SIMULATION OF HALO FORMATION IN BREATHING ROUND BEAMS IN A PERIODIC FOCUSING CHANNEL*

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Abstract

Halo formation in high-intensity axisymmetric beams in a periodic focusing channel is analyzed using particle-in-cell simulations. In order to explore self-consistently the fundamental properties of breathing round beams propagating in a periodic focusing channel, the initial phase-space distribution of a beam injected into a linac is adopted to be a sufficiently realistic distribution such as Gaussian, waterbag and parabolic. Numerical results such as halo intensity and emittance growth are obtained by means of multiparticle simulations.

1 INTRODUCTION

Recent interest in using high-current ion linacs for production of tritium, the transmutation of nuclear waste, etc. has enhanced activitities for halo study. It is necessary to understand mechanisms of intense-beam losses, especially, beam instabilities and halo formation, because machines must operate with a very low beam losses to avoid serious radioaction.

K-V distribution of particles in transverse phase space is used to adopt to predict the behavior of real beam in most theoretical studies [1,2]. Because K-V beam density is uniform, then space-charge forces are linear. Particle-core model has contributed to an understanding of the underlying causes of halo formation from mismatched beams [3-6]. In order to obtain more meaningful simulation results nonlinear particle-density distributions are adopted in a uniform channel [7-11], the codes calculating space charge have been replaced by those with more simple and accurate representation of practical distributions [12,13]. Moreover, it is important to understand the mechanism of halo formation in a periodic focusing channel, since the periodicity of the external field can cause a strong resonant instability [14,15]. The chaotic behavior caused by structure-driven resonance has recently been studied and connected with halo formation [16-19]. We had investigated the mechanism which enables some particles to escape from deep inside core in a uniform channel [11]. In present paper, we discuss the properities of halo formation in breathing round beams in a periodic focusing channel.

We first describe the simulation method in section 2, then apply the code to the phase-space distributions and obtain some simulation results in section 3.

2 SIMULATION METHOD

The Hamiltonian of the transverse motion is given by

$$H_\perp(r, r_\perp; z) = r_\perp^2/2 + \kappa_z(z) r^2/2 + q\phi(r,z)\big/\left(m\gamma^3\beta^2c^2\right), \qquad (1)$$

where m, q, βc denote, respectively, the ion mass, charge and longitudinal velocity, $\kappa_z(z)$, whose profile is shown in Fig.1, is the periodic function representing the variation of the focusing strength, and z is the distance measured along the beam line. $\gamma = \left(1-\beta^2\right)^{-1/2}$, $r = \left(x^2+y^2\right)^{1/2}$ is the distance from the z-axis in the transverse plane, and $r_\perp = \left(x'^2+y'^2\right)^{1/2}$ is the dimensionless transverse velocity with $x' = \dot{x}/\beta c$, $y' = \dot{y}/\beta c$, $\phi(r,z)$ is the space charge potential, which must meet with the Poisson eqution:

$$\nabla^2\phi(r,z) = -\frac{q}{\varepsilon_0}\iint f(r, r_\perp; z)\, d\vec{r}_\perp , \qquad (2)$$

where $f(r, r_\perp; z) = f(x, y, x', y'; z)$ is the distribution function in the transverse nonrelativistic four-dimensional phase space.

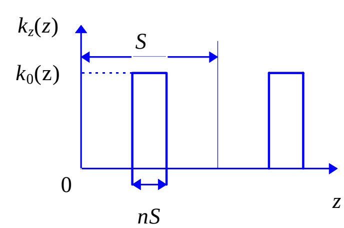

Figure 1: The profile of solenoidal periodic focusing field

All distributions that are function of the transverse Hamiltonian $H_\perp$ are stationary for a uniform focusing channel because $H_\perp$ is a constant of motion in this case. However, in the case of periodic focusing channels $H_\perp$ is no longer constant, and the only stationary state for which an analytic representation could be found is the K-V distribution. For a more general investigation one must rely on numerical simulations by means of adopting nonstationary distributions. Here nonstationary distribution functions used in computer simulation studies are defined as functions of the radius in four-dimensional trace space and not as functions of the Hamiltonian $H_\perp$. For a detailed discussion of nonstationary distributions see [20].

