THE ESTIMATIONS FOR MECHANICAL VIBRATIONS OF STEMS-LIKE ELEMENTS IN RF CAVITIES

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Abstract

In such elements of accelerating cavities as stems, posts, spirals, split rings low frequency (several tens Hz) mechanical oscillations may excite. Analytical expressions for resonant frequencies are presented. If the channel for cooling liquid is placed inside the element, depending on flow parameters the source of noises may exists from turbulence. Estimation for flow parameters and possible spectrum of noises are given.

1 INTRODUCTION

A lot of rf cavities have in the design thin and long stem-like elements. It may be stems for drift tubes support in DTL structure (Fig. 1), post couplers for stabilisation of the accelerating field distribution (Fig. 1), straight (Fig. 2) or circular (Fig. 3) central conductors in double gap bunching cavities. In such thin elements small low-frequency mechanical oscillations may excite. This mechanical oscillations will lead to small changes in the resonant frequency of the cavity and amplitude modulation of accelerating voltage. If the external source of noises exists, it simplifies the excitation of oscillations. One possible source of excitation are the noises due to turbulent flow of the cooling liquid. In this paper the qualitative estimations for own frequencies of mechanical oscillations are presented and parameters of the liquid flow are considered.

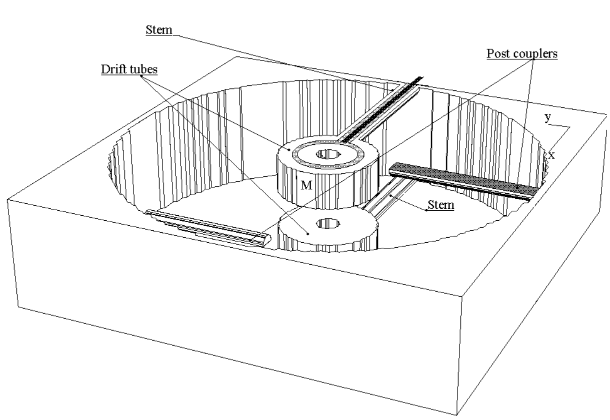

Figure 1: A sketch of DTL structure

2 MECHANICAL OSCILLATIONS

Let consider the stem with the mass m rigidly fixed at one end. At the another end of the stem the drift tube with the mass M are fixed. Two cases should be considered.

2.1 The heavy drift tube

If the mass of the drift tube $M \gg m$, we can neglect the mass of the stem m and consider oscillations of the heavy solid body at the weightless elastic stem. Supposing y is the displacement in the direction perpendicular to the stem axis one get [1] equation for small transverse oscillations:

$$I_c E \frac{d^2 y}{d^2 x} + M(l-x)(\frac{d^2 y}{d^2 t})_{x=l} = 0, \qquad (1)$$

where x is the current coordinate along the stem axis. I_c is the moment of inertia of cross section:

$$I_c = \int_S y^2 dS, \qquad (2)$$

E is the Young modulus of the material of the stem, l is the length of the stem. We suppose the mass M is concentrated in a point at the end of the stem. The eigenfrequency of this oscillations f_h:

$$f_h \approx \frac{1}{2\pi}\sqrt{\frac{3 I_c E}{M l^3}}, \qquad (3)$$

It should be pointed out here strong dependence of f_h from the length of the stem.

To simplify manufacturing procedure, DTL cavities are usually at constant radius and the radius of drift tubes is constant also. So, the stems have the same length, but the mass of drift tubes (together with the length of tubes) changes. For typical 200 MHz DTL, for example for $0.04 \leq \beta \leq 0.4$ with copper hollow stems (outer radius of the stem $R_2 = 20$ mm, the inner one $R_1 = 15$ mm, the length $l = 400$ mm, $I_c = \pi/4(R_2^4 - r_1^4)$) and drift with the mass M from 3.5 kg to 30 kg one will get estimation for $f_h \sim (58 \div 18)$ Hz.

2.2 The stem without drift tube

Another case to be considered is the case $M \ll m$ and we can assume $M = 0$. This case describes oscillations of post couplers (Fig. 1) and may be good approximation for oscillations in central conductors of bunching cavities (Fig. 2, Fig. 3). Considering small transverse oscillations [1]:

$$I_c E \frac{d^4 y}{d^4 x} + S\rho \frac{d^2 y}{d^2 t} = 0, \qquad (4)$$

and eigenfrequencies f_{ln} are:

$$f_{ln} \approx \frac{a_n^2}{2\pi l^2}\sqrt{\frac{I_c E}{\rho S}}, \qquad (5)$$

where ρ is the density if the stem material, S is the square of the stem cross section, a_n are the roots of the characteristic equation:

$$cos(a_n)ch(a_n) = -1, \qquad a_1 = 1.876, a_2 = 4.675.... \quad (6)$$

For the hollow copper post coupler with outer radius $R_2 = 40$ mm, the inner one $R_1 = 30$ mm, the length $l = 400$ mm one will estimate $f_{l1} \approx 140$Hz.

For circular central conductor of bunching cavity estimation (5) may be applied, if $R \gg R_2$, with transformation $l = \pi R$ (Fig.3).

