ANALYTIC MODEL OF ION EMISSION FROM THE FOCUS OF AN INTENSE RELATIVISTIC ELECTRON BEAM ON A TARGET

George J. Caporaso and Yu-Jiuan Chen
Lawrence Livermore National Laboratory, Livermore, California 94550 USA

Abstract

Advanced radiographic systems for stockpile stewardship require very small x-ray sources to achieve the required resolution. Focusing multi-kiloampere beams to diameters on the order of 1 mm onto a Bremsstrahlung target leads to the generation of axial electric fields on the order of several MV/cm which act to extract ions out of the surface plasma and accelerate them upstream into the beam. These backstreaming ions act as a distributed electrostatic lens which can perturb the focus of the electron beam in a time varying manner during the pulse. An analytic model of the ion extraction is presented for a particular target geometry along with scaling laws for the perturbation of the focal spot.

1 INTRODUCTION

High resolution x-ray radiography requires the production of a small (≈1 mm diameter) spot on the surface of a Bremsstrahlung converter target by a relativistic electron beam of at least several kiloamperes [1]. A mechanism that might possibly disrupt the focal spot was proposed by D. Welch [2]. Bombardment of the target by a high power electron beam would lead to the rapid formation of a surface plasma. A large axial electric field would appear at the surface due to the charge redistribution on the target arising from cancellation of the beam's *radial* electric field. This axial field would expel the ions into the beam. These *backstreaming* ions would acquire energies on the order of the space charge depressed potential of the beam and would propagate upstream at very high speeds where they would act as an electrostatic focusing lens. The focusing due to these moving ions would cause the electron beam to pinch upstream of the target and then rapidly diverge. The result would be a spot size that would rapidly increase in time at the converter target.

An analytic model is presented for a "beer can" geometry in which a close fitting conducting tube surrounds the beam right up to the target. A beam envelope equation is used to derive scaling laws for the effect of the backstreaming ions on the focal spot size at the target.

2 TARGET GEOMETRY AND MODEL

We will model the "beer can" geometry shown in Figure 1. In this target arrangement, a conducting tube with the same radius as the electron beam is connected to the target. The presence of the tube limits the space charge depression of the beam which will in turn reduce the emitted ion current.

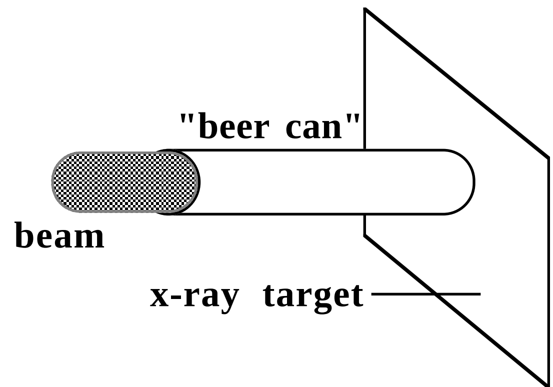

Fig. 1. "Beer can" geometry proposed to reduce the space charge depressed potential of the beam which would reduce the backstreaming ion current.

We will assume that the target surface is sufficiently rich in ions that flow will be space charge limited. The steady state emission is determined by Poisson's equation for the electrostatic potential (in c.g.s. units)

$$\nabla^2\Phi = -4\pi\rho \qquad (1)$$

where ρ is the sum of the beam's charge density and the density of the emitted ions. Since the target (and the tube) are assumed to be grounded, we may use the conservation of energy to obtain the ion velocity as

$$v_i = \sqrt{-2e\Phi / M} \qquad (2)$$

where M is the ion mass and e is the ion charge. The ion charge density is given by

$$\rho_i = J(r) / v_i \qquad (3)$$

where J(r) is the ion current density.

Equation [1] is two dimensional (r and z). A great simplification is made possible by choosing the beam profile to be of the form

$$\rho_b = -\rho_o J_o(\alpha r) \qquad (4)$$

where J_o is the zeroth order Bessel function and $\alpha = x_{01} / a$. Here a is the radius of the beer can, x_{01} is the first root of J_o and $-\rho_o$ is the on-axis charge density of the beam.

Let us seek solutions which have the following form:

$$\Phi(r,z) = -\psi(z)J_o(\alpha r) \qquad (5)$$

and

$$J(r) = \Lambda_o J_o^{3/2}(\alpha r) \tag{6}$$

where ψ(z) and Λ_o are to be determined.