In order to compare different distributions on the same basis, we consider rms-equivalent beams which have the same perveance, rms radius, and rms emittance. To obtain the rms radius, we introduce the envelope eqution:

$$\frac{d^2a}{ds^2}+\kappa(s)a-\frac{\Lambda}{a}-\frac{1}{a^3}=0\ , \tag{3}$$

where a, Λ and $\kappa(s)$ are dimensionless variables: $s=z/S$, $\Lambda=KS/\varepsilon$, K is the generalized perveance, ε the emittance and S is the periodic length of a single focusing cell, and $\kappa(s)=\kappa_z(z)S^2$. The matched normalized radius $a_0(s)$ can readily be derived from Eq.(3) when Λ and $\kappa(s)$ are determined. The vacuum phase advance over one axial period of such a focusing field is approximately given by $\sigma_0=\left[\int_0^1\kappa(s)ds\right]^{1/2}=\left[nk_0(z)\right]^{1/2}$. Then we notice that $a_0(0)$ corresponds to the minimum radius of a matched beam because the original coordinate is located at the center of a drift.

We determine the mismatched initial phase-space distribution $a_i=\mu a_0$, $a'_i=a'_0/\mu$, where a_0, a'_0 correspond to the matched one, and mismatch parameter $\mu\le 1$. The radial space-charge field of an axisymmetric beam can be calculated from Gauss law by counting the number of particles in cells of a finite radial grid which extends up to 5 times the beam matched radius. We monitor the total energy through the transport channel, and keep the total energy constant. Here we employ 10^4 particles and 100 radial meshes over the length $a_0=a_0(0)$.

3 NUMERICAL SIMULATION RESULTS

We take into account the transport channel with enough length so that the beam reaches saturated states before arriving at the exit. Here the filling factor n seen in Fig.1 is 40 percent of the length of a single cell.

If an input beam is perfectly matched to a transport system, there is no reason to expect the growth of a halo unless the distribution is intrinsically unstable against perturbation or there is structure-driven resonances. However, it is impossible to provide a perfect beam, actually, there is an inevitable initial mismatch which generates a halo.

We perform multiparticle simulations, to consider a beam as realistic as possible, several different types of initial phase-space distributions such as Gaussian, parabolic and waterbag distributions are adopted.

3.1 Emittance growth of various nonstationary distributions

To consider a beam as realistic as possible, several various initial phase-space distributions such as Gaussian (GA), parabolic (PA) and waterbag (WB) distributions are adopted to estimate emittance growth. Figure 2 shows emittance growth defined as the ratio of the final rms-emittance to the initial rms-emittance vs the cell number of the transport channel for tune depression η=0.4, which is defined by $\eta=\sigma/\sigma_0$, where σ is the space-charge phase advance over one axial period of such a focusing field and σ_0=75°, and different mismatch parameter (a) μ=0.7, (b) μ=0.8. From Fig.2 we can find that the more realistic the distribution is, the smaller the ripple of emittance growth changes through the transport channel.

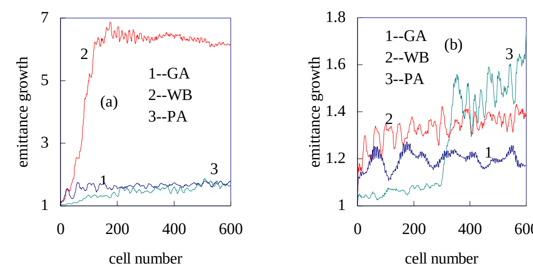

Figure 2: Emittance growth of various distributions vs cell number for (a) μ=0.7, (b) μ=0.8.