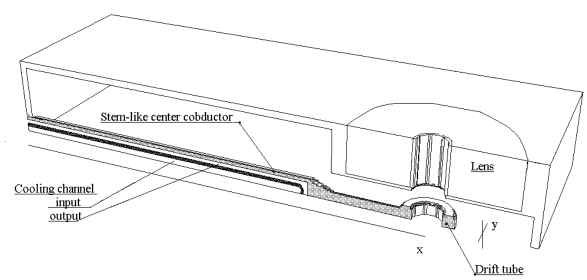

Figure 2: The double-gap bunching cavity with the straight central conductor

2.3 *Oscillations of rotation*

Let consider oscillations of rotation with respect the axis of the stem. Also two cases should be distinguished in comparison of inertia moments of the drift tube I_t and the stem I_s with respect to the stem axis. If $I_t \gg I_s$, the frequency for oscillations of rotation f_{th} is [1], [2]:

$$f_{th} \approx \frac{1}{2\pi}\sqrt{\frac{C}{lI_t}}, \qquad (7)$$

where C is the rotational rigidity, which strongly depends on the shape of cross section of the stem and for the coaxial tube is [1]:

$$C = \frac{\pi E(R_2^4 - R_1^4)}{4(1+\sigma)}, \qquad (8)$$

where σ is the Poisson coefficient. For cylindrical drift tube with radius R_t and the length l_t the inertia momentum I_t needed is [2]:

$$I_t = \frac{M(3R_t^2 + l_t^2)}{12}, \qquad (9)$$

and for the case considered in (2.1) ($l_t = 46 \div 380$ mm, $R_t = 75$ mm) $f_{th} \sim (300 \div 30)$ Hz.

If $I_t \ll I_s$ we can neglect I_t (the case of the post coupler) and the frequency for oscillations of rotation f_{tl} may be estimated as [1], [2]:

$$f_{tl} \approx \frac{1}{2\pi}\sqrt{\frac{C}{lI_s}} = \frac{1}{2\pi}\sqrt{\frac{\pi E(R_2^2 + R_1^2)}{2Ml(1+\sigma)}}. \qquad (10)$$

For the case considered in (2.2) $f_{tl} \approx 450$Hz.

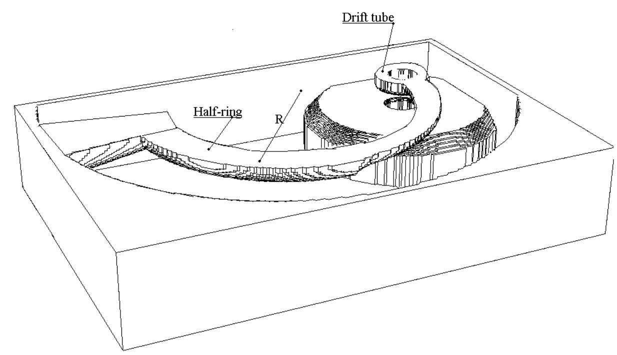

Figure 3: The double-gap bunching cavity with the ring central conductor

3 PARAMETERS OF LIQUID FLOW

It is known well that the character of the liquid flow in the channel is defined by the Reynolds number Re:

$$Re = \frac{Vd}{\eta}, \qquad (11)$$

where V ia the average velocity, d is the characteristic dimension of the channel and η is the kinematic viscosity of the liquid (for water $\eta \approx 10^{-6} m^2/sec$). If $Re \geq Re_{cr}$, the flow in the channel is turbulent. For cylindrical channel $Re_{cr} \approx 1800$, for coaxial one $Re_{cr} \approx 1000$ [3].

The main parameter in the cooling of accelerating cavities is usually the expenditure Q of the cooling liquid, because stabilised temperature of the cavity is needed. So, the average velocity of the flow may be estimated as $V \approx Q/S_c$, where S_c is the square of cross section of the channel. In most practical cases flow is the channel is turbulent , due to limited S and Q given. Another reason is because the heat exchange coefficient for turbulent flow is higher than for laminar one. (It is important to avoid case with $Re \gg Re_{cr}$, because for $Re \geq 6000$ in circular pipe the flow becomes unstable [3]. Due to this reason V is usually not higher than 5m/sec.)

In the turbulent flow the unstability with the frequency $f_{fl} \approx V/d$ can exists in the stream [3]. Estimating f_{fl}:

$$f_{fl} \approx \frac{V}{d} = \frac{\eta Re}{d^2}, \qquad (12)$$

for most practical cases $f_{fl} \geq (100 \div 200)$ Hz.

It means, that for correctly designed cooling channel the possibility of excitation for mechanical oscillation is low enough and this effect is not the main reason.

4 SUMMARY

Analytical estimations for resonant frequencies for different types of oscillations in stem-like elements are presented. It is shown that for typical dimensions of accelerating cavities the frequency range of oscillations may be from several tens Hz to hundreds Hz. This results are useful in the cavity design to take care from possible sources of noises from equipping hardware (cooling, pumping and so on).

5 ACKNOWLEDGMENTS

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