An Idea on Adaptive Optimal Control of SFC

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Abstract

The idea of application of optimal control theory to the computer control system of SFC is presented. The process consists of mathematical modeling of SFC parameter identifying, designing of the fastest response controller and the techniques of realizing adaptive control.

1 Introduction

HIRFL (Heavy Ion Research Facility in Lanzhou) is a large and modern scientific facility in our institute, SFC is the injector of HIRFL. It is very important to match SFC with an automatic control system. But the application of automatic control theory to the control of SFC has not been realized up to now.

This paper presents how I am preparing to employ the control theory in SFC. If a satisfied result is done, I'll expand it to the other parts of HIRFL.

2 Designing step

According to computer control theory, the adoptive control system is shown as Fig.1, where G is the controlled facility, i.e. SFC, D is a controller, X is a observer, and Q is a identifier. The D, X and Q are all programmed in computer.

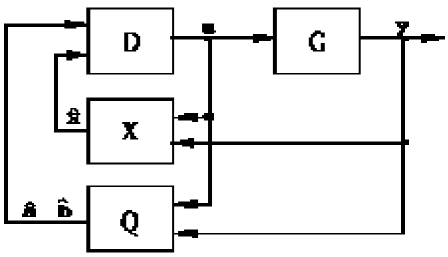

Fig.1 The Adaptive Control System

2.1 Mathematical Modeling of SFC

Before solving the optimal control problem, the mathematical modeling of SFC must be obtained. The mathematical model should be able to represent SFC, or G in Fig.1.

The essential physical principle of SFC is that a charged particle moves in a magnetic and/or electrical field. The ion beam qualities can be adjusted by varying the currents of magnets and quadrapoles. The beam qualities can be measured by diagnostic devices.

In general, the mathematical model for SFC describes the relationship among the beam qualities (outputs) and the adjustable data (inputs) and the states. They are expressed by differential equations or their transformations. The values of variables are small incremental changing near the steady-state operating point, which usually they can be linearized.

The mathematical model derived from the physical formula can not help much in modeling, because it is too complicated to be done. In practical terms, the model obtained from practical experience is very reliable and useful.

The first modeling can be done by some instruments or normal methods. A crude model can be written down when the wave shapes of input and output are being seen on a double trace oscillometer or a set of input and output data are detected. For example, a step function signal is input into a device, then its output is observed, so that the unit transient response does adequately reflect the qualitative behavior of the device considered. The main poles and zeros of its transfer function can be estimated, so an empirical model is obtained. Or we can use a frequency response detector to obtain a frequency response model. The model is expressed as :

$$x'(t) = Ax(t) + Bu(t) \tag{1a}$$
$$y(t) = Cx(t) \tag{1b}$$

The further modeling work for precisely determining the system parameters will be done by identification method of control theory. Basing sequence data of inputs and outputs [u(k),y(k)] recorded by a computer system, a group of prediction values $\hat{y}(k)$ could be computed. To simplify the identification algorithm for real-time application, a recursive least square parameter identification scheme is used. Its cost function is defined as:

$$J(a,b,c) = \sum_{K=1}^{N} \left[y(k) - \hat{y}(k) \right]^2 \tag{2}$$

where a,b,c are the elements of matrices A, B, C respectively. By means of computer, a group of parameters a,b,c may be selected when the cost function J achieves a minimum, which is called a curve-fitting method.

A computer control system is a discrete time system. The computer system samples measured qualities, make calculations, and transmits adjusting data in time sequence.

Equations (1) can be changed into the following different equations:

$$x(k+1) = Ax(k) + Bu(k) \tag{3a}$$
$$y(k) = Cx(k) \tag{3b}$$

where k is the sampling-adjusting ordinal number for variables u,x,y in Fig.1, the control vector u is l dimensional, the measurement vector y is m dimensional, the state vector x is n dimensional, here $n \geq l \geq m$, A is an $n \times n$ matrix, B is an $n \times l$ matrix, and C is an $m \times n$ matrix.

The elements of the state vector x consist of the qualities of the ion beam everywhere, and some physical quantities reflected the operating states of SFC, while some of them are employed for conventional mathematical treatment.