3.2 The structure-driven resonance

The periodicity of the external field can cause a strong resonant instability. Since the unstable particles can easily escape from the core getting a large betatron amplitude, it is necessary to investigate halo formation mechanism in the structure-driven instability. The instability growth rate increases with increasing σ_0, and at sufficiently high values of σ_0 there is an intensity threshold beyond which the beam is unstable for all values of $\sigma\to 0$ [20], that is to say, the second-order even mode occurs from the Vlasov equation perturbation analysis. For σ_0>90° and sufficiently large Λ, the envelope oscillations become chaotic for some mismatched beams [17].

Figure 3(a) shows emittance growth of rms-matched beams with Gaussian distribution, the phase advance without space charge is fixed at 105^0. We find emittance growth rises rapidly from η=0.23 to η=0.25, there is the region where the second-order even mode exists. The beam is trapped by the second-order resonance in the phase-space configuration shown in Fig.3(b).

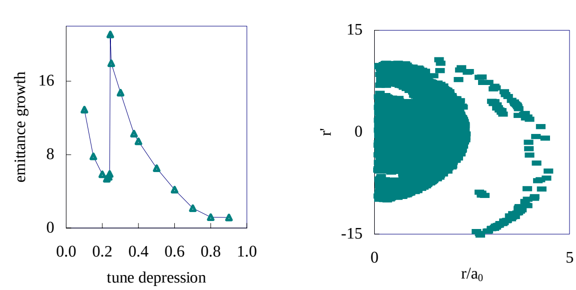

Figure 3: (a) Emittance growth of rms-matched Gaussian beams vs η and (b) phase-space distribution of rms-Gaussian beam at cell number=600 and η=0.245.

3.3 Halo intensity and the maximal radial extent

We firstly consider mismatched beams with Gaussian distribution, the phase advance without space charge is 65°,75°, 85°, respectively. The number of particles which go into the halo seen in Fig.4, gets rather small as mismatch parameter μ tends to 1.0. Here we define halo intensity h as the number of particles outside the boundary r_b=1.75a_0 divided by all of particles we employ.

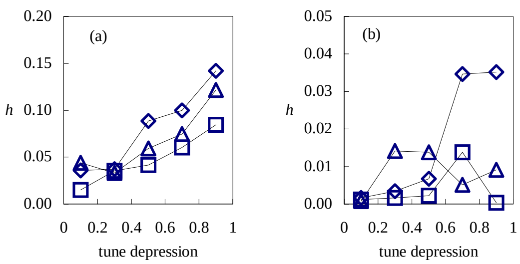

Figure 4: Halo intensity vs η for mismatch parameter (a) μ=0.6, (b) μ=0.8, with different σ_0: ◊-85°, Δ-75°, -65°.

In addition, let us look at Fig.5 where the maximal radial extent r_{max} has been displayed. It is obvious that the maximal radial extent is almost independent of σ_0, but the maximal radial extent is larger as mismatch parameter is lower than 1.0.

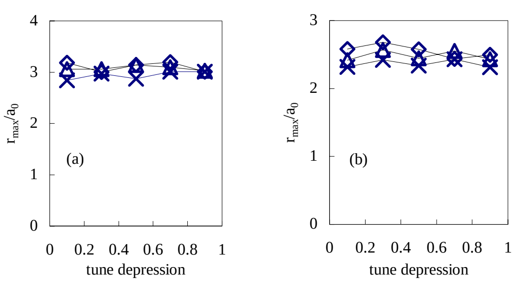

Figure 5: Ratio of the maximal radial extent to radius of matched beam vs η for mismatch parameter (a) μ=0.6, (b) μ=0.8, with different σ_0: ◊-85°, Δ-75°, $\times$-65°.

4 CONCLUSIONS

It has been confirmed that the periodicity of the channel induces resonant instibility in some region. There is no prominent emittance growth in the region when σ_0<90°, however, the strong instibility, especially the second-order resonance, occurs when σ_0>90°, and emittance growth is very large. Therefore, we do our best to set σ_0 at a value below 90° in a linear transport design. We can also set σ_0 above 90°, but we need to select the region where there is no resonance.

Simulation results show halo intensity and maximal radial extent are more increased as the magnitude of initial mismatch increases, and they is not dependent of the tune depression when σ_0<90°.

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