]. We use the superscript (e) for the even problem and the superscript (o) for the odd problem. The field matching is performed at the radius of the inner conductor (liner) in the opening. We call the region inside the inner conductor $r \le a$ the "pipe region" and the region outside the inner conductor $a \le r \le b$ the "coaxial region".

3 THIN WALL ANALYSIS

3.1 Odd Part

We now assume that the hole dimensions are small compared to the wavelength, and we can use the quasi-static solutions for the field components in the vicinity of the hole. For the odd part of the impedance we obtain

$$\frac{Z_\parallel^{(o)}}{Z_0} = -\frac{jk}{8\pi^2 a^2}\chi\Big[1 + \frac{\chi}{8\pi^2}W\Big], \tag{6}$$

with W being defined as

$$W = \sum_n \Big(\int dq [q^2 P_n(q) - \frac{n^2 k^2}{\kappa^2 a^2} Q_n(q)] \Big), \qquad (7)$$

where κ, defined by $\kappa^2 = k^2 - q^2$, is the radial propagation constant, χ is the electric polarizability of a hole, and symbol n stands for the azimuthal index of the modes. Functions P_n and Q_n contain Bessel functions and their derivatives and can be found in [2]. For a small hole, the term proportional to χW will be small compared to 1 and we can write an expression for the admittance as

$$Z_0 Y_\parallel^{(o)} = \frac{j 8\pi^2 a^2}{k\chi} - \frac{j a^2}{k} W. \qquad (8)$$

3.2 Even Part

Similarly to the odd part we obtain

$$\frac{Z^{(e)}}{Z_0} = \frac{jk}{8\pi^2 a^2} \psi_\theta \Big[1 - \frac{\psi_\theta}{8\pi^2} V \Big], \qquad (9)$$

where ψ_θ is the transverse magnetic susceptibility of a hole, and V is defined as

$$V = \sum_n \Big(\int dq [k^2 P_n(q) - \frac{n^2 q^2}{\kappa^2 a^2} Q_n(q)] \Big). \qquad (10)$$

As before, for $\psi_\theta V \ll 1$ we can write

$$Z_0 Y_\parallel^{(e)} = -\frac{j 8\pi^2 a^2}{k\psi_\theta} - \frac{j a^2}{k} V. \qquad (11)$$

We can also combine Eqs. (6) and (9) to obtain the expression for the total impedance

$$\frac{Z_\parallel}{Z_0} = \frac{jka}{8\pi^2 a^3} \Big(\psi - \chi - \frac{1}{8\pi^2} (\psi^2 V + \chi^2 W) \Big). \qquad (12)$$

The structure obtained in Eq. (12) is similar to the one presented in [4], but we now have additional contribution from the modes in the coaxial region.

3.3 Discussion

We now examine our results in Eqs. (8) and (11). One sees that each admittance separates into a part which includes the geometry of the hole (parameters ψ and χ), and the term involving the pipe and the coaxial region. In the present case the real part includes dependence on parameters b and a (where b is the radius of the outer pipe), and is present even below all possible cutoffs because of the existence of the TEM mode in the coaxial region. Additional energy is lost when other outgoing propagating modes are generated in the "coaxial" and the "pipe" region. In fact, experience with [1] suggests that these expressions are valid in the region of the mode cutoffs where ka and $k(b-a)$ are of the order of 1. This speculation is confirmed by the numerical studies [5].

3.4 Numerical Implementation

It is clear from Eq. (12) that the departure from the usual small hole (Bethe) approximation is contained in the quantities W and V defined in Eqs. (7) and (10). The real parts of W, V contribute to the imaginary part of the impedance. From numerical studies [5] we find that for low frequencies $V_r, W_r \sim k^2$, and that below all possible cutoffs imaginary part of $V, W = \pi/[ln(b/a)]$, with additional energy being lost when other outgoing propagating modes are generated.

