# COMPUTING EIGENMODES IN HIGHLY LOSSY ACCELERATING STRUCTURES

S. Setzer*, T. Weiland
Darmstadt University of Technology, Fachbereich 18,
Fachgebiet Theorie Elektromagnetischer Felder, Schlossgartenstrasse 8, 64289 Darmstadt, Germany

*Abstract*

The numerical calculation of eigenvalues in structures containing high loss dielectric and permeable materials is of importance in the field of accelerators as well as in many other high frequency applications. While satisfying algorithms exist for loss free and small-loss problems, the numerical problem of highly lossy material insertion is still a big challenge. We examine the Jacobi-Davidson method that proves to be a rather suitable method for calculating a set of eigenvalues even in the case of highly absorbing materials. Furthermore, unlike the commonly used sub-space methods, this algorithm is not limited to calculate extreme eigenvalues only. It is also capable of finding the eigenfrequencies located around any user specified frequency. Another practical advantage of this method is the absence of parameters such as the upper limit for an eigenvalue spectrum.

## 1 INTRODUCTION

Many problems in the field of electroctromagnetic design require the determination of a set of eigenvalues and corresponding eigenvectors. In the absence of lossy dielectric and permeable material or even for small loss problems, existing algorithms are functioning satisfyingly. In the presence of highly lossy material new methods like the Jacobi-Davidson technique as shown in [1] must be used. This algorithm has been implemented in the eigenvalue module of the electromagnetic CAD software MAFIA which is based on the FI-Technique. Besides the basic theory we present results for typical accelerator components such as the preliminary model of a high order mode damped accelerating cavity built at Fermilab.

## 2 BASIC CONCEPTS

### 2.1 The FIT-Method

The formulations of the Finite Integration Technique (FIT) according to Weiland [2] provides a general spatial discretization scheme usable for different electromagnetic applications of arbitrary geometry. The so called Maxwell Grid Equations and the material relations are given in the following notation:

$$C\hat{e} = -\dot{\hat{\tilde{b}}} \quad (1)$$

$$\tilde{C}\hat{h} = \hat{\tilde{j}} + \dot{\hat{\tilde{d}}} \quad (2)$$

$$S\hat{\tilde{d}} = q \quad (3)$$

$$\tilde{S}\hat{\tilde{b}} = 0 \quad (4)$$

$$\hat{\tilde{d}} = \hat{\tilde{D}}_\varepsilon \hat{e} \quad (5)$$

$$\hat{\tilde{b}} = \hat{D}_\mu \hat{h}. \quad (6)$$

The geometry is discretized on a dual orthogonal grid system with the vectors of the electrical grid voltage $\hat{e}$ and the magnetic flux $\hat{\tilde{b}}$ located on the normal Grid $G$ while the vectors of the electrical flux $\hat{\tilde{d}}$ and the magnetic grid voltage $\hat{h}$ are based on the dual Grid $\tilde{G}$. The analytical curl operator results in the curl matrices ($C$,$\tilde{C}$) and the divergence operator in the source matrices ($S$,$\tilde{S}$).
The presence of lossy dielectric and permeable materials can be included by introducing complex diagonal material matrices $\underline{\hat{\tilde{D}}}_\varepsilon$ and $\underline{\hat{D}}_\mu$. The eigenvalue equation can be obtained by combining equations 1 and 2 in the frequency domain with the now complex grid voltages $\underline{\hat{e}}, \underline{\hat{h}}$:

$$C\underline{\hat{e}} = -i\omega \underline{\hat{D}}_\mu \underline{\hat{h}} \quad ; \quad \tilde{C}\underline{\hat{h}} = i\omega \underline{\hat{\tilde{D}}}_\varepsilon \underline{\hat{e}}$$

$$\Rightarrow \underline{\hat{\tilde{D}}}_\varepsilon^{-1} \tilde{C} \underline{\hat{D}}_\mu^{-1} C\underline{\hat{e}} = \omega^2 \underline{\hat{e}} \quad (7)$$

### 2.2 The Jacobi-Davidson Method

The Jacobi-Davidson method as described in [3] is applicable to the standard as well as to the generalized eigenvalue problem with a complex system matrix. Here, we will only concentrate on the standard eigenvalue problem as it arises when using the FIT as presented above.
The idea of Jacobi-Davidson techniques is to generate a search subspace on which the projected eigenvalue problem of a now much smaller dimension is solved. This is the "Davidson" part and leads to an approximation for the eigenvalue and eigenvector of the unprojected problem. In the Jaccobi part of the algorithm, a correction equation is solved defining an orthogonal correction for the current eigenvector approximation. This correction is also used to expand the orthogonal search subspace.
Although no estimations of extreme eigenvalues are needed, there is a practical problem with the Jacobi-Davidson algorithm, namely the convergence towards a

* Work supported by DFG (GK-GRK 410/1)

specific eigenvalue. Usually one is interested in calculating a set of eigenvalues located around a user specified target which is more effectively done by using a modified method, the JDQR algorithm, as described in [4]. Here the Jacobi-Davidson method is used to generate a partial Schur-form for the standard eigenvalue problem. A explicit deflation technique is introduced which leads to a modified correction equation. After identifying the first eigenvalue, the speed of convergence towards the succeeding ones is increased by reusing already generated subspace information.