Substitution of Equations [2] through [6] into Equation [1] yields

$$\psi - \frac{d^2\psi}{d\zeta^2} = -\frac{4\pi\rho_o}{\alpha^2} + \frac{4\pi\Lambda_o}{\alpha^2}\sqrt{\frac{M}{2e}}\frac{1}{\sqrt{\psi}} \tag{7}$$

where we have defined a dimensionless axial coordinate ζ=αz. If we multiply Equation [7] by dψ/dζ we can obtain a first integral

$$\left(\frac{d\psi}{d\zeta}\right)^2 = \psi^2 - \frac{8\pi\rho_o}{\alpha^2}\psi + \frac{16\pi\Lambda_o}{\alpha^2}\sqrt{\frac{M}{2e}}\sqrt{\psi} \tag{8}$$

where we have used the condition for space charge limited emission to eliminate the constant of integration (i.e., dψ/dζ=0 at the emitting surface ζ=0 where ψ=0).

To proceed further we define a dimensionless variable Ω and a dimensionless constant μ as

$$\Omega \equiv \sqrt{\alpha^2\psi / 8\pi\rho_o} \tag{9}$$

and

$$\mu \equiv \frac{16\pi\Lambda_o}{\alpha^2}\sqrt{\frac{M}{2e}}\left(\frac{\alpha^2}{8\pi\rho_o}\right)^{3/2}. \tag{10}$$

With these definitions we may solve Equation [8] as (we choose the positive root since we expect ψ to increase with ζ)

$$\int_0^{\Omega}\frac{\sqrt{\Omega'}}{\sqrt{\Omega'^3 - \Omega' + \mu}}d\Omega' = \frac{\zeta}{2}. \tag{11}$$

We expect that as $\zeta \to \infty$, Ω will approach a finite asymptotic value corresponding to the space charge depressed potential of the beam. Thus μ must have a value such that the integral in Equation [11] $\to \infty$ as $\Omega \to \overline{\Omega}$ the asymptotic value. We note that the radical in the denominator of Equation [11] must be real for a physical solution to exist. Let us find its minimum. Defining the radicand as χ we have

$$\chi \equiv \Omega^3 - \Omega + \mu \tag{12}$$

and

$$d\chi / d\Omega = 3\Omega^2 - 1 = 0 \tag{13}$$

so that

$$\Omega_o = \pm\sqrt{1/3}. \tag{14}$$

From the derivative of Equation [13] we see that the positive root of Equation [14] will correspond to a minimum of the radicand

$$\chi_{\min} = -2/3\sqrt{3} + \mu. \tag{15}$$

Note that if this minimum value is greater than zero then the integral will be finite regardless of the upper limit of integration in Equation [11] and thus will not be a solution. Therefore μ must have a value such that $\chi_{\min} = 0$. That is, we must have

$$\mu = 2/3\sqrt{3} \quad \text{and} \quad \overline{\Omega} = \Omega_o = 1/\sqrt{3}. \tag{16}$$

Using this result we can solve Equation [11]. The solution is shown in Figure 2. Note that the potential changes rapidly over a distance of order the beam radius.

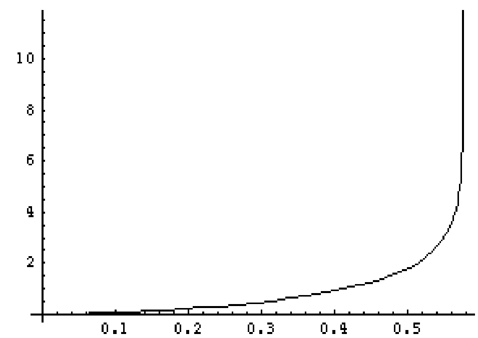

Fig. 2. Solution of Equation [11]. ζ is plotted vs. Ω.

With the solutions given by equation [16] we can immediately determine the asymptotic potential as

$$\psi \to \psi_{\max} = 8\pi\rho_o / 3\alpha^2 \tag{17}$$

and the ion current constant as

$$\Lambda_o = \frac{2}{3\sqrt{3}}\frac{\alpha^2}{16\pi}\sqrt{\frac{2e}{M}}\left(\frac{8\pi\rho_o}{3\alpha^2}\right)^{3/2} \propto \frac{\psi_{\max}^{3/2}}{a^2}. \tag{18}$$

Note that the final result for the ion current resembles the classical Child-Langmuir law for a diode with a potential given by the beam potential and an "A-K gap" given by the beam radius.