To compare the frequency behavior of the admittance of a small hole with the one presented for a pill-box [1], we perform numerical calculations for the square hole with the following parameters $b/a = 1.3125$, $w/a = 0.25$. Results are shown in Figs. 1 and 2 for the imaginary and real part of the admittance, respectively. Figure 2 clearly shows the steps which occur as ka passes the cutoffs corresponding to the TM amd TE modes. For the odd part one sees the cutoffs corresponding to the $\mathrm{TE}_{11}^{(coax.)}$, $\mathrm{TE}_{21}^{(coax.)}$, $\mathrm{TE}_{11}^{(pipe)}$, $\mathrm{TE}_{31}^{(coax.)}$ modes, while for the even part one sees the cutoff corresponding to the $\mathrm{TM}_{01}^{(pipe)}$ mode. This leads to the conclusion that, for a small hole, the odd (electric) part primarily couples to the TE modes, while the even (magnetic) part primarily couples to the TM modes near the cutoffs. This behavior can be also shown analytically (see for example [6]). We also confirmed numerically that this behavior holds for slots with $l/a < 1$ and $kl < 1$, in agreement with [6]. If one goes to very high frequency ($kl \sim \pi$), the imaginary part of the admittance in Fig. 1 eventually crosses the zero axis and changes sign. This behavior corresponds to a resonance due to the hole length. Note that the resonances occur at different frequencies for the odd and even parts.

4 THICK WALL ANALYSIS

When the field is incident on a hole in a wall of finite thickness the usual approach is to split it into two components: one with an asymmetric potential and one with a symmetric potential about the midpoint of the wall [7]. The parameters ψ and χ come from both the symmetric and antisymmetric problems. We then define the magnetic susceptibility and electric polarizability seen within the liner as

$$\psi_{in} = \psi^s + \psi^a, \;\; \chi_{in} = \chi^s + \chi^a, \qquad (13)$$

while the susceptibility and polarizability outside the liner are defined by

$$\psi_{out} = \psi^s - \psi^a, \;\; \chi_{out} = \chi^s - \chi^a. \qquad (14)$$

4.1 Even Part

For the even part of the impedance we obtain [5]:

$$\frac{Z^{(e)}}{Z_0} = \frac{jk}{8\pi^2 a^2} \psi_{in} \Big[1 - \frac{\psi_{in}}{8\pi^2} \tilde{V} \Big], \qquad (15)$$

where

$$\tilde{V} \;\; = \;\; \sum_n \int dq \Big[k^2 \Big(\frac{J_n'(\kappa a)}{\kappa a J_n(\kappa a)} - \frac{\psi_{out}^2}{\psi_{in}^2} \frac{F_n'(\kappa a)}{\kappa a F_n(\kappa a)} \Big)$$

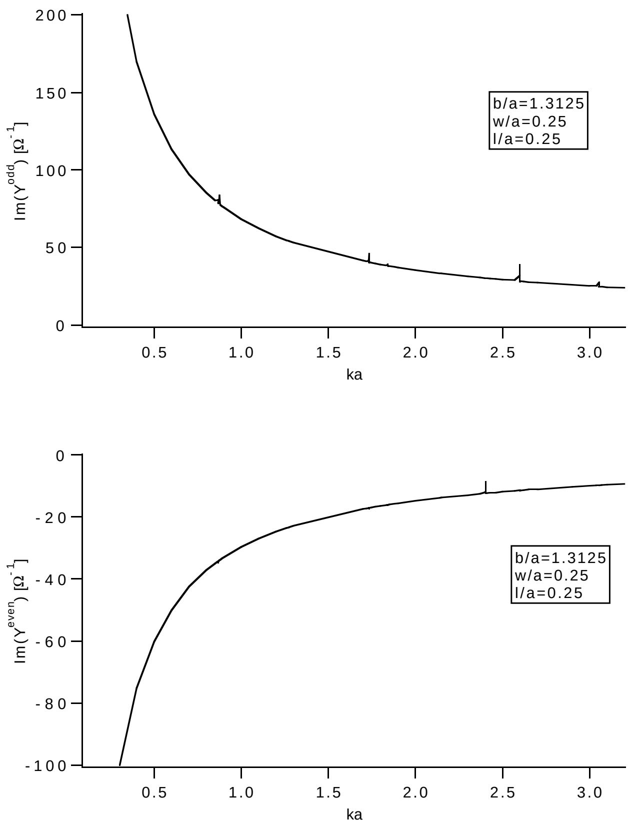

Figure 1: Imaginary part of the admittance near the cutoff frequencies.