2.2 The Fastest Response Controller Design

The expected beam-quality vector is expressed as follows:

$$r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix}$$

where r is an m-dimension step-function vector, and r_1, r_2, ..., r_m are constants.

The function of the fastest response controller is to bring the beam qualities from the initial vector y(0) = 0 to the expected vector y(N) = [r_1,r_2,...,r_m], with the minimal sample-control times and without the occurrence of oscillations.

The fastest response controller can be designed as follows.

Let

$$F_1 = \exp(AT)$$

$$F_2 = \int_0^T \exp(AT)dt \cdot B$$

where T is the sample period. Then

$$\begin{bmatrix} u(1) \\ u(2) \\ \vdots \\ u(N) \end{bmatrix} = \begin{bmatrix} CF_1^{N-1}F_2 & CF_1^{N-2}F_2 & \cdots & CF_2 & 0 \\ AF_1^{N-1}F_2 & AF_1^{N-2}F_2 & \cdots & AF_2 & B \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$e(k) = r - y(k) = r - \sum_{i=0}^{k-1} C\left[F_1^{k-i-1}F_2 u(i)\right]$$

The two response of u(k) and e(k) is transformed into z-transfer function U(Z) and E(Z). Finally, the z-transfer function matrix representation of the fastest and non-oscillation controller is obtained:

$$D(z) = U(z) \cdot E^{-1}(z)$$

2.3 Observer X

SFC is operated in the presence of strong electromagnetic field. For this discussion, SFC is assumed as a linear discrete-time stochastic system. The noise spectrum of the stochastic disturbances is assumed as white noise and independent of any initial states.

Considering stochastic disturbances, the mathematical model of SFC has the following form:

$$x(k+1) = A \cdot x(k) + B \cdot u(k) + H \cdot w(k)$$

$$y(k) = C \cdot x(k) + v(k)$$

where the definition of x,u,y,A,B,C and k have the same meaning as those in (2); Two stochastic variables, w and v, are white noise sequences with an average of zero, and the covariance matrices of w and v are W and V,respectively.

Then, an on-line Kalman filter can be established as follows:

$$\hat{x}(k+1) = A \cdot \hat{x}(k) + B \cdot u(k) + K(k+1) \cdot \left[y(k+1) - C(A \cdot \hat{x}(k) + b \cdot u(k))\right]$$

$$K(k+1) = P(k+1) \cdot C' \cdot \left[CP(k)C' + v(k)\right]^{-1}$$

$$P(k+1) = AG(k)A' + HW(k)H'$$

where $\hat{x}$ is state estimation of state vector x, that $\hat{x}$ is gradually approaches x, G(k) is the covariance matrix of state estimation $\hat{x}$.

The initial estimate of the unknown quantities $\hat{x}(0)$ and can be approximated by prior knowledge of the characteristics of SFC.

2.4 The Realization of Adaptive Control

It should be pointed out that a designed optimal controller could actually be changed into a non-optimal controller over a long period of time. This would be due to the parameters of SFC gradually shifting after a long period of operation.

To prevent this, we must consider the use of adaptive control techniques. Recently, a number of adaptive control algorithms have been proposed. The dynamic parameters of SFC would be updated in every sampling period.

Here An Adaptive Observer and Identifier can be considered.

The mathematical model of SFC can be transformed into the following standard form:

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} a_1 & 1 & 1 & \cdots & 1 \\ a_2 & -r_2 & 0 & \cdots & 0 \\ a_3 & 0 & -r_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_n & 0 & 0 & \cdots & -r_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} x$$

In Fig.1, the output of its adaptive observer X and identifier Q, the estimate vectors $\hat{x}, \hat{a}, \hat{b}$, all gradually approach the real-value vector x,a,b. But the methods only solve problems for SISO system. For a MIMO System the state equation of the system model ought to be transformed into a diagonal form, then these adaptive control methods could be applied appropriately.

3 Project

This engineering adaptive optimal control in SFC is expected to be started next year. Firstly, get familiar with the present state of our beam tuning. Secondly, try to set up the model of SFC. Thirdly, design the controller, observer and identifier on computer. Fourthly, try and fail, until success. I hope we may get a satisfied result and expand to other parts of HIRFL. I believe that proper control theory application would result in significant benefits.