By integrating over the beam profile we find that

$$\psi_{\max} = \frac{4}{3}\frac{I}{x_{01}J_1(x_{01})c} \cong 32.04 I_{kA} \text{ (kV)} \tag{19}$$

and that the asymptotic neutralization fraction of the beam by the ions is

$$\rho_{ion}(r, z \to \infty) / |\rho_{beam}(r)| = 1/9\sqrt{3} = 0.064. \tag{20}$$

The potential gives rise to an asymptotic ion speed given through Equation [2] as

$$v_{\max} = 2.48x10^8\sqrt{I_{kA}Z/A} \text{ (cm/sec)} \tag{21}$$

where Z and A are the charge state and atomic number of the ions respectively.

3 FOCUSING EFFECTS OF BACKSTREAMING IONS

Examination of Equation [21] reveals that substantial motion of backstreaming ions is possible during a single electron beam pulse. For example for a 4 kA beam protons will propagate approximately 50 cm upstream after 60 ns, singly charged carbon ions would travel 14 cm in the same time and singly charged tantalum ions (a typical target material) would move only about 3.5 cm.

These propagation distances are comparable to the focal length of the final focusing lens in radiography systems and so would be expected to exert a substantial focusing force on the electron beam.

An estimate of the effects of these ions can be obtained by using an envelope equation for the beam. By computing the radial electric field produced by the ions and averaging the product of this field with radius over the beam profile it is possible to derive a simple equation for the rms (root mean square) radius of the beam [3]. Assuming a uniform distribution for the beam profile and using the Lapostolle emittance (E) we can then obtain an equation for the edge radius R of the beam (without space charge) as

$$R'' = \frac{E^2}{R^3} - \frac{2 f_n I}{\gamma\beta^2 I_O R} \qquad (22)$$

where f_n is the neutralization fraction (given in the "beer can" model by Equation [20]), $I_o = mc^3/e \approx 17$ kA and E is the Lapostolle emittance.

As the ions propagate upstream we expect the type of behavior shown in Figure 3a to occur. Equation [22] can be solved for different "slices" of the beam corresponding to different distances from the head of the beam. Each slice will experience an ion column of different length and so will have a different history of R vs. z. If the head of the beam is arranged to hit the target at a waist then ion backstreaming will initially lead to a smaller spot on target as the additional electrostatic focusing in close proximity to the target pinches the beam. However, as the ions move further upstream this pinching will occur progressively farther upstream leading to a divergent beam at the target. The behavior of the focal spot at the target as a function of time is shown in Figure 3b. The time at which the spot radius equals R_s, the spot size at the head of the beam is the *disruption time* τ_d.

By numerically solving a dimensionless, scaled version of Equation [22] the length of uniform ion column required to disrupt the focal spot is found to be

$$z_o \approx R_s \sqrt{\pi\gamma\beta^2 I_o / f_n I} \qquad (23)$$

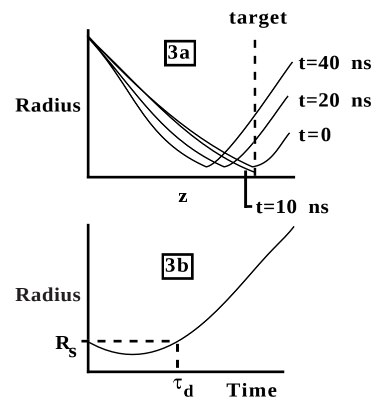

Fig. 3. (a) Trajectories of different beam "slices". (b) Radius as a function of time at the target plane.

and the disruption time follows from Equations [21] and [23] as

$$\tau_d = z_o / v_i \approx 29.5 R_s \sqrt{\gamma\beta^2 A / f_n Z} / I_{kA} \text{ (ns)}. \qquad (24)$$

For example if R_s=0.05 cm, I=2kA and γ=12.7 we find that z_o=3.64 cm and τ_d=10.4 ns for proton emission.

4 CONCLUSIONS

We have provided an exact analytic solution to the problem of the space charge limited flow of ions off the surface of a target surrounded by a tight fitting cylindrical tube of the same diameter as the electron beam. The effects of these ions have been treated with an envelope equation and the scaling laws for the disruption of the focal spot have been derived.

5 ACKNOWLEDGMENTS

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6 REFERENCES

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