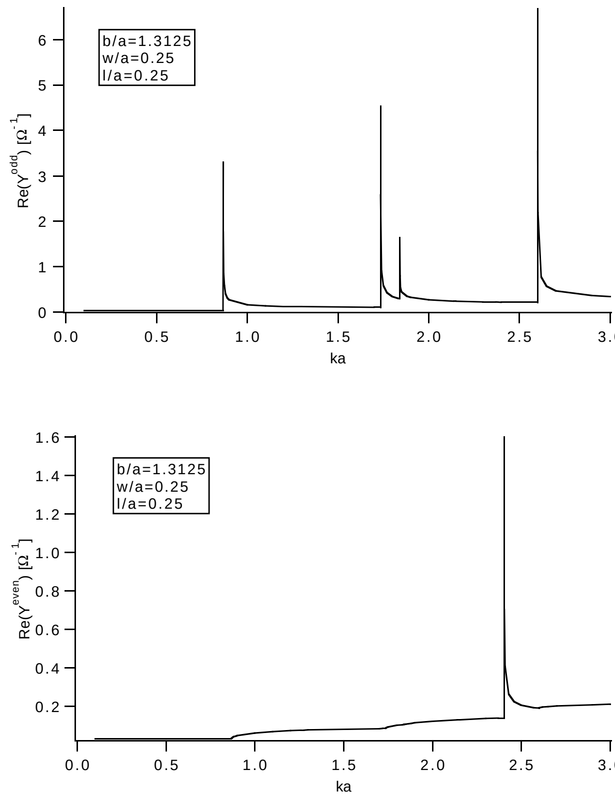

Figure 2: Real part of the admittance near the cutoff frequencies.

$$- \quad \frac{q^2n^2}{\kappa^2a^2}\Big(\frac{J_n(\kappa a)}{\kappa a J_n'(\kappa a)} - \frac{\psi_{out}^2}{\psi_{in}^2}\frac{G_n(\kappa a)}{\kappa a G_n'(\kappa a)}\Big)\Big]. \qquad (16)$$

4.2 *Odd Part*

For the odd part of the impedance we obtain [5]:

$$\frac{Z^{(o)}}{Z_0} = -\frac{jk}{8\pi^2a^2}\chi_{in}\Big[1 + \frac{\chi_{in}}{8\pi^2}\tilde{W}\Big], \qquad (17)$$

where

$$\tilde{W} = \sum_n \int dq \Big[q^2\Big(\frac{J_n'(\kappa a)}{\kappa a J_n(\kappa a)} - \frac{\chi_{out}^2}{\chi_{in}^2}\frac{F_n'(\kappa a)}{\kappa a F_n(\kappa a)}\Big) - \frac{k^2n^2}{\kappa^2a^2}\Big(\frac{J_n(\kappa a)}{\kappa a J_n'(\kappa a)} - \frac{\chi_{out}^2}{\chi_{in}^2}\frac{G_n(\kappa a)}{\kappa a G_n'(\kappa a)}\Big)\Big]. \qquad (18)$$

Expressions for $\tilde{V}$ and $\tilde{W}$ include the parameters ψ and χ; and, therefore, for the case of a wall of finite thickness, the admittance does not separate into a part which includes only the geometry of a hole and a term which includes the geometry of a pipe. But this separation is still valid for a thin wall ($\chi_{out} = \chi_{in}$, $\psi_{out} = \psi_{in}$) and for a very thick wall ($\chi_{out} = \psi_{out} = 0$).

5 SUMMARY

We obtained analytic expressions for the impedance of a small hole which includes effects of energy propagation along the inner beam pipe and/or outer coaxial pipe. This allow us to understand the structure of the impedance in the frequency region near the cutoffs of the inner beam pipe and outer coaxial structure. We then extended our analytic treatment to a wall of finite thickness and discussed the resulting expressions, and concluded that the admittance could no longer be separated into a part depending only on the pipe geometry and a part depending primarily on the hole geometry.

6 REFERENCES

[1] R.L. Gluckstern and F. Neri, in Proceedings of the 1989 Particle Accelerator Conference (IEEE, Chicago, IL, 1989), p. 1271.

[2] A.V. Fedotov and R.L. Gluckstern, Phys. Rev. E, 56, N. 3, 3583 (1997).

[3] A.V. Fedotov and R.L. Gluckstern, Phys. Rev. E, 56, N. 6, 7217 (1997).

[4] S.S. Kurennoy, R.L. Gluckstern and G.V. Stupakov, Phys. Rev. E, 52, N. 4, 4354 (1995).

[5] A.V. Fedotov and R.L. Gluckstern, Phys. Rev. ST Accel. Beams 1, 024401 (1998).

[6] G.V. Stupakov, in Proceedings of the 1995 Particle Accelerator Conference (IEEE, Dallas, TX, 1995), p. 3306.

[7] R.L. Gluckstern and J.A. Diamond, IEEE Trans. on Mic. Theory and Techn., 39, N. 2, 274 (1991).