## 3 NUMERICAL RESULTS

The JDQR-algorithm, a modified version of the Jacob-Davidson method, has been implemented for rz-problems in the eigenvalue module of the electromagnetic CAD software MAFIA. As an example the preliminary design of a high order mode damped cavity developed at Fermilab [5] has been chosen. The damping of the unwanted higher modes excited by the beam itself is achieved by inserting rings of lossy dielectric and permeable material with $\underline{\varepsilon} = 5 - 5i$ and $\underline{\mu} = 100 - 3i$. The geometry of the cavity is shown in figure 1.

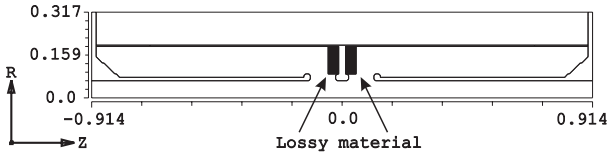


Figure 1: High order mode damped accelerating cavity

The first ten eigenmodes and the corresponding quality factors have been calculated , the results are summarized in table 1. To ensure that the lowest ten modes are identified, the user given eigenvalue target has been set to zero.

The speed of convergence for each eigenmode is depicted in figure 2. Usually, the identification of the first eigenfrequency takes more iteration then for the higher modes. This is due to the fact that subspace information generated for the first eigenvalue can be used to speed up the convergence of the following ones.

| $Mode$ | $f_{re}/MHz$ | $f_{im}/MHz$ | $Q$ |
|---|---|---|---|
| 1 | 54.608 | 0.657 | 41.56 |
| 2 | 63.723 | 15.888 | 2.01 |
| 3 | 64.833 | 16.357 | 1.98 |
| 4 | 87.578 | 1.902 | 23.02 |
| 5 | 98.203 | 36.505 | 1.35 |
| 6 | 98.278 | 36.606 | 1.34 |
| 7 | 137.547 | 53.701 | 1.28 |
| 8 | 137.630 | 53.829 | 1.28 |
| 9 | 171.255 | 52.872 | 1.62 |
| 10 | 171.340 | 53.066 | 1.61 |

Table 1: First ten Eigenmodes of a high order mode damped cavity

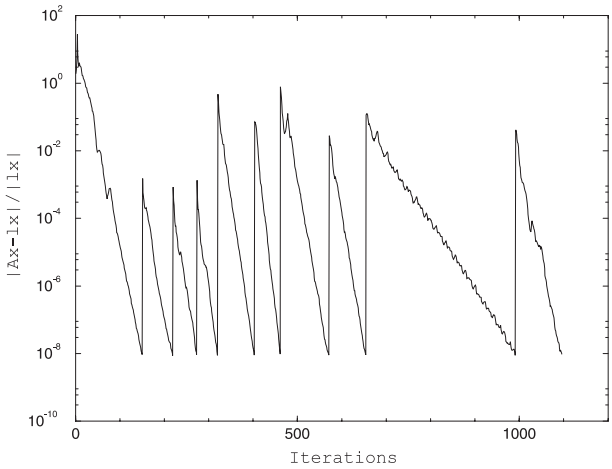


Figure 2: Relative error of eigenvalues vs. number of iterations

## 4 CONCLUSION

The Jacobi-Davidson method has proved to be a successful approach to the identification of resonating modes in structures containing highly absorbing materials. The algorithm is also capable of finding the eigenfrequencies located around an user given frequency which is useful for the examination of special frequency ranges. Another practical advantage of this method is the absence of parameters such as the upper limit for an eigenvalue spectrum.

## 5 REFERENCES


[1] S. J. Cooke, B. Levush,"Eigenmodes of Microwave Cavities containing High-Loss Dielectric Materials",Proceedings of the Particle Accelerator Conference PAC 97, Vancouver, to be published

[2] T. Weiland, "A Discretization Method for the Solution of Maxwells Equations for Six-Component Fields", Electronics and Communication (AEÜ), Vol. 31, pp. 116-120, 1977

[3] G. L. G. Sleijpen , H. A. van der Vorst, "A Jacobi-Davidson iteration method for linear eigenvalue problems",SIAM J. Matrix Anal. Appl., 17(2), pp. 401-425,1996

[4] D. R. Fokkema, G. L. G. Sleijpen, H. A. van der Forst,"Jacobi-Davidson style QR and QZ algorithm for the partial reduction of matrix pencils", Preprint 941, Department of Mathematics, University Utrecht, Janur 1996, to appear in SIAM J. Sc. Comput.

[5] W. Chou, private communication